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Kjerstin TORRE

ABSOLUTE AND RELATIVE TIMING IN BIMANUAL COORDINATION : CONTRIBUTION OF SERIAL CORRELATION ANALYSIS AND IMPLICATIONS FOR MODELING

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Composition of the jury :

Didier DELIGNIÈRES	(Supervisor)	Université Montpellier 1
Viktor JIRSA	(Examiner)	Université de la Méditerranée Marseille
C. E. (Lieke) PEPER	(Reviewer)	VU University Amsterdam
Jean-Jacques TEMPRADO	(Examiner)	Université de la Méditerranée Marseille
Eric-Jan WAGENMAKERS	(Examiner)	University of Amsterdam
Pier-Giorgio ZANONE	(Reviewer)	Université Paul Sabatier Toulouse

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SUMMARY

1 GENERAL INTRODUCTION

Articulating bimanual coordination theory and a serial correlation perspective

Perspectives on bimanual coordination and the role of timing control

Bimanual coordination may be among the most investigated paradigms in motor behavior research. Specific insights have been provided by various entry points (including neurobiology, behavioral studies, modeling, pathology and aging), each likely to be in line with a conceptual framework that proposes a specific way of doing experiments, analyzing and modeling data. Ideally, these diverse insights should *in fine* come together to form a coherent and comprehensive picture of the organizing principles of coordination. Yet a unified understanding of motor behavior, built on a foundation of complementary findings across approaches, remains uncommon as the aim of many studies is to generalize their findings, with little interest in reframing these findings with regard to the specific procedures that were implemented.

Dynamical systems and information processing perspectives have given contrasting accounts of bimanual coordination. The first conceptualizes it in terms of emergent behavior in self-organized systems, (*e.g.*, Kelso, 1995) and the second in terms of representation of timing goals ordering the movements of effectors (*e.g.*, Semjen & Summers, 2002). The divergence in perspective between the two approaches is in part historically grounded (see Krampe, Engbert & Kliegl, 2002) and the differences in methodology are such that the results often are not directly comparable.

On the one hand, the dynamical systems approach initially focused on interlimb coordination, as assessed by stability properties and qualitative changes in collective behavior (differential stabilities of intrinsic patterns, spontaneous phase transition from anti-phase to in-phase coordination with an increase in oscillation frequency). The dynamics of collective behavior (expressed by the order parameter) is assumed to be a property emerging from the continuous, state-dependent interactions within a multi-scaled, self-organized system (Haken, Kelso & Bunz, 1985; Kugler & Turvey, 1987; Schöner & Kelso, 1988). Coupling between limbs has been a key notion that was further applied to the interplay between the neural level and motor behavior (Beek, Peper, & Daffertshofer, 2002; Peper, Ridderikhoff, Daffertshofer & Beek, 2004) and to synchronization tasks formalized as coupling between an oscillating system and the external stimuli (*e.g.*, Assisi, Jirsa, & Kelso, 2005; Chen, Ding, & Kelso, 1997; Fink, Foo, & Jirsa, 2000). Within this theorization of coordination, the relative timing between two components emerges, yet does not derive from the operation of a timekeeping entity that regulates the absolute timing of the components' movements. Component dynamics has mainly been addressed from the perspective of accounting for collective behavior.

Since the HKB model in its coupled oscillator form (Haken et al., 1985), a focus on the kinematic properties has generally been adopted by modeling the components as limit-cycle oscillators (Haken et al., 1985; Kay, Saltzman, Kelso, & Schöner, 1987). With few exceptions, most researchers using the dynamical systems approach have essentially investigated continuous oscillatory tasks and reasoned on the basis of the phase plane representation of the component dynamics. A notable exception is the ground-breaking work of Yamanishi, Kawato and Suzuki (1980). In general, time has thus been normalized and ruled out, and attention has been directed to the components' within-cycle behavior. Here, the succession of cycles in rhythmic coordination allows the consistent assessment of global stability properties in coordination or component dynamics, but the serial cycle-to-cycle dynamics has been concealed. The timing of individual oscillators mainly results from nonlinear properties in coupling processes, and issues of unimanual, self-paced timing have been relatively sparsely addressed (for exceptions, see Daffertshofer, 1998, Schöner, 2002).

On the other hand, the timing control hypotheses in bimanual coordination extend the research on unimanual timing. The Wing and Kristofferson (1973) model has allowed to account for major empirical observations in unimanual self-paced tapping. In this two-level model, a stochastic timekeeper is responsible for the abstract mental representation of time, independently of the peripheral level of motor implementation. This framework was further extended to synchronized rhythmic movement (Schulze & Vorberg, 2002; Vorberg & Schulze, 2002; Vorberg & Wing, 1996) and bimanual coordination. From this perspective, coordination is greatly influenced by an independent cognitive level, and relative timing is in some way related to the absolute timing representation, both in the assumption of an unique timekeeper (*e.g.,* Semjen & Summers, 2002; Turvey, Schmidt, & Rosenblum, 1989; Vorberg & Hambruch, 1978, 1984) and the hypothesis of multiple timing processes operating in parallel or in an integrated manner (*e.g.*, Helmuth & Ivry, 1996; Ivry & Richardson, 2002; Krampe, Kliegl, Mayr, Engbert, & Vorberg, 2000).

Studies in line with the timing control hypotheses have mainly used rhythmic tasks with brisk discrete movements, so that the influence of spatial constraints and the interactions arising from overlapping component trajectories are minimized. In these conditions, the temporal ordering of specific motor events becomes evident, and coordination or the component dynamics has mainly been characterized using the variability (*e.g.*, Helmuth & Ivry, 1996; Ivry & Richardson, 2002; Semjen & Ivry, 2001; Semjen, 2002) or correlation (*e.g.*, Semjen & Summers, 2002) properties of inter-event intervals across or within components. Studies applying the notion of timing control to continuous tasks have tended to imply a dissociation between the complementary processes of coupling in the time domain and the trajectory domain (*e.g.*, Beek et al., 2002; Semjen, 2002; Ridderikhoff, Peper, & Beek, 2005).

Variant conceptualizations of coordination have actually been based on variant (emblematic) task designs including, for instance, continuous or discontinuous movements (such as bimanual oscillation vs. bimanual tapping), iso- or multi-frequency coordination, and the use or not of an external pacing signal. Different accounts, mainly focusing on either spatio-temporal coupling or timing, have often been neither directly opposable nor easily comparable. Obviously, the role of timing control in coordination is an outstanding issue among the conceptually divergent viewpoints.

Rhythmic movement tasks prevail as the empirical basis for studying coordination dynamics whatever the conceptual framework, and coordination basically has been defined by a strong temporal congruence of effector movements (similar frequencies, stable relative phase). This refers to the *relative* timing between the components, *i.e.*, the dispersion of motor 'events' between the effectors. However, although in rhythmic coordination the successive motor events of each effector mark out the time of performance, effector movements have not systematically been assumed to be timed in *absolute*[1].

First, the relative timing between components can be assumed to remain stable by mutual adjustments to the variations in their respective temporal patterns. Here, the components' absolute timing is essentially a consequence of the state-dependent, spatial and/or temporal coupling that yields timing coherence between them. In this view, the absolute timing of components is 'non-specific' in the sense that it is not a determinant degree of freedom for the coordinative behavior. This has been an unspoken assumption of most of

[1] Throughout this document we refer to *absolute* and *relative timing* considering that absolute timing governs the production of movements with stable and reproducible temporal patterns, without interaction with other timed movements and without an external prescription of time intervals. Relative timing, in contrast governs the production of stable and reproducible coordination patterns between multiple component movements, or between a movement and external events (sensorimotor synchronization).

the studies that frame coordination issues in terms of *coordinative systems* and *unitary motor action* and focus primarily on collective behavior (*e.g.*, Kelso, 1995).

One can also assume that rhythm introduces temporal (and spatial) organization to movement and thus provides a 'timing frame' for coordination. In this case, timing control in motor behavior, *i.e.*, the ability to produce consistent time intervals is distinct from the coordinative function itself, since it applies to the performance of 'uncoupled' rhythmic movements as well. In this view, variations in the components' patterns are no longer considered inherent 'errors', nor manifestations of coupling, but rather a potentially meaningful temporal distribution that the coordination might be patterned on.

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Rhythmic bimanual coordination tasks basically have two interrelated goals: an absolute timing goal defined by the maintenance of a regular rhythm by the two limbs, and a relative timing goal defined by the maintenance of a stable phase relationship between the limbs. However, the dynamics at the relative timing (coordinative) level has mostly been studied separately from the within-limb (component) dynamics, and some authors have stressed the need to establish links between these two levels (*e.g.* Peper, Ridderikhoff, Daffertshofer, & Beek, 2004; Riley, Santana, & Turvey, 2001; Schöner, 2002).

In a more or less unstated way, timing issues in coordination have often been conceived as implying hierarchical organization and 'prescription', both of which are attributed to more cognitive approaches, and have received much less attention from dynamical systems approaches. To address the role of timing in bimanual coordination, an appropriate approach should probably take into account the methods, tasks and theoretical perspectives that usually have had little in common across the different frameworks, and bring in new criteria for a more 'colorless' perspective on the issue of timing control in coordination.

According to either approaches examination of variability in coordination performance has constituted a key-point for theorizing bimanual coordination. This makes that, by naming *variability* in the field of bimanual coordination, one is quite automatically attached to either of two notional bodies including, on the one hand, *stability, dynamics, critical fluctuations* and, on the other hand, *Weber's law, delays, component analysis.* Although the dynamical systems and the representational frameworks differ in the way they deal with the characteristics of temporal variability, either approach connects variability with the notions of *perturbation* and *white noise* or random fluctuations, disregarding its temporal structure. Even the Wing and Kristofferson's framework, although it bases on the evidence of negative lag 1 autocorrelation between successive time intervals produced in rhythmic tapping, is in line with this common parallel between variability and white noise as it deals with the negative correlation as the simple consequence of the association of two terms of white noise inherent in motor implementation.

In the following, in contrast, we outline why and how assessing the temporal structure of variability, *i.e.* characterizing serial correlation, would provide a pertinent and valuable entry point into the issue of timing control in bimanual coordination.

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Dealing with variability, serial correlation, 1/*f* noise

Variability describes a situation where successive observations of a same phenomenon could be expected identical but are not. Variability is most of time noticed and acknowledged as an unavoidable outcome of any data generating mechanisms. The part of variability which remains unaccounted for by these mechanisms – or which one chooses to leave unaccounted in view of a given scientific approach – is then considered as *noise* inherent to the system's functioning and likely to grow with system's perturbation. Then, in the perspective of describing and modeling a phenomenon, collected data are commonly filtered to eliminate noise. Addressing variability in the perspective of inferential statistics, an experimental effect or a specific state of behavior, etc., may be evidenced *in spite of* noise which affects both the underlying mechanisms and the measurement itself. That is, one may consider (i) that the state of an observed phenomenon or system can be made out and refined by repeated observation and in spite of the variability that accompanies repetition, or (ii) that different states (of amplitude) of variability observed in different conditions contain information about the functioning or the structure of the system. Back to bimanual coordination research, let us outline a brief example of the ways either of the two theoretical perspectives has dealt with variability.

The core assumption of the representational perspective is that there is a level of control responsible for the abstract representation of timing and coordination. This level functions regardless of which are the components executing the movement, and is theorized in terms of a stochastic timekeeper. This assumption holds whatever the variants and different extensions developed within the representational perspective, notably those considering either a unitary central clock (*e.g.* Wing & Kristofferson, 1973; Vorberg & Shulze, 2002; Vorberg & Wing, 1996), or multiple coupled (*e.g.* Helmuth & Ivry, 1996; Ivry & Richardson, 2002) or partly parallel (*e.g.* Krampe, Kliegl, Mayr, Engbert, & Vorberg, 2000; Pressing, Summers, & Magill, 1996) timekeeping processes.

The hierarchical two-level organization entailed by this assumption has essentially been based on (and, reversely, continued to inspire) a way of investigating properties of variability in performance, focusing notably on the empirical evidences for phenomena like the Weberian increase of variability with the duration of intervals to estimate/to produce, the reduction of variance (so-called *multieffector advantage*) in the within-hand time intervals during bimanual tapping, and the negative lag 1 auto-correlation in tapping time intervals.

In sum, according to the representational approach of bimanual coordination, noise in performance can originate from different independent sources of variability involved in bimanual coordination, the variabilities inherent to the timing component and the motor component being duration-dependent and duration-independent, respectively. Then, the empirical variability properties are thought as possible insight into the different sources.

Figure 1 shows a representative example of such component analysis of temporal variability in line with the representational perspective on coordination. Ivry, Richardson, and Helmuth (2002) analyzed the variability of time intervals produced in bimanual in-phase coordination at different tempi, and decomposed the variability into a central source and a motor source.[2] Results presented in this graph support the hypothesis of two central integrated timers in bimanual coordination: (i) the central variability increases significantly with the duration of produced time intervals while the motor variability remains almost constant; (ii)

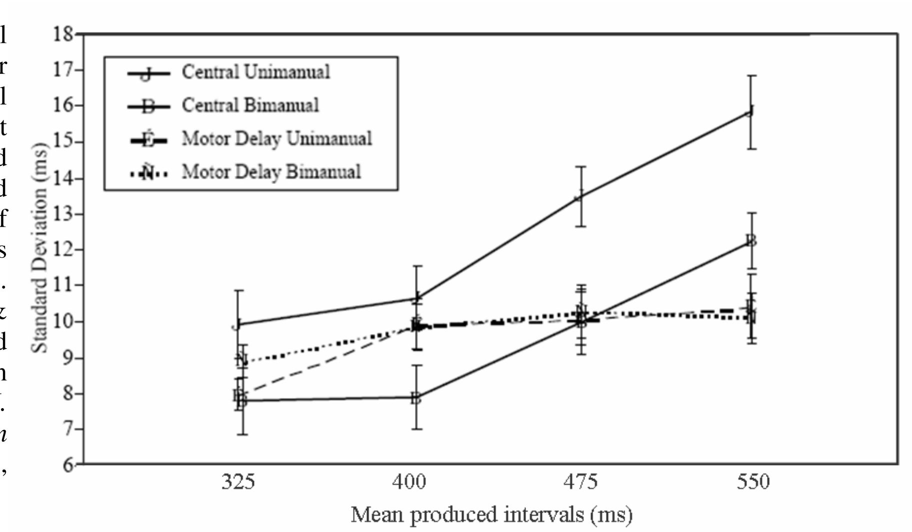

Figure 1. Estimates of central and motor variability for unimanual and bimanual tapping tasks at four target durations. Data are averaged over the left and right hand and plotted as a function of the mean produced intervals rather than the target values. From Ivry, Richardson, & Helmuth (2002): Improved temporal stability in multieffector movements. *J. Exp. Psych. Human Perception and Performance*, *28*, 72-92.

[2] To distinguish central and motor variability the authors combined three methods based (i) on the Wing-and Kristofferson model and the estimation of lag 1 autocorrelation due to the motor delays, (ii) on slope analysis of the function relating standard deviation produced of time intervals to their duration which gives an estimation of the central variability, and (iii) on the analysis of the covariance function of between-hand time intervals following the procedure proposed by Vorberg & Hambruch (1984).

there is an interaction effect between task (unimanual/bimanual) and component (central/motor): the central variability only is smaller in the bimanual condition than in the unimanual condition.

According to the dynamical systems perspective, coordination properties emerge from nonlinear interactions of components and processes at all levels in the system. The approach builds on the idea that coordination is better featured by its dynamical properties than by its static properties. The description of rhythmic coordination through phenomenological models relies on the characteristics of temporal variability (or stability) and qualitative changes in coordination performance. Here, noise is considered one of the emerging properties and is assessed globally at the macroscopic, behavioral observation level.

Coordination dynamics is characterized by the differential stability of anti-phase and in-phase coordination, in-phase coordination being intrinsically more stable than anti-phase. Continual fluctuations bring the system to (re)organize into stable states: with increasing movement tempo there is a loss the (relative) stability of anti-phase, featured by an increase in the amplitude of performance variability (critical fluctuations) which causes a qualitative change, or phase transition, to in-phase coordination. Number of studies have then addressed the organization of bimanual coordination by examining the evolution of these stability properties as a function of perturbation introduced into the system, and diverse stabilizing or destabilizing factors like attention (Monno, Temprado, Zanone, & Laurent, 2002; Temprado, Zanone, Monno, & Laurent, 1999), learning (Temprado, Monno, Zanone, & Kelso, 2002; Zanone & Kelso, 1992), movement perception and sensory information (Fink, Foo, & Jirsa, 2000; Kelso, Fink, DeLaplain, & Carson, 2001; Mechsner, Kerzel, Knoblich, & Prinz, 2001; Wilson, Collins, & Bingham, 2005), asymmetry (de Poel, Peper, & Beek, 2006), neuromuscular and spatial constraints (Carson, Riek, Smethurst, Lison-Parraga, & Byblow, 2000; Temprado, Salesse, & Summers, 2007), etc.

In sum, variability has been thought as essentially featuring the coordination dynamics but also as a ‘motor’ of these dynamics, seeing that the system organizes via instability. Figure 2 shows a typical example of stability properties in coordination dynamics with increasing movement frequency. The participant begins performing anti-phase coordination (relative phase of about 180°); coordination becomes less stable, *i.e.* standard deviation increases, until reaching the critical frequency (here about 2.4 Hz) where the transition from anti-phase to in-phase coordination (relative phase of about 0°) occurs spontaneously.

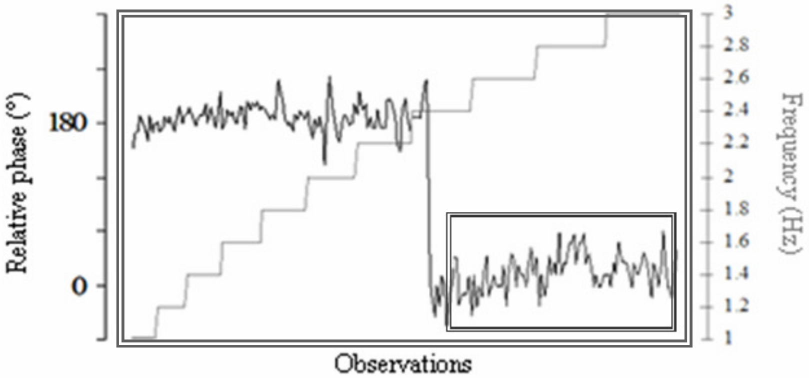

Figure 2. Bimanual coordination dynamics in one participant following the frequency dictated by a metronome. The frequency increases by increments of 0.2 Hz each 10 seconds. The relative phase is determined using point estimate (personal data).

While inferences from the analysis of variability diverge in the two perspectives on bimanual coordination, both limit themselves to a Gaussian approach: by featuring fluctuations only in terms of amplitude (standard deviation) they conceal any form of dependence in time. That is, the two approaches meet at the basic assumption that performance variability is uncorrelated white noise, which reflects perturbation of the actual state of behavior and remains a part of data that is dissociable from the significant signal according to the common equation: data = signal + noise. In the example of the representational approach (Figure 1), the assessment of different states of variability amplitude allows to test hypotheses about the two-level architecture of bimanual coordination. In the dynamical systems approach (Figure 2) the Gaussian properties constitute a grain of analysis that is sufficient to assess the macroscopic dynamics of coordination and reveal, in particular, its nonlinearity in critical condition, to the extend that the mean of relative phase measurements defines a coordination pattern and critical fluctuations are featured by amplitude. In contrast, when focusing on the stable states of coordination, for instance at comfortable movement frequency, the Gaussian properties are invariant and provide little information about the organization of coordination. However, while variability appeared worthy of examination in the vicinity of phase transition, why would it become unimportant and meaningless in the stable regimes of coordination?

> *« Variability is worthy of study in its own right, and examination of variability leads to insights that might have been missed had we focused all of our attention on the trend of the data »* (Gould, 2004)

The analysis of variability in steady state data needs using a time series approach that addresses the temporal structure of fluctuations; here, noise in itself becomes an object of investigation. Most studies in the fields of experimental psychology, movement science, or behavioral neuroscience use data collected from individuals performing blocks of trials in a given experimental condition. While such design provides repeated measures for refining the assessment of the observed variable, it also yields variability in performance data that is likely to be correlated to the extend that there is some consistence in the successive performances done by a same 'system'. In that case, variability appears to be more than featureless background fluctuations likely to grow with perturbation. Performance history is likely to contain information about the data generating mechanisms (even more interesting in cases where these correlations cannot be assumed to be due to the implication of explicit memory), and the characterization of noise structure allows to determine what is, and what is not plausible in terms of organization of the underlying mechanisms (Lambert, 2000).

Actually, it appears that temporal variability in natural time series is rarely random, and white noise is more an exception than the rule (Slifkin & Newell, 1998). Frequently, time series contain transient or *short-range* correlations, easily formalized by the class of auto-regressive moving average (ARMA) processes which applies across several disciplines. ARMA processes determine the relationship between two - or a very few - successive states of variable. In the auto-regressive processes written: $y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t$, the current state of the variable depends partly on the previous state(s). In the moving average process, written: $y_t = \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t$, the current state of the variable depends on the random noise that affected the previous state(s).; p and q determine the order of ARMA processes, the parameters ϕ_i and θ_i determine the magnitude of dependences, and ε is white noise. These simple processes have the advantage that they allow quite intuitive interpretations, at least for the low parameter orders: basically, the state of a given variable may either be conserved (auto-regression) or adapted (moving average), successive realizations may be negatively correlated if corrected, or positively correlated (depending on signs of ϕ and θ) if there is some persistent mechanism that exerts its influence over time being likely to lead to trends in series.

Short-range processes directly determine the time, or number of realizations, over which there is a significant dependence between performances. That is, the series is featured by a characteristic time scale that specifies the width of the window of correlation between data beyond which the series ceases to be auto-correlated. Typically, the auto-correlation function of short-range correlated series exhibits a rapid exponential decay over time.

However, a huge interdisciplinary literature has demonstrated that the serial correlation pattern of many variables differs from such short-range processes in that correlations are not transient. Instead, series display correlations over multiple time scales and are composed of an infinite continuum of high-frequency and small amplitude fluctuations nested within low-frequency and high-amplitude fluctuations. This makes the series *self-similar*, meaning that there is no characteristic time scale that could inform us about the scale at which the series is observed, and causes the so-called *long-range correlation*, meaning that correlation between data persists whatever the lag between them. Typically, the autocorrelation function exhibits a power-law decay. Such series are alternatively said long-range correlated, or *fractal*. A straightforward procedure has often been to notice the complexity of such time series assuming that the generating mechanisms must be complex in accordance, and to aggregate multiple simple processes with different characteristic time scales and different strengths of correlation, to account for the observed correlation patterns. However, although such aggregation of short-range processes may sometimes mimic long-range correlation, this procedure is neither parsimonious nor necessarily pertinent with respect to the object under study. Moreover, it does not confer series with 'genuine' long-range correlation properties.[3]

In the category of fractal, or long-range correlated series, *1/f noise* designates a very specific process defined by a proportional relationship between the amplitude and the period of the fluctuations that compose the series. It is situated at the frontier between stationary series (fractional Gaussian noises) and non-stationary series (fractional Brownian motions)[4]. One very remarkable – probably also the most popular - property of 1/*f* noise is its widespread occurrence.

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'1/f noise is ubiquitous.' Once this very common statement acknowledged one may oscillate between the appeal of this intriguing statistical phenomenon that features as well data of solar activity as of DNA sequences, stock market fluctuations, human cognition, meteorological phenomena, heart beat rhythm, semiconductors, etc., and the question of what could be the advantage of inquiring into a so ordinary property. Should the appeal of 1/*f* noise get over this first hesitation one would rapidly realize that 1/*f* noise is not as neutral and unbiased as one could expect from a statistical property; the finding of 1/*f* noise is intimately related to contrasted theoretical perspectives, to methodology used for data analysis, and to experimental setting. While the coexistence of divergent theoretical perspectives does not help clarifying the yet puzzling character and the potential impact of the 1/*f* phenomenon, the fact that the findings of 1/*f* noise have not been totally independent of the methodology of analysis and data collection may even lead to question their actual consistence. On top of this,

[3] The distinction between short-range and long-range correlation constitutes a methodological issue that is addressed in Chapter 2.

[4] As the purpose in this introductory part was not to provide extensive information regarding 1/*f* noise, please consider details about the statistical properties and theory related to 1/*f* noise throughout the document, notably in the methodological Chapter 2, as in Chapter 3, and Chapter 4 which deals specifically with theories and models for 1/*f* noise.

because almost no model that has not especially been developed in the view of accounting for 1/*f* noise is likely to predict 1/*f* correlation, the interpretation and significance given to the phenomenon remain vulnerable and its consideration moderate.

However, evidencing long-range correlation or 1/*f* noise is not necessarily at odds with current theories and models in the field of bimanual coordination, even though they do not account for it in their actual formalization. In fact, once the gap between different perspectives on bimanual coordination and serial correlation approach has been bridged, the characterization of serial correlation is likely to provide original criteria for questioning theory, in particular our present issue of the role of timing processes in coordination.

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Bridging the gap between bimanual coordination and serial correlation, 1/*f* noise

Since 1/*f* noise is everywhere it is very likely to occur in bimanual coordination too. However, as the interest in assessing 1/*f* noise in bimanual coordination would not be in providing one further evidence of its widespread occurrence, let us outline what beyond this ubiquity may instigate to address the issue of timing control in bimanual coordination from the perspective of serial correlation analysis and 1/*f* noise.

The first connection between bimanual coordination and serial correlation analysis is straightforward. Bimanual rhythmic coordination has often been addressed in terms of component level and collective level; this component - collective approach can easily be transposed in terms of within-hand, or absolute timing and between-hand, or relative timing. While serial (long-range) correlation have not received much attention from coordination studies, research on the timing of unimanual rhythmic movement, in contrast, have dealt with long-range correlation for about 10 years, notably since the article by Gilden, Thornton and Mallon (1995) which revealed that time intervals produced in self-paced finger tapping contain 1/*f* noise.

In the whole timing literature dealing with long-range correlation, one may discern in particular two bodies of work of which considerations could be usefully integrated into bimanual coordination study. The first group of studies has been interested in the origins and alterations of 1/*f* noise in timing tasks. The purpose of these studies has been to uncover the processes responsible for 1/*f* noise and to account for the quantitative and qualitative changes of correlation properties under the influence of factors like the duration of time intervals to produce, or the external pacing of movements (Chen, Ding, & Kelso, 1997, 2001; Chen, Repp, & Patel, 2002; Ding, Chen, & Kelso, 2002; Gilden et al., 1995, Madison, 2001, 2004; Pressing & Jolley-Rogers, 1997). On the assumption that the coupling between hands performing coordinated rhythmic movements is also likely to act on, or be acted by processes which are responsible for 1/*f* noise in unimanual timing tasks, assessing the changes or invariance of correlation properties in bimanual coordination may inform about the contributions of timing and coordination processes.

The second group of studies has been in line with the previously established theoretical distinction between two forms of timing control, namely *event-based* timing (or *explicit* timing) and *emergent* timing (or *implicit* timing), which are involved as a function of the discontinuous (*e.g.* finger tapping) *versus* continuous (*e.g.* oscillation or circle drawing) character of movements, respectively (Ivry, Spencer, Zelaznik, & Diedrichsen, 2002; Robertson et al., 1999; Spencer & Ivry, 2005; Spencer, Zelaznik, Diedrichsen, & Ivry, 2003; Zelaznik, Spencer, & Doffin, 2000; Zelaznik, Spencer, & Ivry, 2002). The characterization of serial correlation revealed specific correlation properties that allow discriminating between the two forms of timing control: while both forms of timing yield similar long-range correlations (1/*f* noise) in produced time intervals, they cause distinctive short-range correlations (Delignières, Lemoine, & Torre, 2004; Delignières, Torre, & Lemoine, 2008; Lemoine & Delignières, in press). Here, as specific correlation properties indicate the involvement of different timing processes, analyzing what happens to these signatures in bimanual coordination may help appraising the role of timing in coordination.

The second connection between bimanual coordination theory and long-range correlation, especially 1/*f* noise, is conceptual. A number of notions including pell-mell *complexity*, *contingence*, *non-linearity*, *self-organization*, *emergence, multiple interacting components*, *stability* and *adaptability*, *metastability*, etc., pervade as well bimanual coordination research (especially in the dynamical systems approach) as theory of 1/*f* noise; a conceptual closeness that may prompt some broad propositions like *"movement systems may apply [this] fractal principle whenever and wherever the challenges of coordination require it."* (Turvey, Schmidt, & Beek, 1993).

Let us take *metastability* as a telling example. Metastability expresses a balanced interplay between the functioning of a system as a whole and the functioning of its components as individual systems nested within the global system. This balance between opposing tendencies yields 'quasi-stable' or transient states of behavior. This organization principle has been alternatively accounted for in terms of *relative coordination*, *magnet* and *maintenance* effect (von Holst, 1973), or *integration* and *segregation* (*e.g.* Bressler & Kelso, 2001; Friston, 1997), providing a strong framework for thinking about the dynamics of spatio-temporal patterns across different research fields including notably inter-limb coordination, neuro-physiology and cognition. This relative functional instability provides the system with the necessary adaptive features to persist facing extrinsic or intrinsic changing constraints (Kelso, 1995).

In other respects, metastability has also been assumed to constitute a transdisciplinary, nomothetic account of the pervasive occurrence of 1/*f* noise. Metastable patterns of neural, behavioral, etc., dynamics emerge from the interactions between components near to 'critical points' between strongly coordinated and quasi uncoordinated activity. 1/*f* noise is a general feature of fluctuations near such critical points (Bak, Chen, & Creutz, 1989; Kello, Beltz, Holden, & Van Orden, 2007; Usher, Stemmler, & Olami, 1995); accordingly it has been considered *"a universal earmark of metastability"* (Kello, Anderson, Holden, & Van Orden, in press). That is, 1/*f* noise refers basically to the same organization principles of complex systems as metastability. Moreover, the occurrence of 1/*f* noise has usually been associated with the same functional properties as metastability. As 1/*f* noise situates at the frontier between stationary and non-stationary processes, it has been considered the statistical signature of an optimal compromise between stability and flexibility of a system, and its finding is typically associated with healthy or unconstrained systems. In contrast, 1/*f* noise has been shown to be altered in the case of systems' 'dysfunction' (for examples of alteration of fractal properties with human aging and disease, see Goldberger, 1997; Hausdorff, Mitchell, Firtion, Peng, Cudkowicz, Wei, & Goldberger, 1997; Goldberger, Peng, & Lipsitz, 2002; Iyengar, Peng, Morin, Goldberger, & Lipsitz, 1996; Peng, Havlin, Hausdorff et al., 1995).

In sum, the connection between bimanual coordination and serial correlation, especially 1/*f* noise, appears twofold (Figure 3): on the one hand, via previous investigation of timing processes and the occurrence of serial (long-range) correlation in unimanual timed movements and, on the other hand, via a shared conceptual framework for thinking about the organization principles of complex systems. Even though the characterization of serial (long-range) correlation has been considered by neither of the two – representational and dynamical systems - perspectives on coordination it seems however relevant to both approaches.

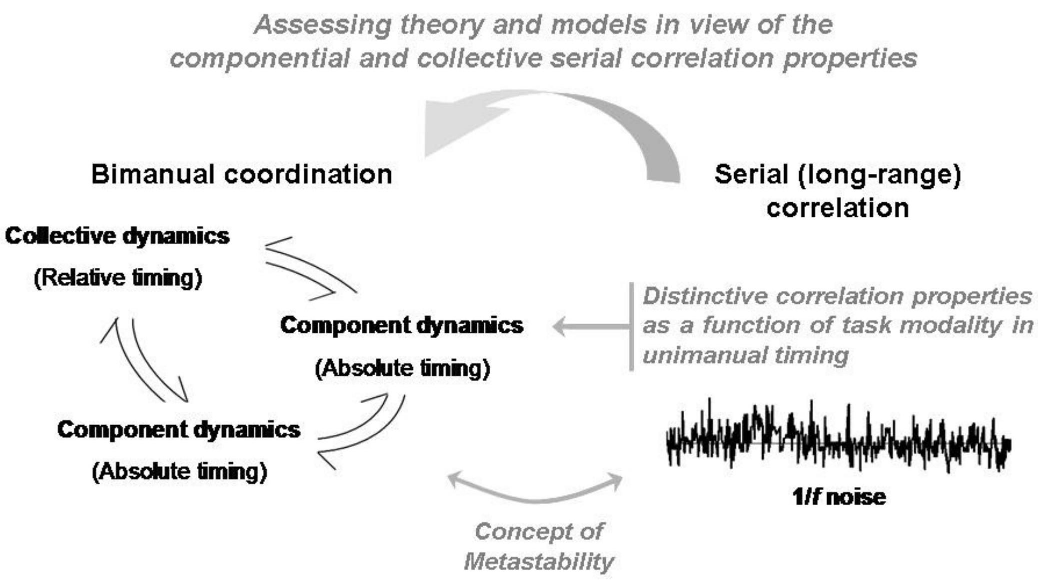

Figure 3.

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The purpose of this thesis has been to use serial correlation analysis for providing original criteria to assess theory and related models for the role of timing processes in bimanual coordination.

Therefore, **Chapter 2** first presents a methodological contribution for the detection and characterization of long-range correlation in experimental time series. This step allows to demonstrate, in **Chapter 3**, the presence of 1/*f* noise in series of relative phase performed in a classical bimanual coordination task. This result can actually be interpreted following two divergent perspectives on the 1/*f* phenomenon: a nomothetic or a mechanistic perspective. This is why the purpose of **Chapter 4** is to face these two perspectives and to underscore the relevance and the usefulness of mechanistic accounts, meaning simple, domain-specific models for 1/*f* noise. In **Chapter 5** we follow this mechanistic approach and focus on unimanual timing to propose an account of the qualitative change of correlation properties induced by the synchronization of movements with a metronome. Finally, in **Chapter 6** we integrate these contributions into an analysis of absolute and relative timing in eight unimanual and bimanual coordination task modalities (tapping and oscillations, self-paced and in synchronization with a metronome). We support the idea that the different unimanual timing processes which, we show, persist at the component level in bimanual coordination constitute a basis on which coordination builds. **Chapter 7** gives some concluding comments to this work.

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2 A METHODOLOGICAL STEP

Detection of long-range dependence and estimation of fractal exponents through ARFIMA modelling

Preliminary comments

The best known signature of $1/f$ noise is obvious in the log-log power spectrum of series: it is featured by a linear regression slope of –1, corresponding to a spectral index β of 1 in the case of 'perfect' $1/f$ noise. Usually, however, one considers a broader definition of $1/f$ noise, for β comprised between 0.5 and 1.5, which motivates the common notation: $1/f^{\beta}$ noise.

Several methods and improved procedures apart from spectral analysis have actually been used in literature for assessing the fractal structure of time series and quantifying the intensity of long-range correlations. Among these methods figure, without being exhaustive, *Detrended Fluctuations Analysis*, (Peng et al., 1993), *Rescaled Range Analysis* (Hurst, 1965; Caccia et al., 1997), *Dispersional Analysis* (Basingthwaighte, 1988; Caccia et al., 1997), *Maximum Likelihood Estimation* (Deriche & Twefik, 1993), *Scaled Windowed Variance* methods (Cannon et al., 1997), *Higushi*'s (1988) method, etc.

These methods alternatively work in the temporal domain, in the frequency domain, or they assess the geometric fractal properties of series. All are characterized by different advantages, drawbacks like over- or underestimation tendencies, and limitations with regard to the properties required for the series to analyze (for details on the methodology of fractal analysis, see for example Delignières, Ramdani, Lemoine, Torre, Fortes, & Ninot, 2006; Eke, Herman, Bassingthwaighte, Raymond, Percival, Cannon, Balla, & Ikrényi, 2000; Jennane, Harba, & Jacquet, 2001; Pilgram & Kaplan, 1998). The specific indexes provided by each method are however interrelated; they can be converted through quite simple equations into a single index, and it has been recommended to perform multiple estimations associating different methods to obtain the most reliable estimation of the 'true' fractal index (Delignières, Torre, & Lemoine, 2005; Rangarajan & Ding, 2000).

Moreover, given the known drawbacks of each method, the choice of the method to apply can also depend on whether the purpose of study is to compare statistically the intensity of long-range correlation in different experimental conditions or groups (in which case one would favor the methods with the minimum variability of estimations), or to determine the exact intensity of long-range correlations (in which case one would favor the accuracy of estimations), or to provide evidence for a given class of fractal processes like $1/f^{\beta}$ noise in the variable under study.

Now, all above listed methods present a same limitation: an index of long-range correlation is computed given the initial presumption that the series do indeed contain long-range correlation. However, the presence of some form of correlation in series does obviously not imply that these correlations are long-ranged as, for instance, we mentioned in introduction that the association of short-rang processes may sometimes mimic long-range correlations. This issue has motivated the development of inferential procedures (Farrell, Wagenmakers, & Ratcliff, 2007; Thornton & Gilden, 2005; Wagenmakers, Farrell, & Ratcliff, 2004) which aim at testing statistically for the presence of long-range correlation before estimating a fractal index.

In this regard, some findings of $1/f^{\beta}$ noise reported in literature actually appear questionable. As a matter of fact, fractal methods always give a numeric result: Figure 4 shows a caricatural but striking example to illustrate this issue. Although the series does obviously not contain long-range correlation, the power spectrum displays the typical features of $1/f^{\beta}$ noise, with a linear regression and a spectral index close to 1.

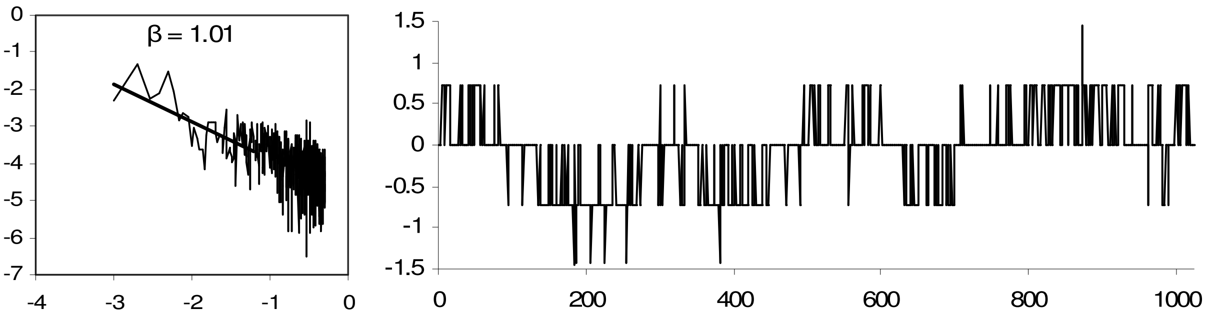

Figure 4. Time series obtained from a random distribution of 5 values; the correspoding power spectrum computed using ${}^{low}PSD_{we}$ shows power as a function of frequencies in log-log coordinates.

The following article deals with the issue of the detection and the characterization of long-range correlation in experimental time series.

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Detection of long-range dependence and estimation of fractal exponents through ARFIMA modelling

Kjerstin Torre, Didier Delignières, & Loïc Lemoine

EA 2991 Motor Efficiency and Deficiency
University Montpellier I, France

Abstract: The aim of this paper was to evaluate the performances of ARFIMA modelling for detecting long-range dependence and estimating fractal exponents. More specifically, our aim was to test the procedure proposed by Wagenmakers, Farrell and Ratcliff (2005), and to compare the results obtained with the Akaike Information Criterion (AIC) and the Bayes Information Criterion (BIC). The present studies show that ARFIMA modelling is able to adequately detect long-range dependence in simulated fractal series. Conversely, this method tends to produce a non-negligible rate of false detections in pure ARMA series. Generally, ARFIMA modelling presents a bias favouring the detection of long-range dependence. AIC and BIC gave dissimilar results, due to the different weights attributed by the two criteria to accuracy and parsimony. Finally, ARFIMA modelling provides good estimates of fractal exponents, and could adequately complement classical methods, such as spectral analysis, detrended fluctuation analysis or rescaled range analysis.

Introduction

Some recent experiments, generally underlain by the framework of dynamical systems theory, tried to analyze the dynamical structure of time series of psychological or behavioural variables. A quite intriguing result in these experiments was the recurrent discovery of fractal properties in the studied series. Fractals were evidenced, for example, in self-esteem (Delignières, Fortes, & Ninot, 2004), in mood (Gottschalk, Bauer, & Whybrow, 1995), in serial reaction time (Gilden, 1997; van Orden, Holden, & Turvey, 2003), in finger tapping (Gilden, Thornton, & Mallon, 1995; Delignières, Lemoine, & Torre, 2004), in stride duration during walking (Hausdorff, Peng, Ladin, Wei, & Goldberger, 1995), or in relative phase in a bimanual coordination task (Schmidt, Beek, Treffner, & Turvey, 1991).

All these variables were previously conceived as highly stable over time, and fluctuations in successive measurements were considered as randomly distributed, and uncorrelated in time. As a consequence, a sample of repeated measures was assumed to be normally distributed around its mean value, and noise could be discarded by averaging. This methodological standpoint was implicitly adopted in most classical psychological research (for a deeper analysis, see Gilden, 2001; Slifkin & Newell, 1998). In other words, temporal ordering of data points was ignored and the possible correlation structure of fluctuations was clearly neglected.

Fractal analysis focuses, in contrast, on the time-evolutionary properties of data series and on their correlation structure. Fractal processes are characterized by a complex pattern of correlations appearing following multiple interpenetrated time scales. In such processes, the value at a particular time is related not just to immediately preceding values, but also to fluctuations in the remote past. Such series are said to present long-term memory, or long-range dependence. This property is typically revealed by a very slow decay over time of the auto-correlation function, which tends to follow a power law.

Detecting fractal properties in empirical time series could have important theoretical implications. Fractals are considered as the natural outcome of complex dynamical systems behaving at the frontier of chaos (Bak & Chen, 1991; Marks-Tarlow, 1999). Psychological variables should then be conceived as the macroscopic and dynamical products of complex systems composed of multiple interconnected elements. Moreover, psychological and behavioural

Send correspondence to:
Kjerstin Torre
Faculty of Sport Sciences – University Montpellier 1
EA 2991, "Motor Efficiency and Deficiency"
700 Avenue du Pic Saint Loup
34090 Montpellier – France
Email: ktorre@univ-montp1.fr

time series often present fractal characteristics close to a very special case of fractal process, called 1/*f* or *pink* noise. With regard to the power spectrum of such time series, '1/*f* noise' signifies that each frequency has power proportional to its period of oscillation. As such, power is distributed across the entire spectrum and not concentrated at a certain portion. Consequently, fluctuations at one time scale are only loosely correlated with those of another scale. This relative independence of the underlying processes acting at different time scales suggests that a localized perturbation will not necessarily alter the stability of the global system. In other words, 1/*f* noise renders the system more stable and more adaptive to internal and external perturbations (West & Shlesinger, 1989).

The fGn/fBm model

A deeper presentation of fractal processes could be necessary to ensure a better understanding of the following parts of this article. A good starting point for this presentation is Brownian motion, a well-known stochastic process that can be represented as the random movement of a single particle along a straight line. Mathematically, Brownian motion is the integration of a white Gaussian noise. As such, the most important property of Brownian motion is that its successive increments in position are uncorrelated: each displacement is independent of the former, in direction as well as in amplitude. Einstein (1905) showed that, on average, this kind of motion moves a particle from its origin by a distance that is proportional to the square root of the time.

Mandelbrot and van Ness (1968) defined a family of processes they called *fractional Brownian motions* (fBm). The main difference with ordinary Brownian motion is that in an fBm successive increments are correlated. A positive correlation signifies that an increasing trend in the past is likely to be followed by an increasing trend in the future. The series is said to be persistent. Conversely, a negative correlation signifies that an increasing trend in the past is likely to be followed by a decreasing trend. The series is then said to be *anti-persistent*.

Mathematically, an fBm is characterized by the following scaling law:

$$\langle \Delta x \rangle \propto \Delta t^{H} \qquad (1)$$

which signifies that the expected displacement $\langle \Delta x \rangle$ is a power function of the time interval (Δt) over which this displacement is observed. H represents the typical scaling exponent (or *Hurst* exponent) of the series and can be any real number in the range $0 < H < 1$. Ordinary Brownian motion corresponds to the special case $H = 0.5$ and constitutes the frontier between anti-persistent ($H < 0.5$) and persistent fBms ($H > 0.5$).

Fractional Gaussian noise (fGn) represents another family of fractal processes, defined as the series of successive increments in an fBm. Note that fGn and fBm are interconvertible: when an fGn is cumulatively summed, the resultant series constitutes an fBm. Each fBm is then related to a specific fGn, and both are characterized by the same H exponent. These two processes possess fundamentally different properties: fBm is non-stationary with time-dependent variance, while fGn is a stationary process with a constant expected mean value and constant variance over time.

Several methods have been proposed for quantifying the fractal properties of a given series. The most often used is the *Power Spectrum Density* (PSD) method, which exploits the property that in fractal processes, spectral power is a power function of the corresponding period:

$$S(f) \propto 1/f^{\beta} \qquad (2)$$

In this equation, f represents frequency and $S(f)$ the corresponding squared amplitude. This relationship is revealed by the obtaining of a linear regression in the double-logarithmic plot of the power spectrum. β can be easily estimated by calculating the negative slope ($-\beta$) of the line relating log (S(f)) to log f. As previously evoked in the introduction, this property led to the designation of '1/*f* noise' which corresponds to the special case $\beta = 1$. The definition is not so strict, nevertheless, and generally one considers a broader family of processes, called $1/f^{\beta}$ noises, β ranging from 0.5 to 1.5.

Another set of methods, such as *Detrended Fluctuation Analysis* (DFA) *or Rescaled Range Analysis* (R/S) exploits a corresponding relationship, in the time domain, which states that in a fractal process variance is a power function of the time over which it is computed:

$$\mathrm{Var}(x) = \Delta t^{2H} \qquad (3)$$

This equation derives directly from Eq. 1, H representing the Hurst exponent. Time-domain methods generally aim at estimating variability over a number of intervals of different lengths, and then to estimate H through the double logarithmic plot of variability against interval length. If the series under study is a fractal process, a linear regression is expected.

Long-range and short-range dependence

Long-range dependence, nevertheless, is not the only conceivable model of dependence in times series. More simply, one could conceive the current observation as being only influenced by the preceding one, or by a limited set of preceding observations. For instance, this kind of short-term dependence could be conceptualized in terms of feed-back processes: during the repetitive performance of a given task, any information concerning the previous trial can be exploited for improving the current realization (Delcor, Cadopi, Delignières & Mesure, 2003; Spray & Newell, 1986). Another kind of short-term dependence was evidenced by Fortes, Delignières and Ninot (2004), which showed that the evolution of self-esteem over time could be adequately modelled by a short-term process, according to which any perturbation is partly corrected during the next assessment, in order to allow self-maintenance. The well-known timing model of Wing and Kristofferson (1973) also suggests a kind of short-term dependence in serial finger tapping: in this model, serial dependence is not determined by a feed-back process, but just by the contamination of contiguous inter-tap intervals by a common motor error term. Pressing and Jolley-Rogers (1997) proposed a modified version of the previous model for accounting for serial dependence in synchronization tapping. This model included an error correction process, each asynchrony between signal and tap being compensated at the following tap. This typical short-term dependence model was shown to adequately fit experimental data. Another well-known family of short-term dependence models gathers the sequential sampling models, developed in the domain of choice reaction time tasks (Laming, 1979; Ratcliff & Smith, 2004).

The most important point to note is that long-range dependence is not the only available hypothesis for accounting for serial dependence in psychological time series. Short-term dependence models offer an alternative framework, allowing a suitable fitting of empirical data, and supporting interesting and innovative models of psychological processes.

Statistical limitation of classical methods

As previously indicated, the identification of fractal properties with classical methods is only based on the visual inspection of the power spectrum or the diffusion plot, in bi-logarithmic coordinates. Generally authors are satisfied with obtaining an approximate linear fit to their data (see, for example, Yamada, 1995). Nevertheless, this visual evaluation remains qualitative, and there is no statistical test for judging if a regression line is appropriate or not (Wagenmakers, Farrell & Ratcliff, 2005).

Another problem is that short-term dependence processes can sometimes mimic the spectrum or the diffusion plot of a 1/*f* series (Farrell, Wagenmakers & Ratcliff, 2005; Rangarajan & Ding, 2000; Wagenmakers, Farrell & Ratcliff, 2004). Wagenmakers et al. (2004) proposed a number of examples of such ambiguous results obtained with short-range dependence processes. An auto-regressive process, for example, is supposed to present in the log-log spectral power a typical flattening in the low frequency region, reflecting the fact that there is no long-range dependence in the series. In a wide range of frequencies, nevertheless, the power spectrum also presents a 1/*f*-like linear trend: short-range dependence (*i.e.*, a relation between the value at time t and time t-1) leads to the occurrence of low frequency components that have more power than high frequency components (Wagenmakers et al., 2004). The difference from a genuine 1/*f* spectrum often lies in two or three ambiguous points in the lowest frequencies.

Rangarajan and Ding (2000) called for the complementary use of different methods, in the frequency and time domains, in order to avoid false conclusions. They present a number of simulated examples in which spectral and rescaled range analyses gave dissimilar results, suggesting the limitation of the use of a unique method. Their approach, nevertheless, remains qualitative and the conclusions drawn could stay ambiguous.

Some authors proposed the application of a so-called *surrogate data test* (Theiler, Eubank, Longtin, Galdrikian, & Farmer, 1992; Hausdorff, Peng, Ladin, Wei, & Goldberger, 1995). This method consists in randomly shuffling data sets and estimating the fractal exponents of the obtained series. The expected mean scaling exponent for these surrogate data sets is about 0.5. An inferential test is then applied to compare the exponents obtained from the original series and the exponents obtained from the surrogate data sets. The aim of this method was to determine whether the detected fractal behaviour reflected reality or was due to chance or to the applied methods. Nevertheless, the null hypothesis that is tested in this procedure is the absence of correlation in the series. This null hypothesis is surely not the most relevant, as the absence of correlation (purely white noise) in psychological time series should be considered more as an exception than as the rule (Slifkin & Newell, 1998). According to Wagenmakers et al. (2004) the fundamental question is about the nature (short-term vs long-term) of dependencies in the series. Clearly, classical methods are unable to adequately answer this question. Spectral analysis, DFA or R/S analysis could be relevant for quantifying long-range dependence, when long-range dependence is a priori supposed to be present. But they seem per se unable to validate such a presence. Wagenmakers et al. (2004) and Farrell et al. (2005) recently proposed an inferential test for the presence of long-range dependence in time series, based on ARFIMA (auto-regressive fractionally integrated moving average) modelling.

Modelling short-range and long-range dependence

In short-range dependence processes, each value can be predicted by a limited set of preceding values. Box and Jenkins (1970) introduced a family of linear models, called ARIMA (for auto-regressive, integrated, moving average), intended to represent a variety of short-term relationships in time series.

ARIMA models are potentially composed of three components. The auto-regressive component suggests that the current observation y_t is determined by a weighted sum of the p previous observations, plus a random perturbation ε_t:

$$y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t \quad (4)$$

In this equation ϕ_i represent the influence of the i^{th} previous value, and is assumed to progressively decay over time.

The moving-average component supposes that the current observation depends on the value of the random perturbations that affected the q preceding observations, plus its own specific perturbation:

$$y_t = \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t \quad (5)$$

The integrated component of the model determines whether the observed values are modelled directly, or whether the differences between consecutive observations are modelled instead. The differencing parameter d indicates the number of differencing that should be applied to the series before modelling.

An ARIMA model is a combination of these three components, and can be designated by the respective orders of the three combined processes as (p,d,q). As an example, a (1, 1, 1) model should obey the following equation:

$$y_t - y_{t-1} = \phi_1(y_{t-1} - y_{t-2}) + \theta_1\varepsilon_{t-1} + \varepsilon_t \quad (6)$$

ARIMA models could be more conveniently expressed using the so-called *backshift operator*, defined as:

$$By_t = y_{t-1} \quad (7)$$

The generic ARIMA (p,d,q) model can then be rewritten as:

$$\phi(B)(1 - B)^d = \theta(B)\varepsilon_t \quad (8)$$

where $\phi(B)$ and $\theta(B)$ are, respectively, the auto-regressive and the moving average operators, represented as polynomials in the backshift operator:

$$\phi(B) = 1 - B\phi_1 - B^2\phi_2 - \ldots - B^p\phi_p \quad (9a)$$

and

$$\theta(B) = 1 - B\theta_1 - B^2\theta_2 - \ldots - B^q\theta_q \quad (9b)$$

Granger and Joyeux (1980) showed that it is possible to provide this model with long-range dependence properties by allowing the differencing parameter d to take on fractional values, thereby obtaining an ARFIMA model. ARFIMA models provide a very parsimonious account for long-range dependence, by the addition of a single parameter to classical ARMA models. Importantly, they allow the simultaneous modelling of short-term processes (by the combination of the p and q parameters), and long-range dependence through the d parameter, and as such the isolation of their respective effects. Finally, the ARFIMA parameters can be estimated using exact maximum likelihood, allowing the significance of the difference of d from 0 to be tested. As such, ARFIMA modelling can effectively be used for detecting the presence of long-range dependence in the series. Moreover, the estimation of d allows the quantification of the intensity of the long-range correlations within the series, as d is related to the spectral exponent β by the simple equation:

$$\beta = 2d \quad (10)$$

Note that d is bounded within the interval [-0.5; 0.5]; that is, ARFIMA modelling can only take stationary signals with spectral exponents β between –1 and +1 (fGns) into account. For fBm signals, it is possible to apply ARFIMA to the corresponding fGn (obtained by differentiation). One can then estimate the theoretical fractional parameter of the fBm series by adding 1 to the d value obtained from the fGn (Diebolt & Guiraud, 2005).

Wagenmakers et al. (2005) recently proposed a complete inferential procedure, based on ARFIMA modelling, for ascertaining the presence of long-range dependence in time series. Their method consists in fitting 18 models to the studied series. Nine of these models are ARMA (p,q) models, p and q varying systematically from 0 to 2. These ARMA models do not contain any long-range serial correlation. The other nine models are the corresponding ARFIMA (p,d,q) models, differing from the previous ARMA models by the inclusion of the fractional parameter d representing persistent serial correlations. One supposes that if the series contains long-range dependence, ARFIMA models should present a better fit than the transient ARMA models. Note that in a previous paper, Wagenmakers et al. (2004) proposed to only contrast an ARMA (1,1) and an ARFIMA (1,d,1) model. The present approach, based on the test of a wider range of ARFIMA models, constitutes an interesting improvement, as the correct specification of parameters p and q allows a better estimation of the long-range estimator d (Taqqu & Teverovsky, 1998; Wagenmakers et al., 2005).

The determination of the best model is not so straightforward. The examination of the likelihood scores provided by the fitting procedure is not sufficient, as the capacity of models to account for the data is partly related to their number of free parameters. The selection of models has to be based on a trade-off between accuracy and parsimony: the best model is the one that gives a good account of the data with a minimum number of free parameters.

One of the most popular methods for combining accuracy and parsimony is the Akaike Information Criterion (Akaike, 1973), which can be computed according to the following equation:

$$\mathrm{AIC} = -2\log L + 2k \quad (11)$$

where L represents the maximum likelihood for the model under study, and k is the number of free parameters in the model, plus an additional parameter for the variance of the error series (*i.e.* for an ARMA(p,q), $k = p + q + 1$, and for an ARFIMA(p,d,q), $k = p + q + 2$). As can be seen, the first term rewards accuracy, and the second penalizes the lack of parsimony. The lower the AIC, the better the model is supposed to be.

An alternative criterion, the Bayes Information Criterion, is often used for model selection. BIC is defined as :

$$\mathrm{BIC} = -2\log L + k\log N \quad (12)$$

where N represents the number of observations in the series. BIC differs from AIC with the second term, which invokes a different penalty for parsimony: one can easily conclude from Eq. 11 and 12 that BIC should penalize complex models more severely than AIC, especially for long series.

The raw values of these criteria remain difficult to interpret and to compare between models. Wagenmakers and Farrell (2004) proposed a convenient transformation of the raw values into weights. Consider that the goal is to select the best model among m candidates. The first step is to compute the difference, for each model, between the criterion for this model and for the best model. That is, for the AIC criterion and the i^{th} model :

$$\Delta_i(AIC) = AIC_i - \min AIC \quad (13)$$

This difference in AIC can then be converted into an estimate of relative likelihood through the following transform :

$$L_i(AIC) \propto \exp\left\{-\frac{1}{2}\Delta_i(AIC)\right\} \quad (14)$$

Finally, these relative likelihoods are transformed into weights by normalization (*e.g.* by division by the sum of the relative likelihood of all models) :

$$w_i(AIC) = \frac{L_i(AIC)}{\sum_{j=1}^{m} L_j(AIC)} \quad (15)$$

$w_i(AIC)$ can be conceived as the probability for the i^{th} model being the best model, given the data and the set of candidate models (Wagenmakers & Farrell, 2004). Similar calculations can be performed on BIC, leading to specific weights $w_i(BIC)$.

On the basis of these weights, two criteria could be proposed for detecting the presence of long-range dependence in the series : (1) the best model (*i.e.* the model with the highest weight) should be an ARFIMA (p, d, q), d being significantly different from 0, and (2) the sum of the weights of the ARFIMA models should be higher than the sum of the weights of the ARMA models. Note that the weights computed among a given set of models sum to one. The sum of ARFIMA models weights represents the overall probability of ARFIMA models to overcome their ARMA counterparts.

The aim of the present study was to test the capacity of ARFIMA modelling to detect the presence of long-range dependence in simulated fractional Gaussian noise series. We limited the analyses to stationary processes, which represent the generally expected behaviour of psychological variables. We tested this capacity over a wide range of H exponents, from 0.1 to 0.9. We also tested the effect of the length of the series, and the effect of the addition of white noise to the studied series. Secondly, we submitted simulated ARMA series to the same procedure, in order to analyze the occurrence of false detections of long-range dependence in such series.

Thirdly, we analyzed the performance of ARFIMA modelling in estimating the H exponent. In this final step, we tested again the effect of series length, and the effect of the addition of white noise. In all cases, we compared the respective performances of AIC and BIC. Finally, we applied the method to a set of empirical series, collected in an experiment on bi-manual coordination.

Detection of long-range dependence in simulated fGn series

We used the algorithm proposed by Davies and Harte (1987), for generating fGn series of known H exponent (for a detailed presentation, see Caccia, Percival, Cannon, Raymond, and Bassingthwaigthe, 1997). We generated 40 fGn series of 2048 data points for each of 9 values of H ranging from 0.1 to 0.9 by steps of 0.1. In order to test the effect of series length, we applied the analysis on the entire series (2048 points), and then on the first 1024 points, the first 512 points, the first 256 points, and finally the first 128 points (*i.e.* series of 2^{11}, 2^{10}, 2^9, 2^8 and 2^7 points).

In a second step, we added to each original series a white noise series (fGn with H = 0.5). The added white noise series were different for each fGn series. We tested four noise/signal *SD* ratios: 0.00 (no added white noise), 0.33, 0.66, and 1.00 (equal variance for white noise and signal). The analysis was then applied to these contaminated signals. These tests were performed for a single series length (1024 points).

Models' fitting was conducted using the ARFIMA package (Doornik & Ooms, 1999; Ooms & Doornik, 1998) for the matrix computing language Ox (Doornik, 2001). We used, with some minor adaptations, the Ox code provided by Simon Farrell, available at the following web address: http://eis.bristol.ac.uk/~pssaf/ (for details, see Farrell et al., 2005).

Figure 1 reports the percentage of correct specifications (*i.e.* the best model was an ARFIMA model, with parameter *d* significantly different from zero), according respectively to AIC and BIC. Generally fGn series were recognized as long-range dependence processes, except for H = 0.5 (in this case, the series were simply white noises). Note, nevertheless, that in this case the best model was generally an ARFIMA model (and not as expected an ARMA (0,0) model), but the coefficient *d* was not significantly different from 0. Generally both methods selected ARFIMA models (for 100.0% of the series for BIC and 95.4% for AIC). The simplest models (0,*d*,0), (1,*d*,0) and (0,*d*,1) represented 99.1% of the selected models for BIC but only 60.7% for AIC.

Clearly BIC produced a better percentage of correct detections than AIC. For the longest series (1024 and 2048 points), the detection appeared perfect for BIC. The percentage of errors was higher for AIC, even for long series. The number of errors increased when series length decreased, especially for weakly persistent fGn (H = 0.6 or H = 0.7). Misspecifications with AIC were due in part to the selection of complex ARMA models, such as (2,2) or (1,1), and in part to the selection of complex ARFIMA models, such as (2,*d*,2) or (1,*d*,1), *d* being in this case not significantly different from zero.

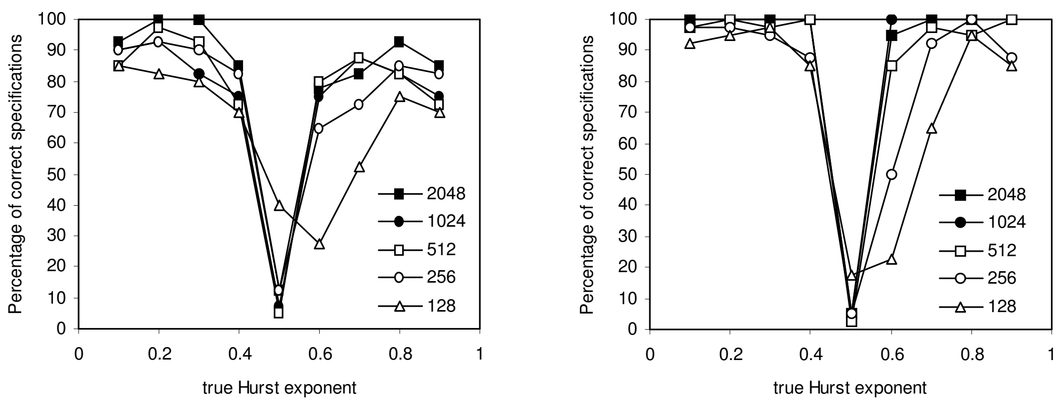

Figure 1: Percentage of correct specifications, according to AIC (left) and BIC (right). fGn series, from 2048 to 128 data points

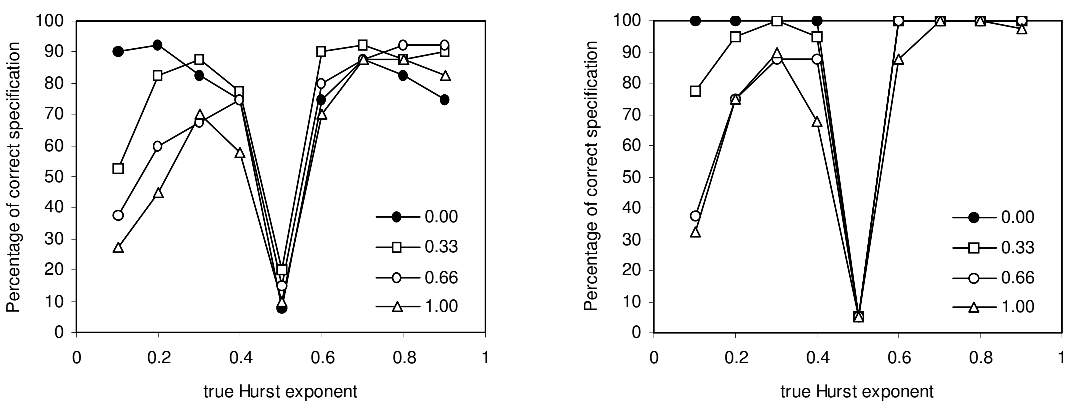

Figure 2: Percentage of correct specifications, according to AIC (left) and BIC (right). fGn series (1024 data points) with added noise.

Figure 2 indicates the percentage of correct specification for series of 1024 points contaminated by white noise. The effect of noise was clearly different for anti-persistent and persistent fGns: the addition of noise didn't affect the performances of BIC for persistent series, and even tended to enhance the performance of AIC. On the contrary, the addition of white noise generated a number of incorrect specifications for anti-persistent series, especially for the lowest *H* values (*H* = 0.1 and *H* = 0.2). Generally, BIC gave better results than AIC.

In these tests ARFIMA models represented 99.8% of the models selected by BIC, and 94.8% of the models selected by AIC. BIC selected the simplest model (0,*d*,0) for 96.6% of the series, and AIC only for 45.5%. Misspecifications were generally due to the selection of complex ARFIMA models, with parameter *d* not significantly different from zero. Note that the increase of misspecification rate in anti-persistent fGns with BIC was due to the appearance of non significant values for *d* in very simple models, such as (0,*d*,1).

Similar results were obtained considering the sums of ARFIMA weights (Figure 3 and 4). Generally these sums were higher for BIC than for AIC. As can be seen in Figure 3, the sums remained close to 1 for the longest series (2048, 1024, and 512 points), and tended to decrease for the shortest series, especially for weakly persistent noises (*H* = 0.6 and *H* = 0.7). The same phenomenon was observed for AIC, but the sums were generally lower. Note that in both cases a mean sum of ARFIMA weights of about 0.6 was obtained for series with *H* = 0.5.

Figure 3 also reports the variability of the obtained sums. The most interesting observation is the very low variability of BIC sums, for the longest series (2048 and 1024 points). The sums obtained were much more variable with AIC, even with long series.

Figure 4 reports the effect of the addition of white noise on the sums of ARFIMA weights. Here also, the effect of noise was clearly different for anti-persistent and persistent fGns: the addition of noise didn't affect the performances of BIC for persistent series, except for *H* = 0.6. The performances of AIC were degraded for *H* = 0.6, but enhanced for the highest *H* values. As previously, the addition of white noise induced a decrease of the sums of ARFIMA weights for anti-persistent series, especially for the lowest *H* values (*H* = 0.1 and *H* = 0.2). Generally, BIC gave better results than AIC.

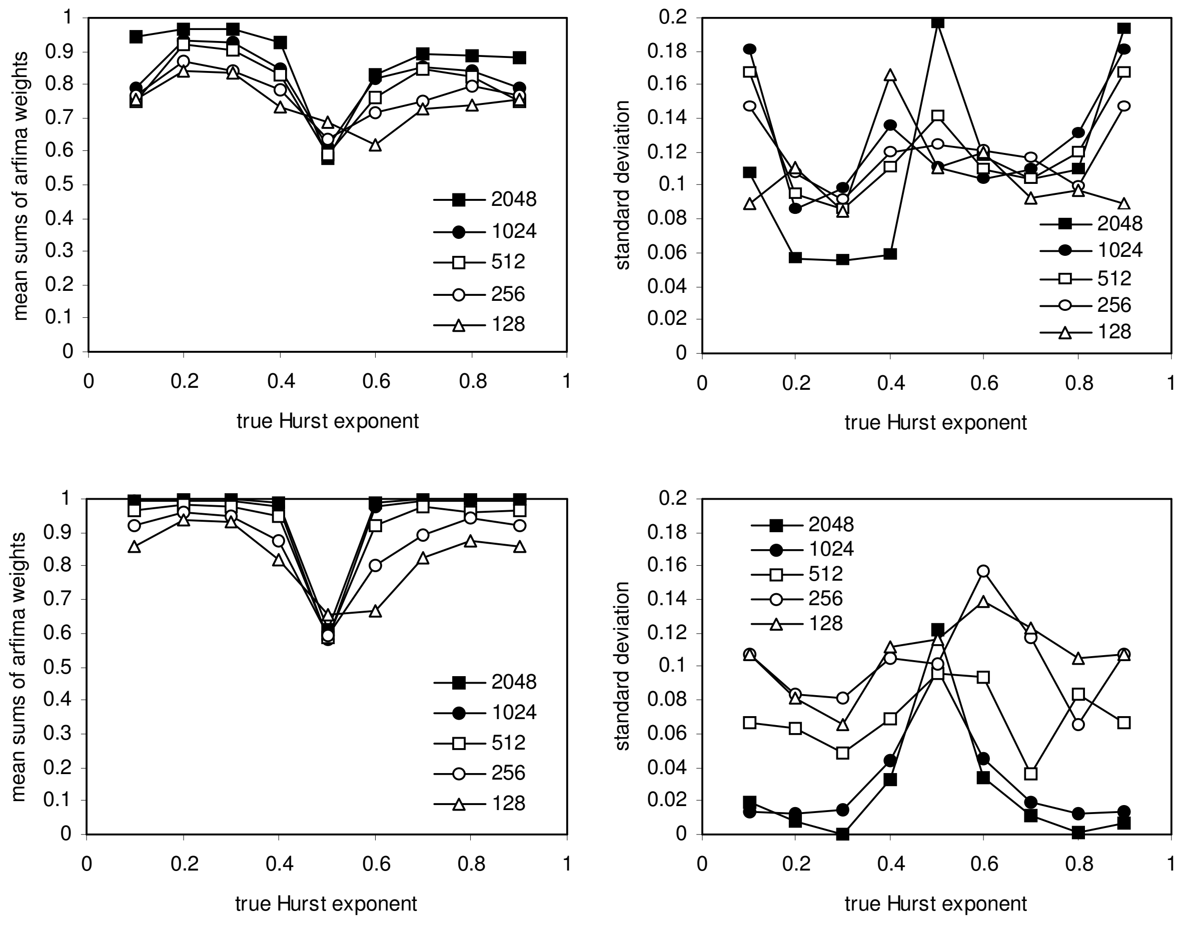

Figure 3: Mean (left panels) and standard deviation (right panels) of the sums of weights of ARFIMA models, according to AIC (top) and BIC (bottom). fGn series, from 2048 to 128 data points.

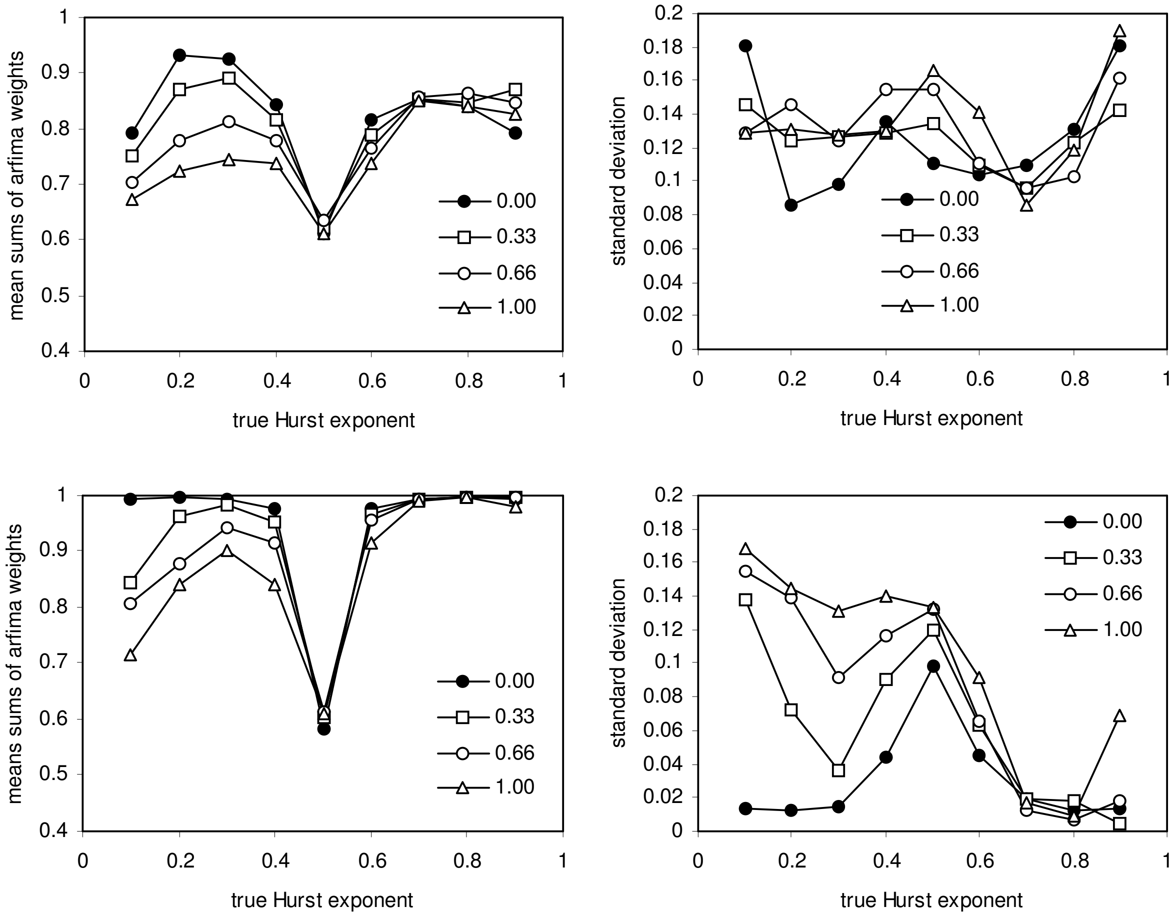

Figure 4: Mean (left panels) and standard deviation (right panels) of the sums of weights of ARFIMA models, according to AIC (top) and BIC (bottom). fGn series (1024 data points) with added noise.

Figure 4 also indicates the effect of white noise on the variability of the obtained sums. Variability remained weakly affected for BIC, concerning persistent series. Conversely, the addition of noise dramatically increased sums variability for anti-persistent series. Variability appeared higher with AIC, whatever the H value or the percentage of contamination by white noise.

False detections in simulated short-range dependence series

We simulated AR(1) and MA(1) series, according to Eq. (4) and Eq. (5), respectively, ϕ and θ both varying from 0.1 to 0.9 by steps of 0.1. Forty series were simulated in all cases. In order to test the effect of series length, we applied the analysis on the entire series (2048 points), and then on the first 1024 points, and the first 512 points. We report in Figure 5 the percentage of false detections of long-range dependence (*i.e.* the best model was an ARFIMA model, with coefficient d significantly different from 0).

Whatever series length, long-range dependence was erroneously detected for a number of series. The percentage of misspecification remained limited (around 10%) for BIC, when ϕ or θ exceeded 0.5. In this range of values, the use of AIC resulted in slightly higher levels of false detections. The percentage of misspecification dramatically increased for the lowest values of ϕ and θ. BIC seemed particularly affected, with percentages of false detections between 40 and 60% when ϕ or θ equalled 0.1.

The most frequently selected model was the ARFIMA (0,d,1) for MA(1) series (for 56.6% of the series with AIC, and 75.9% with BIC), and the ARFIMA (1,d,0) for AR(1) series (for 60.6% of the series with AIC, and 79.5% with BIC). The ARMA model that was actually underlying the series was rarely selected as the best model: the model (0,1) was selected by AIC for 1.0% of the MA(1) series, and by BIC for 2.5%, and the model (1,0) was selected by AIC for 1.3% of the AR(1) series, and by BIC for 2.0%.

Estimation of H exponent through ARFIMA modelling

In this part we used again the simulated fGn series, with H varying from 0.1 to 0.9. We fitted the nine ARFIMA models (p, d, q), p and q belonging to {0, 1, 2}, and we retained the coefficient d obtained from the best model. d was converted into H according to the following equation:

$$H = \frac{2d+1}{2} \quad (16)$$

as $\beta = 2d$, and, for fGn, $H = (\beta + 1)/2$.

Figure 6 indicates the H estimations obtained, according to AIC and BIC, as a function of the true exponent. As can be seen, both methods gave rather good mean estimates, at least for the longest series (2048 and 1024 points), despite a slight underestimation of H for AIC all along the continuum, and for BIC for anti-persistent fGn ($H < 0.5$). The underestimation dramatically increased for AIC for the shortest series, and especially for $H > 0.3$. On the contrary, BIC seemed to function quite well with short series, except for the shortest ones (128 points), for which an underestimation tendency appeared for $H > 0.3$.

Figure 6 also indicates that both criteria differed in terms of estimation variability. BIC provided very consistent assessments with long series (1024 and 2048 points), except for highly anti-persistent fGns ($H = 0.1$). The variability in H estimation increased when series length decreased, especially for persistent fGn. Variability was clearly higher for AIC, even with long series.

The ARFIMA models (0,d,0), (1,d,0) and (0,d,1) represented more than 99% of the models selected by BIC, but in contrast, only 64% of the models selected by AIC. More complex models, as (1,d,1) or (2,d,2), were selected by AIC for 8.5% and 16% of the series, respectively. Note that these percentages were not affected by series length, suggesting that the degradation of the results with the decrease of series length, in terms of bias or variability, cannot be attributed to a poorer model selection.

Finally we compared the present results with those obtained by analysing the same series with some classical methods in fractal analysis. We applied three analyses: $^{low}PSD_{we}$, DFA, and R/S-detrended analysis, selected for the quality of their performances in H estimation (Delignières et al., 2005).

$^{low}PSD_{we}$ is an improved version of spectral analysis, proposed by Fougère (1985) and modified by Eke et al. (2000). This method uses a combination

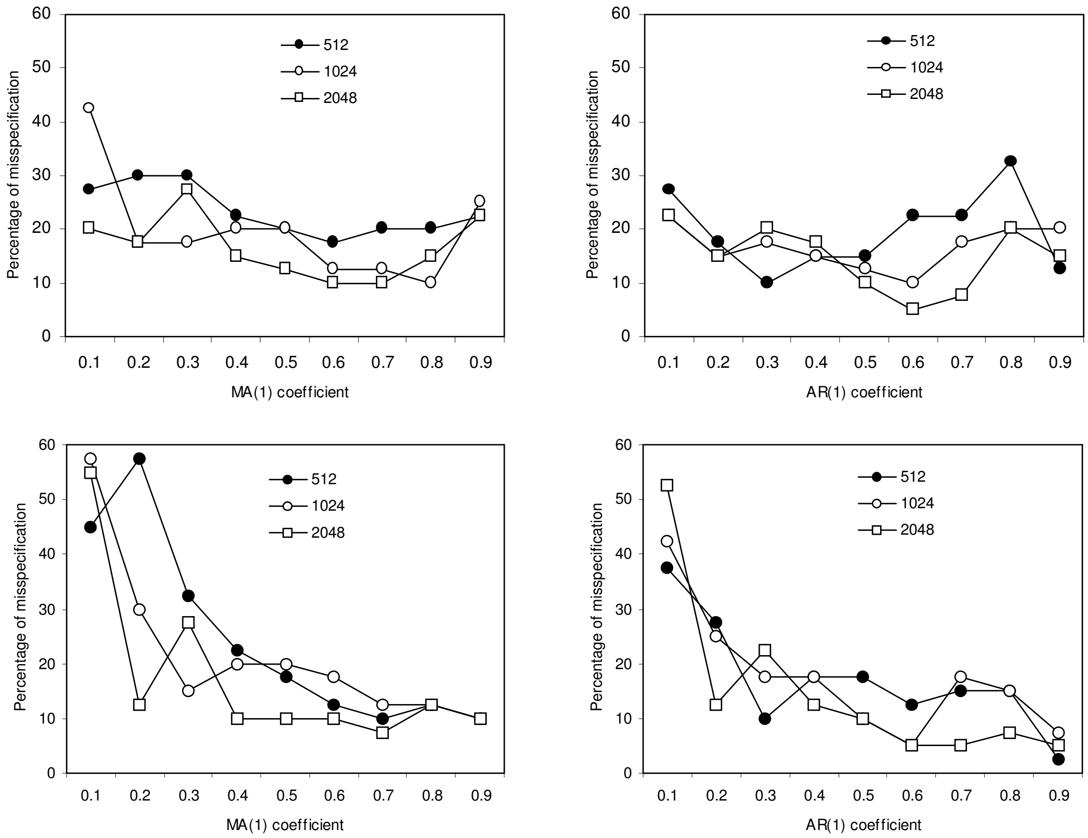

Figure 5: Percentage of misspecifications of MA(1) series (left) and AR(1) series (right). Top panel: AIC; bottom panel: BIC.

of preprocessing operations: Firstly the mean of the series is subtracted from each value, and then the series is tapered by the application of a parabolic window. Thirdly a bridge detrending is performed by subtracting from the data the line connecting the first and last point of the series. Finally the fitting of β excludes the high-frequency power estimates ($f > 1/8$ of maximal frequency). This method was proven by Eke et al. (2000) to provide more reliable estimates of the spectral index. β can be converted into H according to the following equations, which holds for fGn series (Eke et al., 2000):

$$H = \frac{\beta + 1}{2} \tag{17}$$

Detrended fluctuation analysis (DFA) is a well-known method, initially proposed by Peng et al. (1993). This method computes the standard deviation of an integrated, and locally detrended version of the original series, over intervals of increasing length. DFA allows estimation of an exponent α, ranging from 0 to 2. fGn are characterized by α exponents ranging from 0 to 1, and in this range α corresponds to H.

R/S-detrended analysis is an improved version of the classical Hurst method, proposed by Caccia, Percival, Cannon, Raymond, and Bassingthwaigthe. (1997). This method differs from the traditional algorithm by the application of a local detrending of the series of cumulative sums before the calculation of the local range. This method was proved to give more reliable estimates of the fractal exponent than the classical R/S analysis (Caccia et al., 1997; Eke et al., 2000).

Figure 7 allows comparison of the results obtained with ARFIMA modelling (using BIC) and these three methods. In terms of accuracy, ARFIMA modelling gave satisfactory results, as compared with the other methods. The best one was clearly DFA, which provided unbiased estimations, whatever the true underlying H and the length of the series. $^{low}PSD_{we}$ tended to underestimate H for anti-persistent fGns, even for the longest series. For shorter series, a progressive underestimation tendency appeared, for $H > 0$. R/S analysis was characterized by a known bias of overestimation for $H > 0.4$ (Caccia et al., 1997), and a slight underestimation for $H = 0.9$. ARFIMA modelling, albeit presenting its own biases, could support the comparison with the other methods.

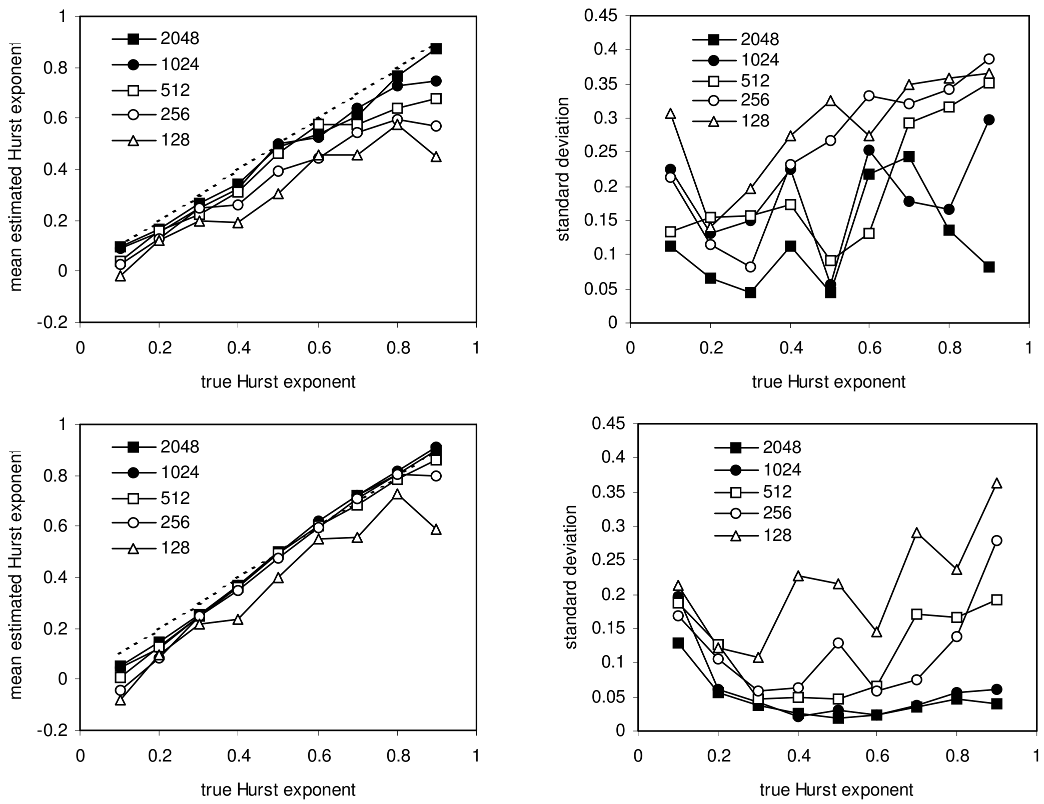

Figure 6: Estimation of the Hurst exponent by ARFIMA modelling, according to the AIC criterion (top panel) or to the BIC criterion (bottom panel). Simulated series of fGn with lengths of 2048, 1024, 512, 256 and 128 data points were used. Left panels report the estimated mean exponent against the true exponent. The dashed line indicates the theoretical equality between estimated and true exponents. Right panels report the variability (standard deviation) of estimation, against the value of the true exponent.

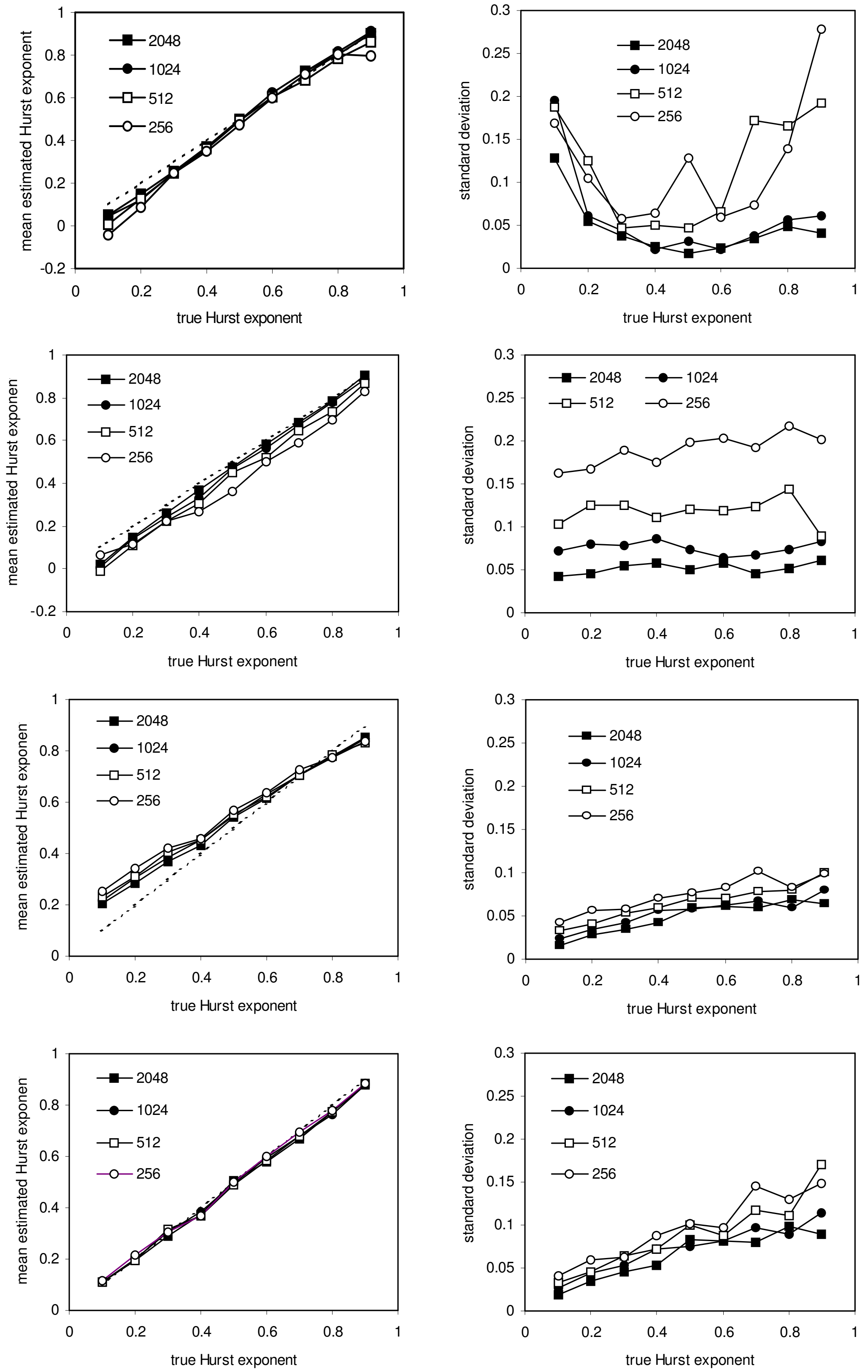

Figure 7: Estimation of Hurst exponent, according, from top to bottom, to ARFIMA modelling (BIC criterion), $^{low}PSD_{we}$, R/S analysis, and DFA. Simulated series of fGn with lengths of 2048, 1024, 512, and 256 data points were used. Left panels report the estimated mean exponent against the true exponent. The dashed line indicates the theoretical equality between estimated and true exponents. Right panels report the variability (standard deviation) of estimation, against the value of the true exponent.

In terms of variability, the performances of the four methods were very different. As can be seen, $^{low}PSD_{we}$ was very affected by series length, with a dramatic increase of variability with the shortest series (512 and 256 points). R/S analysis produced the lowest variability, and was moderately affected by the decrease of series length. Variability, nevertheless, tended to increase with increasing values of *H*. DFA presented a similar pattern of results with, nevertheless, a slightly higher level of variability, especially for persistent fGn. ARFIMA modelling was characterized by the highest levels of variability (1) for short series (512 and 256 points), and (2) for highly anti-persistent fGn (H = 0.1 and H = 0.2). Conversely, for the longest series (2048 and 1024 points), and for fGn with $H > 0.2$, ARFIMA modelling produced very consistent estimations, comparable in variability to those of R/S analysis.

An empirical example

Finally, we tried to apply ARFIMA modelling to series collected during a recent experiment focusing on the time-evolutionary properties of bi-manual coordination. Thirteen participants performed simultaneous oscillations of the forearms, according to two coordination patterns (in-phase and anti-phase), and following two frequencies (low vs high). The variable of interest was the relative phase between the two effectors, computed punctually at each initiation of the cycle of the dominant limb. Each participant performed these four conditions twice, allowing the collection of 104 series of 1024 points.

We applied ARFIMA modelling to these series, following the previously described procedure. AIC preferred an ARFIMA model for 72 series (70%), and BIC for 92 series (93%). The mean weight for these best ARFIMA models was respectively 0.32 for AIC, and 0.68 for BIC. The mean sum of weights for ARFIMA models was 0.74 for AIC and 0.93 for BIC. Only 4 series were simultaneously classified as ARMA processes by the two criteria (AIC and BIC). Considering the previous indications from our simulation studies, these results provided quite strong evidence for the presence of long-range dependence in these series of relative phase.

We then estimated *d* from the best model selected among the nine ARFIMA candidate models. According to AIC, we obtained a mean value of 0.263 (*SD* = 0.167), corresponding to an *H* value of 0.763, and according to BIC, a mean value of 0.258 (*SD* = 0.098), corresponding to an *H* value of 0.756. These mean *H* values were consistent with those obtained with the other methods ($^{low}PSD_{we}$: H = 0.723; DFA: H = 0.786, R/S analysis: H = 0.785). The samples of exponents obtained with ARFIMA modelling were significantly correlated with the exponents obtained from DFA (AIC: r = 0.476; BIC: r = 0.494), and from R/S analysis (AIC: r = 0.462; BIC: r = 0.481). With $^{low}PSD_{we}$ exponents, the correlation was significant for BIC (r = 0.472) but not for AIC (r = 0.157).

Discussion

A recurrent result, in this paper, was the better performances of BIC, as compared to AIC, in the detection of long-range dependence as well as in the estimation of fractal exponents. Clearly the penalty imposed by BIC on complex models led to a better estimation of the nature of the underlying processes. In most cases the best models for BIC were very simple: often the model didn't contain any short-term processes ((0,*d*,0) models), and in the others cases auto-regressive and moving average processes were rarely combined, and their orders never exceeded 1 ((1,*d*,0) or (0,*d*,1) models). In contrast, AIC generally preferred more complex models, using auto-regressive and moving average terms in combination and with higher orders (e.g. (2,*d*,1) model). As pointed out in our first study, a number of errors with AIC was due in part to the selection of complex ARMA models, such as (2,2) or (1,1), and in part to the selection of complex ARFIMA models, such as (2,*d*,2) or (1,*d*,1), *d* being in this case not significantly different from zero. This suggests that a combination of auto-regressive and moving average terms could mimic long-term dependence and capture an important part of the variance contained in the series. Several authors have demonstrated the possibility of mimicking fractal behaviour by the aggregation of a limited set of short-term processes (Granger, 1980; Hausdorff & Peng, 1996; Pressing, 1999). As such, one could characterise the potential problem posed by AIC in favouring complex models: the combination of auto-regressive and moving average terms seems able in some cases to completely hide the presence of long-term dependence in the series, or to lead to a non-relevant estimation of the parameter *d*.

Nevertheless, it is important to keep in mind that our simulation studies were performed with simulated series of pure fGn, supposed to contain only long-range processes. The ARFIMA model supposed to adequately fit the series should thus be very simple, and this could explain why BIC, in this particular case, gave better results. In contrast, empirical psychological series could be composed of a combination of short-range and long-range processes (Wagenmakers et al., 2004), and AIC could possibly perform better with such series. Additional studies are necessary to test this hypothesis. However at this point, our results should not be considered as definitely banishing the use of AIC, as the overall superiority of BIC could be related to the nature of our simulated series.

We limited our investigations in the present paper to the two classical model selection rules used by Farrell et al. (2005). A number of alternative criteria have been proposed, such as minimum description length (MDL; Rissanen, 1978) or combined information criterion (CIC; Broersen, 2000). Additional investigations should be necessary for assessing the performance of these model selection criteria for the present purpose. Some improved versions of our classical criteria, such as the corrected AIC (Hurvich & Tsai, 1989), have also been proposed for the analysis of short data records. A 'short series', nevertheless, is defined by the fact that the number of observations is not much larger than the number of parameters in the candidate models. This was clearly not the case in the present study, and we didn't consider it necessary to use these corrected criteria.

The first study suggests that ARFIMA modelling performs quite well for detecting long-range dependence, at least when series have a sufficient length (1024 or 2048 points). These lengths correspond to those generally used in psychological experiments, even if on some occasions shorter series could be encountered. (*e.g.* Delignières et al., 2004; Kadota, Kudo & Ohtsuki, 2004). Whatever the method, nevertheless, its seems unreasonable to use series shorter than 512 points, and 1024 points seem to represent the best compromise between the requirements of the methods and the inherent limitations of psychological experiments (Delignières et al., 2006). The shortest series we studied here (256 and 128 points) should only be considered as formal examples, aiming at analyzing the behaviour of methods in extreme conditions (for a notable exception, see Madison, 2004).

The two criteria tested in this study (the percentage of ARFIMA models selection and the mean sum of ARFIMA models weights) gave similar results. Nevertheless, they should not be considered as absolutely redundant: for example, the selection of an ARMA as best model can be associated with a sum of weights in favour of ARFIMA models. These occasional discrepancies are not important when an extensive set of series is analyzed, as in the present studies. But when the analysis focuses on a single series, the sum of ARFIMA model weights should represent a better indication of the underlying presence of long-range dependence than the nature of the best model.

The performance of ARFIMA modelling in detecting long-range dependence appeared weakly affected by the presence of noise, at least for persistent fGns. This result is important, because noise is often present in the time series collected in experimental setting, and also because series collected in psychological experiments generally fall in this range of persistent fGns (Wagenmakers et al., 2004).

One can be surprised, in contrast, by the poor performance of ARFIMA modelling in the analysis of pure ARMA series. An important rate of false detections of long-range dependence was observed, especially when the auto-regressive or moving average coefficients were low. This result was particularly evident for BIC, with error rates exceeding 50% for the lowest p and q values. In most cases, the best model was not an ARMA but an ARFIMA model (although d was often not statistically different from 0, except for the lowest p and q values), and the sum of weights of ARFIMA models was always higher (around 0.65) than its ARMA counterpart. Note that when an ARFIMA model was selected with a significant d, this parameter was always close to zero (especially for series of 2048 points), and clearly outside the typical $1/f^{\beta}$ range which should begin at $d = 0.25$ (*i.e.* $\beta = 0.5$).

These results point out a tendency of the method to favour the selection of ARFIMA models. Surprisingly, Wagenmakers et al. (2004) showed an opposite pattern of results: in their simulation experiment ARMA models were mistakenly identified as ARFIMA models in only 7.5% of the simulated series, whereas ARFIMA models were mistakenly identified as ARMA models in 26.2% of the simulated series, suggesting a bias favouring the selection of ARMA models. The method they used, however, compared only two models, an ARMA (1,1) and an ARFIMA (1,d,1), according to AIC, and their simulated series were for a first set pure $1/f$ noises (contaminated by white noise), and for the second set ARMA (1,1) processes. The relatively high rate of misspecification for the first set could be explained by the fact that the most plausible model - ARFIMA (0,d,0) – was not tested by the authors. On the other hand, the low rate of misspecification for the second set could be related to the presence, in the two tested models, of the ARMA (1,1) that was actually used for generating the series.

Our present results, based on the test of a wider set of candidate models, suggest clearly a bias favouring ARFIMA models. Fractional integration seems to endow ARFIMA models with a greater flexibility than their ARMA counterparts, leading to important rates of spurious detections of long-range dependence. These observations suggest avoiding testing a unique series for the presence of long-range dependence but, rather, to systematically collect a set of series obtained in similar conditions. The rates of misspecification observed in our simulation studies could provide some useful guidelines in the interpretation of results. On the basis of our first simulation study (see Figure 1), one could propose rejecting the hypothesis of the presence of long-range dependence in the series if the percentage of identification of an ARFIMA model is inferior to 90% using BIC, and 70% using AIC. Similar standards could be proposed for the mean sums of ARMIFA models weights, which should approximately reach 0.9 with BIC and 0.7 with AIC for accepting the long-range dependence hypothesis. The set of empirical series we tested in our final study satisfied these two criteria. Wagenmakers et al. (2005) performed the same analyses on the original series collected by van Orden et al. (2003) in a reaction time task and in a word naming task. In the reaction time task, AIC and BIC selected an ARFIMA model for 7 and 4 series over 10, respectively (70 and 40%), and in the word naming task, for 14 and 13 series over 20, respectively (70 and 65%). These results were obviously less convincing than those reported in our final study, and offered more limited support for the presence of long-range dependence in reaction time series.

ARFIMA modelling also appears as a valuable tool for the estimation of fractal exponents. This method gives acceptable results, as compared with classical methods, with limited biases, and a low variability, at least with relatively long series (1024 and 2048 points). Here also, BIC seemed to provide better and less variable estimates than AIC. One can conclude that BIC selects the model comprising the most relevant p and q parameters, allowing a more accurate estimation of d (Taqqu & Teverovsky, 1998).

Obviously, the different methods didn't give exactly the same results, in terms of H exponents, for a given series. The final study showed that with real series, the correlations between the samples of exponents remains moderate, albeit significant. It is important to keep in mind that each method gives an estimation of the fractal exponent. All these methods are supposed to converge toward a common value, with increasing series length. Nevertheless, these methods exploit different properties of fractal series and are based on different algorithms: as such, and with relatively short series, differences in estimated exponents were not surprising. An interesting strategy is to average the estimates obtained from different methods, exploiting different algorithms, in order to obtain a more accurate estimation of the true H value (*e.g.* Schmidt et al., 1991). ARFIMA modelling could, in this respect, offer a valuable additional estimate.

In conclusion, the method proposed by Wagenmakers et al. (2005) and Farell et al. (2005) constitutes a quite appealing solution for statistically testing for the presence of long-range dependence in times series. A number of problems remain, nevertheless, due to the tendency of the method to favour the selection of ARFIMA models. Further methodological efforts are needed for improving the suitability of the method and allowing less ambiguous identification.

Secondly, ARFIMA modelling provides an interesting method for estimating fractal exponents. Often researchers limit their investigations to a limited set of well-known methods (spectral analysis, DFA, or R/S analysis). ARFIMA modelling constitutes a valuable alternative and a complementary solution for the problem of fractal estimation.

Finally, one could consider ARFIMA as a useful tool for deriving suitable models for the systems underlying the analyzed series. Nevertheless, it's important to keep in mind that a number of models are able to generate series possessing $1/f$ properties, such as, for example, the aggregation of multiple auto-regressive processes (Granger, 1980), the summation of moving-average processes on different time scales (Wing, Daffertshofer, & Pressing, 2004), stochastic delay differential equations (Chen, Ding, & Kelso, 1997), or self-organized critical models (Davidsen & Schuster, 2000). $1/f$ fluctuations represent a ubiquitous phenomenon, and one could doubt that a unique model could work for the diversity of biological or physical systems exhibiting such long-range dependencies (Wagenmakers et al., 2004). A relevant model should present theoretical and empirical plausibility, with regards to current theories and knowledge about the system under study. In this respect, ARFIMA models may not necessarily constitute the most appropriate candidates.

Authors' notes

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Transition part

The ARFIMA/ARMA modeling method has been introduced into the field of experimental psychology (Wagenmakers et al., 2004) to answer the common pitfall of 'classical' fractal methods which provide estimations of fractal exponents but are unable to give evidence for the actual presence of long-range correlation in series. The above evaluation study confirms that this method constitutes a quite satisfactory framework for inferentially testing for serial long-range correlation. Its worthiness is notably in that it focuses on the actual question of what is the type of correlations in series (short-range or long-range), instead of whether fluctuations are correlated or random; an issue that classical methods cannot answer consistently.

However, ARFIMA/ARMA modeling should certainly not be taken as providing infallible statistical evidences regarding the nature of correlation in series. In particular, this method cannot be reliably used to conclude with the presence or absence of long-range correlation in a single time series. The method needs to be applied on a set of series in a given experimental condition to determine (i) the percentage of series better fit by an ARFIMA model and the (ii) mean sum of weights captured by ARFIMA models which allow to reject or accept the hypothesis of long-range correlation with some reliability. Note that applying ARFIMA/ARMA to experimental and simulated time series has allowed to question some claims for $1/f^{\beta}$ noise (and ensuing theories) in literature (*e.g.*, Delignières et al., 2008; Wagenmakers, Farrell, & Ratcliff, 2005).

One potential limitation of the ARFIMA/ARMA modeling method, of which one should at least be aware, is the form of inductive bias due to the presupposition that one of the two alternative model classes is relevant to the data under study. This poses the question of whether ARMA models constitute a pertinent null hypothesis (Thornton & Gilden, 2005). Importantly with regard to the theoretical approach of serial (long-range) correlation and generating mechanisms, one may consider in contrast that this method is in accordance with the idea that short-range and long-range processes are not mutually exclusive. This idea has been supported by some empirical studies and used for theoretical interpretation, notably in the field of timing control (*e.g.*, Delignières et al., 2004, 2008; Gilden et al., 1995; Torre & Delignières, in press).

In sum, ARFIMA/ARMA modeling constitutes an interesting tool for the detection of long-range correlation which legitimates the estimation of fractal exponents. It is essential, however, to keep in mind the limitations of this method as those of classical fractal methods;

associating different types of analysis and performing multiple estimations remains the best way to dependable conclusions. On this methodological basis, we propose in Chapter 3 to assess the serial correlation properties of relative phase in bimanual coordination. (Experimental data used for this study are available at http://www.edm.univ-montp1.fr/fr/recherche-membre.php?membre=55)

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3 CHARACTERIZATION OF SERIAL CORRELATIONS IN BIMANUAL COORDINATION

$1/f^{\beta}$ fluctuations in bimanual coordination: An additional challenge for modeling

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$1/f^{\beta}$ fluctuations in bimanual coordination: An additional challenge for modeling

Torre, K, Delignières, D., & Lemoine, L.

EA 2991 Motor Efficiency and Deficiency
University Montpellier I, France

Abstract: We analyzed the correlation structure of discrete relative phase series in bimanual in-phase and anti-phase coordination by associating a number of fractal methods and using discrete rather than continuous relative phase measurement. ARFIMA/ARMA modeling provided statistical evidence for the presence of long-range correlation, and the series were unambiguously characterized as $1/f^{\beta}$ noise. Diverging accounts of bimanual coordination are defended in the literature. Because the evidence for $1/f^{\beta}$ noise provides new insight into the properties of stability in coordination, it should be considered as an empirical criterion for determining which mechanisms are likely to be engaged in bimanual coordination models. We discussed some implications for studying the neural basis of coordination, and we tested the performance of three current models in accounting for $1/f^{\beta}$ noise in discrete relative phase. None of these models was proven to generate the expected correlation structure.

INTRODUCTION

The recurring and ubiquitous manifestation of coordination principles across different time scales, organisms and physical systems with little in common suggests that universal and non-specific principles might be engaged. In the domain of human behavior, bimanual coordination tasks represent a dominant experimental paradigm (Kelso, 1995), and the properties of organizing and/or disorganizing processes within a system can be explored as these tasks are performed. The dynamic systems approach that underlies the bimanual coordination paradigm focuses on the variability of relative phase (ϕ), which describes the spatio-temporal relationship between two oscillating limbs. In this approach, the bimanual coordination system is considered to be self-organized, meaning that "order" and spontaneous adaptations emerge from the intrinsic properties of a system that is likely to be constrained by changeable external factors. Self-organizing phenomena necessarily occur in complex systems, with multiple interactions between their components. The emerging behavior possesses specific properties of stability. In the bimanual coordination framework, two preferential coordination patterns, in-phase (ϕ=0°) and anti-phase (ϕ=180°), have been evidenced. These two *attractors* define zones of intrinsic stability. Because the in-phase pattern is intrinsically more stable than anti-phase, a spontaneous transition from anti-phase to in-phase, preceded by a critical increase in relative phase fluctuations, occurs at the individual's critical oscillation frequencies.

Variability analyses have constituted a basis for developing bimanual coordination models, notably with regard to two currently challenging frameworks: the *HKB model* (Haken, Kelso & Bunz, 1985; Schöner & Kelso, 1988) and the *multiple timer model* (Ivry & Richardson, 2002). For both models, however, the characterization of coordination variability is limited to relative phase variance. Thus, variability is only considered in terms of (relative) magnitude and the possible correlation structure of fluctuations in time is completely disregarded. In contrast, we suggest that the focus put on variability in bimanual coordination modeling calls for its unambiguous characterization and, in this regard, white noise is only one particular type of variability that appears as an exception rather than a rule in

Send correspondence to:
Kjerstin Torre
Faculty of Sport Sciences – University Montpellier 1
EA 2991, "Motor Efficiency and Deficiency"
700 Avenue du Pic Saint Loup
34090 Montpellier – France
Email: ktorre@univ-montp1.fr

nature (Slifkin & Newell, 1998). Specifically, a number of systems in steady state have been proven to present $1/f^{\beta}$ fluctuations, *i.e.*, highly structured variability rather than random fluctuations around a mean reference value. Examples include self-esteem (Delignières, Fortes & Ninot, 2004), time-intervals production (Gilden, 2001), stride intervals (Hausdorff et al., 1997), heart rate variability (Peng et al., 1995), and serial force production (Wing, Daffertshofer & Pressing, 2004). A deeper analysis of the dynamics of relative phase during the performance of stable bimanual patterns could be expected to yield similar results.

For most observed phenomena, the collected time series present auto-correlation with either positive (persistent) or negative (anti-persistent) dependence between the successive values. One usually distinguishes between short-range and long-range dependence (LRD). LRD indicates that the auto-correlation function presents a very slow power-law decay, meaning that a given value in the series conserves a statistical memory of very past values rather than being influenced only by a few immediately preceding values. LRD is the fundamental feature of fractal series, and several indexes can be used to assess fractal properties. The most current is the spectral index β, which represents the negative of the linear regression slope in the bi-logarithmic power spectrum of series. The spectral index is related to the Hurst exponent (H), which gives the correspondent characterization of series in the time domain. In the general procedure for characterizing the variability of experimental series (Eke et al., 2000), spectral analysis allows an initial classification by distinguishing two families of fractal processes: (i) fractional Gaussian noises (fGn, for $\beta \in [-1,1]$), representing stationary series, and (ii) fractional Brownian motions (fBm, $\beta \in [1,3]$), representing non-stationary series whose variance increases with time following a $2H$-power law. An fBm is obtained by the integration of an fGn, and both are then characterized by the same Hurst exponent ($H \in [0,1]$), which is used to categorize correlations as persistent (H>0.5) or anti-persistent (H<0.5). White noise (H=0.5) is situated in the center of the fGn family, at the boundary between persistent and anti-persistent processes.

Within this fGn-fBm continuum, $1/f^{\beta}$ noise, in the strict sense, is characterized by $\beta = 1$. However, the strict definition is usually extended so that $1/f^{\beta}$ noise encompasses the range $0.5<\beta<1.5$ (Wagenmakers, Farrell & Ratcliff, 2004). The specificity of $1/f^{\beta}$ noise is that its power is roughly proportionally distributed over the different frequencies, resulting in a regression slope close to -1 in the log-log power spectrum. Thus, dependencies in series arise from the high-frequency, low-amplitude fluctuations that are nested within the low-frequency, high-amplitude fluctuations (Beltz & Kello, 2006). The statistical specificity of $1/f^{\beta}$ noise represents a compromise between stability and variability and is classically found in the global behavior of complex systems. Accordingly, $1/f^{\beta}$ noise cannot be considered as random background fluctuations when evidenced in human behavior. Instead, it appears to play a functional role in the maintenance and adaptability of systems (Goldberger, 1999). As such, the capability of models to account for this special type of fluctuation should constitute an important testing criterion, which could then orient the efforts to improve these models by identifying the mechanisms likely to generate $1/f^{\beta}$ noise. In this view, a number of models that have been proposed to account for the results observed in gait, tapping, or force production have been inspired by the characteristic time-scaling of $1/f^{\beta}$ noise (Gilden, 2001; West & Scafetta, 2003; Wing et al., 2004).

To our knowledge, only one study has addressed the issue of fractal properties in motor coordination. Schmidt et al. (1991) analyzed anti-phase coordination between the wrists oscillating in the sagittal plane. They collected series of continuous relative phase (CRP) and applied spectral analyses to examine the series' fractal properties. The results showed β indexes ranging from 1.64 to 2.96, characterizing CRP as over-diffusive (persistent) fBm. Such a result seems counterintuitive and surprising. Because the participants performed a task using a stable and highly attractive coordination pattern, one would expect a meaningful capture of coordination to yield stationary series. We argue that these results may have been due to the use of CRP, which could be inappropriate for analyzing the functional variability in coordination.

In human behavior, cognition, and biological processes, fractal methods have essentially been used to analyze series of successive discrete performances or events (as in the above-mentioned examples). In those cases, the addressed variable and the task modalities naturally yielded sequences of discrete measures, constituting *event series* rather than genuine time series with a constant interval between measures (Gilden, 2001). In bimanual coordination studies, two types of series are usually computed and used for analysis: continuous relative phase and discrete relative phase (DRP), which is determined once per cycle at the reversal points of oscillations. One thus might wonder which measurement of relative phase is more meaningful and appropriate for addressing the variability properties in coordination.

CRP and DRP are frequently considered to be equivalent (Kelso, 1995), with CRP seen as a higher resolution form of DRP that nevertheless contains similar information. However, Peters et al. (2003) stressed that CRP cannot be used to reliably describe the relationship between two signals in the time domain, especially when the time series are not strictly sinusoidal or when a constant time lag appears between signals. Oscillations in bimanual tasks are known to be not strictly sinusoidal. The typical phase portrait of an oscillator is not circular and, accordingly, limb movements have been modeled using the equation of a hybrid Rayleigh-Van der Pol oscillator (Kay et al., 1987). Moreover, asymmetry (*i.e.*, a slight advance of one oscillator on the other one) is commonly observed in bimanual coordination and is explained by either different eigenfrequencies of the oscillators or asymmetric coupling due to handedness (Byblow et al., 1998). Peters et al. (2003) showed that for such signals a continuous measurement presents important waves and drifts, and they recommended the use of CRP to describe only the relationship between the normalized phase planes of the oscillators, rather than the relationship between the two original time series. We argue that a meaningful measure for revealing the stability of coordination and studying its properties should derive from the relative positions of the oscillators in time, and thus it should be based on DRP. Last, DRP series are not genuine time series but are instead event series that capture the temporal dispersion of specific events in the coordination of the two limbs. Obtaining long-range correlation between successive values is thus not a given per se, because such dependencies would not be simply explicable by the persistent mechanical constraints inherent to movement. As such, it would be functionally meaningful for coordination.

Our main aim in this paper was to provide clear evidence for the presence of $1/f^{\beta}$ noise in DRP series. Generally, $1/f^{\beta}$ noise is assumed to manifest in complex systems that are allowed to fluctuate without extrinsic constraints (Beltz & Kello, 2006). In this regard, a number of studies have reported an alteration in the fractal properties of series in conditions where systems were in difficulty (see for example Chen, Ding, & Kelso, 2001; Hausdorff et al., 1997). In order to avoid similar effects in the present study, participants performed in-phase and anti-phase coordination tasks at a comfortable frequency, to ensure that coordination would be bi-stable. In line with our preceding claims, we compared the temporal structures of CRP and DRP series, expecting to evidence $1/f^{\beta}$ fluctuations only in the latter. Last, we addressed some perspectives for neurobiological coordination studies and implications for bimanual coordination models, and we tested the capability of current models to account for $1/f^{\beta}$ noise in DRP series.

METHOD

Participants and device

Fourteen participants (11 male and 3 female) between the ages of 23 and 46 years took part in the experiment. They declared no particular competence involving specific coordination between the upper limbs and no neurological injury or recent upper limb injury. They gave written informed consent and were not paid for their participation. Participants were seated comfortably with the forearms in a horizontal position. They performed bimanual forearm oscillations holding two wooden joysticks, 15 cm in length, with a single degree of freedom in the frontal plane. The positions of the joysticks were adjusted to each participant's morphology. In-phase and anti-phase oscillations were paced by an auditory metronome. In both conditions participants performed one complete cycle within the given periods, starting with maximal pronation of the dominant hand on the beep. They performed oscillations with an amplitude of approximately 45° on each side of the vertical axis. No physical stop or other systematic feedback was given in order to maintain this amplitude. The angular displacement of the joysticks was captured by two potentiometers. Data were collected via a *Nanologger* (Digimetrie, Perpignan, France) analogue interface. The experiment was approved by the local ethics committee.

Procedure

The experiment comprised two sessions. The aim of the first session was to determine the individual critical frequencies. Participants performed 10 trials following an incremental protocol (initial frequency: 1 Hz, with increments of 0.2 Hz and step durations of 10 seconds). They started in anti-phase and maintained this coordination until a spontaneous transition to in-phase. We instructed them not to resist this transition and we determined the transition frequency for each trial. For each participant, the critical frequency was defined as the median of the observed transition frequencies.

During session 2, participants performed four trials: two in in-phase and two in anti-phase. The two conditions were randomly assigned. All trials were performed with a comfortable oscillation frequency (f) corresponding to 64% of the individual critical frequency for each participant. This percentage was chosen so that for all participants f represented the lowest possible frequency bounded by 1 Hz, which was reported as the minimal frequency of comfort (Monno et al., 2002). The sampling frequency was 500 Hz.

The participants performed at least 1024 cycles per trial. Trial durations thus ranged from 9 to 17 minutes, depending on the imposed frequency. The duration of measurement had to strike an optimal compromise between the constraints imposed on the participants and the minimal time required for reliable fractal analyses. Series of 2^9 or 2^{10} points are currently considered to be an acceptable compromise

(Chen, Ding & Kelso, 1997; Delignières et al., 2005, 2006; Gilden, 2001). Moreover, it is recommended that the estimation of the fractal exponents be repeated for a the same condition, notably by associating different methods (Delignières et al., 2005; 2006; Eke et al., 2000; Rangarajan & Ding, 2002). Therefore, (i) the participants performed each condition twice and (ii) we associated different methods of analysis in order to obtain the most accurate estimations of fractal exponents by averaging (Delignières et al., 2005; Torre et al., 2007).

Data Analyses

We applied a bi-directional low-pass Butterworth filter (cut-off frequency 15 Hz) to the collected data and used an appropriate algorithm for peak detection. DRP values (ϕ, in degree unit) were then computed following the *point estimate* method:

$$\phi_i = \left(\frac{t_{i+1} - \tau_{i+1}}{t_{i+1} - t_i} \right) 360 \qquad (1)$$

where t_i and t_{i+1} represent the timings of two successive inflexion points in the dominant hand pronation, and τ_{i+1} the timing of the corresponding inflexion point in the opposite hand pronation. Unintended errors in coordination were not discarded from analyses. To avoid shifts in relative phase subsequent to such accidents, the following ϕ values were again computed at corresponding right and left reversals.

Means and standard deviations (SD) were computed for each DRP series. In order to test for the effect of experimental conditions on coordination stability, a two-way ANOVA 2 (Coordination) × 2 (Trial) with repeated measures on both factors was applied on SD. A Greenhouse-Geisser correction was applied, and significance was set at .05.

For the computation of the CRP series, we normalized position and velocity within each cycle. CRP was determined as the difference between the phases of the right- and the left-effector within cycles. We retained the first 32,768 points (2^{15}) of the series for analysis.

Fractal analysis of the DRP series. A first classification of the series as either fGn or fBm is an essential step that directs the nature of the following analyses (Eke et al., 2000). We used $^{low}PSD_{we}$, an improvement of the classical spectral analysis that has been shown to give more accurate estimates of the spectral index (Delignières et al., 2006). It differs from classical spectral analysis by three pre-processing operations performed before Fourier transformation: (i) the mean of the series is subtracted from each value, (ii) a parabolic window is applied to taper the series, and (iii) the resulting series is linearly detrended. Finally, only the low-frequency region of the spectrum is taken into account for determining the regression slope. The spectral index β is the negative of the linear slope. As previously indicated, for fGn $-1<\beta<1$, and for fBm $1<\beta<3$.

Identification of a linear regression in the power spectrum has usually been considered a satisfactory criterion to confirm fractal properties. However, the insufficiency of such analyses was recently stressed by Wagenmakers et al. (2004), who questioned interpretations that rely exclusively on qualitative and ambiguous graphical representations. The authors argued that fitting a regression line to such graphs presupposes that the series present LRD. Before determining a regression slope it is thus necessary to provide statistical evidence that such a regression is relevant.

We therefore used ARFIMA/ARMA modeling (Wagenmakers et al., 2004; Torre et al., 2007) to evaluate the statistical evidence for LRD in the series. This method consists of fitting 18 models to the studied series: nine are ARMA (p,q) models, with p and q systematically varying from 0 to 2, and the other nine are the corresponding ARFIMA (p,d,q) models, where d is the fractional integration parameter. The method selects the best model on the basis of a goodness-of-fit statistic that is based on a trade-off between accuracy and parsimony: the best model is the one that gives a good account of the data with a minimal number of free parameters. We used the Bayesian Information Criterion (BIC), which was proven to give the best results for detecting LRD (Torre et al., 2007). The raw BIC values can be conveniently converted into weights (Wagenmakers et al., 2004), and the weight obtained by a given model can be conceived as the probability that the model is the best of the set of candidate models. Note that the weights computed among a given set of models sum to one.

The conclusions about the presence of LRD in the collected DRP series were drawn from two complementary indicators: (i) the nature (ARFIMA or ARMA) of the model presenting the highest weight and (ii) the cumulated weights of the tested ARFIMA models compared with the cumulated ARMA weights. Torre et al. (2007) proposed that the presence of LRD in a set of series can be assumed when at least 90% of the series is best fitted by an ARFIMA model and the cumulated weight for the ARFIMA models exceeds 0.90. The analyses were performed using the ARFIMA package for the *Ox* platform (Doornik 2001, available at http://eis.bristol.ac.uk/~pssaf/).

In order to estimate the fractal exponents, we applied four methods: $^{low}PSD_{we}$, DFA, R/S-detrended analysis, and ARFIMA modeling. Because details concerning the algorithms of these methods can be consulted in the references given for each one, we chose to limit our presentation to general principles and precautions for their application.

The spectral indexes, determined by $^{low}PSD_{we}$ for the preliminary characterization of the series, provided a first estimation of the fractal exponents. For fGn series, spectral indexes can be converted into Hurst exponents by $H = (\beta + 1)/2$.

The DFA (*detrended fluctuation analysis*, Peng et al., 1993) operates in the time domain. This method gives an estimation of an exponent α comprised between 0 and 2. fGn series are characterized by $0 < \alpha < 1$, and α equals the Hurst exponent in this case.

The R/S-detrended analysis (Caccia et al., 1997) provides more reliable estimates of *H* than the original Hurst method. It differs from the original algorithm by locally detrending the integrated series, before determining the local ranges.

Last, we used ARFIMA modeling for a fourth estimation of the fractal exponents. Among the nine tested ARFIMA models, we retained the coefficient *d* determined for the best model; *d* can be converted into *H* by $H = (2d + 1)/2$.

We checked the consistency of the estimations using inter-methods correlations and computed the average *H* for each series. In order to test the effect of experimental conditions on fractality, we performed a two-way ANOVA 2 (Coordination) × 2 (Trial) on the *H* exponents, with repeated measures.

Finally, $^{low}PSD_{we}$ and DFA were applied on the CRP series for comparison with the results obtained on DRP.

RESULTS

The mean relative phase was -3.99 (± 5.95) for in-phase trials and 175.84 (± 9.94) for anti-phase trials. The ANOVA revealed no main effect of Coordination or Trial. The interaction was significant ($F(1,13) = 6.01$, $p<.05$): for the first trial, variability was higher in anti-phase than in in-phase and, for anti-phase, variability was higher in the first trial than in the second.

The power spectra showed a systematic linear trend with negative slope (Figure 1). The estimation of regression slopes yielded β values between -0.01 and 0.98 (mean 0.43). These indexes showed the stationarity of all series ($\beta < 1$), which were thus classified as fGn.

Consequently, we performed ARFIMA modeling on the original series. For 53 of the 56 DRP series (95%), an ARFIMA model was selected, with a fractional integration parameter (*d*) significantly different from zero. The most often selected model was ARFIMA(0,*d*,0), the most parsimonious one. The sum of weights concentrated by all ARFIMA models was 0.93 on average. Only three series were best fitted by an ARMA model. This set of results allowed to conclude that the DRP series presented genuine fractal properties (Torre et al., 2007).

The measures given by the four methods were converted into *H* according to the above given equations. The *H* estimates ranged from 0.50 to 0.99 (mean= 0.72) for $^{low}PSD_{we}$, from 0.54 to 1.09 (mean= 0.80) for DFA, from 0.56 to 1.03 (mean= 0.80) for R/S analysis, and from 0.60 to 0.97 (mean=0.78) for ARFIMA modeling. All samples of estimates were significantly correlated: $r_{54}(^{low}PSD_{we}/DFA) = 0.46$, $r_{54}(^{low}PSD_{we}/R/S) = 0.50$, $r_{54}(^{low}PSD_{we}/ARFIMA) = 0.52$, $r_{54}(ARFIMA/DFA) = 0.63$, $r_{54}(ARFIMA/R/S) = 0.64$, and $r_{54}(DFA/R/S) = 0.96$. The final *H* values were then determined by averaging the four

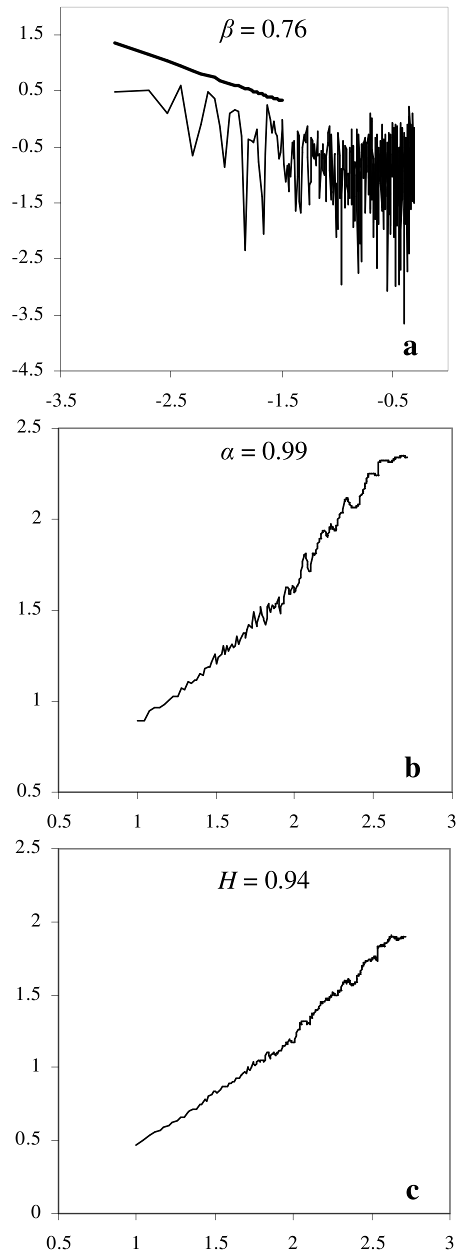

Figure 1. Representative example (in-phase coordination trial) of (a) the individual power spectra, and the individual diffusion plots of (b) DFA, and (c) R/S-detrended analysis, computed on DRP series.

estimations. The mean H ranged from 0.73 to 0.79, in-phase and anti-phase confounded. The ANOVA performed on mean H showed no significant effect of Coordination, Trial, or interaction.

In a last step, we compared indicators for $1/f^{\beta}$ noise in the DRP versus the CRP series. Contrary to the analyses performed on DRP, fractal analyses in the time and frequency domains gave discrepant results for CRP: DFA suggested Brownian motion-like diffusion, with a mean α of 1.50 (± 0.12) and 1.47 (± 0.13) for in-phase and anti-phase, respectively. $^{low}PSD_{we}$ gave a mean β of 3.41 (± 0.92) for in-phase and 3.34 (± 1.02) for anti-phase, situating the CRP series out of the expected range for fractal series.

One might wonder whether finding $1/f^{\beta}$ noise in the DRP but not the CRP series was related to the event-based nature of the former, or to the fact that the two series represent different time scales. In order to examine this latter possibility, we re-sampled the CRP series by picking out the CRP values coinciding with the metronome signals. These new series presented equally spaced values, quite close to the corresponding DRP values. Nevertheless, DFA yielded a mean α of 0.63 (± 0.11) for the re-sampled CRP series versus 0.80 (± 0.12) for the DRP series. The difference was statistically significant ($F(1,11) = 47.82$, $p<0.001$).

DISCUSSION

Our results provided unambiguous evidence for $1/f^{\beta}$ noise in the DRP series. The present discussion is organized into four parts. We first address the methodology and the results of the fractal analyses. Second, we discuss the interest of using DRP instead of CRP for analyzing stability properties in bimanual coordination. We then address some of the theoretical implications of finding $1/f^{\beta}$ noise. Fourth, we test three current bimanual coordination models and discuss some perspectives for further modeling with regard to the evidence of $1/f^{\beta}$ noise.

Fractal analyses. Conclusions about the presence of $1/f^{\beta}$ noise in series have often been drawn on the basis of visual examination of power spectra or diffusion plots, providing more or less equivocal indications for the presence of LRD. Using ARFIMA modeling, we were able to statistically confirm the presence of LRD in our series. The percentage of series best fitted by an ARFIMA model and the sums of ARFIMA weights were clearly above the thresholds proposed by Torre et al. (2007). The fact that the most often selected model was the simplest one, meaning that the fractional parameter was sufficient to account for the correlation structure, gave additional support for the effective presence of LRD.

In order to obtain the most reliable estimation of the fractal exponents, we associated four methods implementing different algorithms. We particularly associated analyses in the frequency and the time domains, because discrepancies between these analyses are likely to reveal that series have no LRD (Rangarajan & Ding, 2000). Individually, all methods suggested the presence of $1/f^{\beta}$ noise, and the estimations of the fractal exponents given by the four methods were consistent and significantly correlated with each other. We believe these precautions are essential for an unambiguous characterization of fractal dependences in experimental series.

The reliability of fractal analyses tends to deteriorate for fewer than 2^9 point series (Delignières et al., 2006). While our present study concentrated on the demonstration of $1/f^{\beta}$ noise, future investigations examining the evolution of fractal exponents as function of different factors should be careful in interpreting the results: one can hardly assert that performing a coordination task in about 12 minutes (especially in anti-phase and/or at high frequency) would induce no fluctuation related to participants' motivation or attention.

Analyzing DRP series. In the introduction we argued for analyzing discrete rather than continuous relative phase. The analyses of the CRP series did not give any support for the presence of $1/f^{\beta}$ noise. In addition to situating series out of the range of $1/f^{\beta}$ noise, DFA and $^{low}PSD_{we}$ gave divergent results, providing a strong indication that LRD was not present (Rangarajan & Ding, 2002). Note also that the correlation structure of CRP was dramatically altered by a re-sampling procedure, a result which further points to the absence of scale invariance in these series.

With spectral indexes above 3.0, the results obtained with CRP series were consistent with those reported by Schmidt et al. (1991), indicating that they were indeed due to the relative phase measurement and not to the nature of the task. The spectral indexes characterized the series as non-stationary, which is inconsistent with the global stability of the performed coordination patterns. Thus, it seems more relevant to examine the stability properties in coordination using DRP series. A discrete measurement minimizes to some extent the direct influence of a coherent movement trajectory, and cycle-to-cycle fluctuations can be considered to represent the "functional variability" of coordination whereas continuous fluctuations would represent an "absolute variability" (Delignières et al., 2005). Our last analysis showed that the presence of $1/f$ noise in the DRP series was related to their event-based nature, and not to their typical observation time.

In the present study, DRP was determined once per cycle, at oscillation peak. One might ask whether the evidenced fractal fluctuations were related to global cycle-to-cycle dynamics or to the specific status of reversal points. In a complementary analysis, we computed series of DRP, determined at diverse phase angles of the dominant hand (from 0° to 315°, by steps of 45°), and we used DFA to estimate the fractal exponents. Figure 2 depicts the results obtained for the in-phase and anti-phase series. An ANOVA revealed a significant effect of phase angle, and post-hoc tests showed that exponents were higher in the peak region (0°, 45° and 315°) than in the rest of the cycle. There was no effect of coordination or interaction.

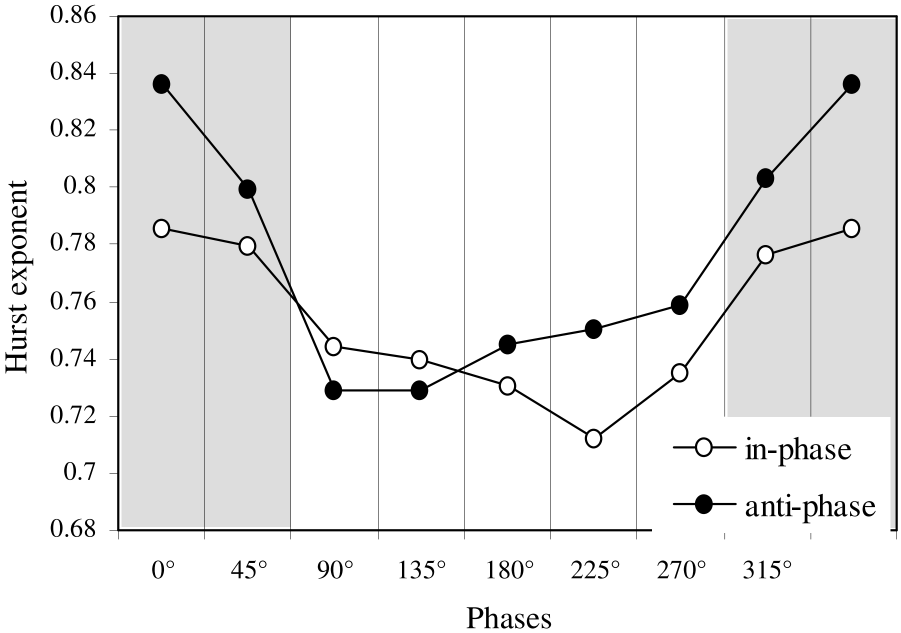

Figure 2. Fractal exponents of DRP series computed by DFA, at different phases in normalized cycles, for in-phase and anti-phase coordination.

Thus, whatever the performed pattern, DRP series were more strongly organized in time at the oscillation peaks. The peak region in our experiment corresponded to the reversals "on" the beep of the metronome. One could assume that this increase in LRD is related to a local stabilization (anchoring) in cycles near the pacing signal (Byblow et al., 1994; Carson, 1995).

We believe, however, that arguing for the use of DRP does not oppose the engagement of continuous coupling processes between oscillators. Such continuous coupling is assumed in the HKB model. This hypothesis has received strong support from numerous studies that have evidenced the role of continuous perceptual coupling or the use of the relative direction of movement in performing bimanual coordination (Huys et al. 2005; Wilson et al. 2005). Nevertheless, coupling can be continuous without being homogeneous over the cycle and without excluding the contribution of other discrete coupling processes. In this regard, note the hypotheses such as timing-process based coordination (*e.g.*, Semjen, 2002) or anchoring with sensory information provided by external pacing. The DRP determined at reversal points may allow the global outcome of all continuous or discrete coupling processes to be captured and shows the highest LRD between cycles.

The finding of 1/f $^{\beta}$ noise in neurobiological coordination research. Stability constitutes a key point in theories about the organization and control of coordination, and 1/*f* $^{\beta}$ noise represents an alternative framework for thinking about stability. Relative phase not only fluctuates around a reference value, but it possesses statistical memory and a certain predictability. The statistical specificity of 1/*f* $^{\beta}$ noise appears to be the expression of a functional variability that guarantees the simultaneous maintenance and adaptability (Goldberger et al., 1999) of "stable" coordination, variability not being synonymous with less or worse controlled coordination (Riley & Turvey, 2002).

We would like to focus on two lines of thinking about this finding in regard to neurobiological coordination research. Among the different accounts of the neural basis for coordination, the *intermanual crosstalk* hypothesis postulates that interaction between functional elements at different levels of organization constitutes a basis for emergent cognitive/behavioral functions (Bressler & Kelso, 2001; Varela et al., 2001). The nervous system integrates multi-level information into globally coherent brain states whose spatio-temporal dynamics have been shown to be linearly related to behavioral dynamics (Banerjee & Jirsa, 2006; Fuchs, Jirsa & Kelso, 2000; Kelso et al., 1998). For example, neural equivalence of the phase transition in bimanual oscillation has been evidenced in the recording of brain activation patterns (Jirsa, Fuchs & Kelso, 1998; Daffertshofer, Peper & Beek, 2005). It therefore seems worthwhile to take into account the specific correlation structures in behavioral state variables when investigating and modeling the connections between the level of neural dynamics and the level of effector dynamics. In this view, for instance, it appears appealing to conceive of $1/f^{\beta}$ noise as having much in common with the idea of *metastability* in neural dynamics (*e.g.*, Bressler & Kelso, 2001; Friston, 1997). Indeed, metastability, defined as a weak interdependency between constituents that gives rise to transient neural states, thereby providing the brain with its adaptive capacity, matches the description of the functional properties that are currently attributed to $1/f^{\beta}$ noise (Beltz & Kello, 2006; Goldberger, 1999; Schmidt et al., 1991). One might also question the contributions of specific brain rhythms and meaningful temporal scales in the dynamics of engaged brain regions to the frequential composition of the produced coordination series. Notably, the alteration of long-range correlation in behavioral outcome due to altered CNS control induced by neurodegenerative disease, as shown in human gait dynamics (Hausdorff et al., 1997), may suggest a direction for exploring the neural basis of coordination.

In another line of thought, the finding of $1/f^{\beta}$ noise offers an original window for examining the relationship between the component level and the collective level in coordination. Generally speaking, coordination engages a partly separate control of intra-limb synergies (Tseng & Scholz, 2005) or intra-limb timing processes (Ivry & Richardson, 2002). Riley et al. (2001) argued that variability in relative phase is likely to have deterministic sources at the oscillator level. Now, it has been shown that the series of periods produced in self-paced unimanual oscillation and tapping present a $1/f^{\beta}$ structure (Delignières et al., 2004). One can then wonder whether $1/f^{\beta}$ noise in relative phase is only the direct consequence of the intra-limb correlation structure. However, when a metronome paces the movement, the $1/f^{\beta}$ structure in periods disappears and becomes anti-persistent noise (Chen et al., 1997; Torre, Lemoine & Delignières, 2006). On the one hand, this raises the question of the actual relevance, and the possible influence on coupling processes, of the common "non-specific" use of the metronome in bimanual coordination experiments. On the other hand, on this basis we propose to examine the coherence of the correlation structures in single limb movement vs. coupled limb movement (article in preparation), in order to evaluate the contribution of the characteristic intra-limb dynamics to coordination and/or, conversely, the modifications induced by coupling.

Bimanual coordination models and $1/f^{\beta}$ noise. If $1/f^{\beta}$ noise represents a functional variability in coordination dynamics, the present results suggest the need to reorient current modeling efforts by considering long-range correlation structure as a crucial criterion. Because $1/f^{\beta}$ noise is quite a ubiquitous manifestation, one can hardly assume that a single model would account for this specific process over the whole range of the biological and physical systems it characterizes. Before aiming to develop new models, it thus seems essential to examine those models that have already received strong theoretical support by research into bimanual coordination.

First, we addressed the HKB model in its coupled oscillators formulation (Schöner & Kelso, 1988). The differential equation of a hybrid Rayleigh-Van der Pol oscillator (Kay et al., 1987) models the limit cycle behavior of each limb:

$$\ddot{x} + \delta\dot{x} + \lambda\dot{x}^3 + \gamma x^2\dot{x} + \omega^2 x = 0 \qquad (2)$$

In this equation, δ is a linear damping parameter, λ and γ are the Rayleigh and Van der Pol parameters, respectively, and ω^2 is a linear stiffness parameter. The two oscillators are coupled via a nonlinear function determined by parameters a and b, as follows:

$$\begin{aligned} &\ddot{x}_1 + \delta\dot{x}_1 + \lambda\dot{x}_1^{\,3} + \gamma x_1^{\,2}\dot{x}_1 + \omega^2 x_1 \\ &\quad = (\dot{x}_1 - \dot{x}_2)[a + b(x_1 - x_2)^2] \\ &\ddot{x}_2 + \delta\dot{x}_2 + \lambda\dot{x}_2^{\,3} + \gamma x_2^{\,2}\dot{x}_2 + \omega^2 x_2 \\ &\quad = (\dot{x}_2 - \dot{x}_1)[a + b(x_2 - x_1)^2] \end{aligned} \qquad (3)$$

The coupling function assumes that the differences between the positions and velocities of the two oscillators are continuously used for coordinating. Intrinsic fluctuations are accounted for by adding a white noise term of variance Q to each equation in (3).

We simulated 100 in-phase and 100 anti-phase series using a 4th order Runge-Kutta routine, with δ = -0.4, λ = 0.001, γ = 0.38, a = 0.1, and b = 0.47. In order to approximate our experimental oscillation frequencies, we set $\omega = 3\pi$. White noise variance was set to Q = 0.5. These parameters allowed to reproduce experimental DRP variability, with SDs of 5.90 (± 0.47) and 9.96 (± 2.90) in in-phase and anti-phase series, respectively. To test the series for the presence of $1/f^{\beta}$ noise, we applied $^{low}PSD_{we}$, DFA, and ARFIMA modeling. For in-phase series, we obtained mean β = 1.10 (± 0.24) and mean α = 0.88 (± 0.13) and for anti-phase series, mean β = 1.73 (± 0.25) and mean α = 1.12 (± 0.11). Accordingly, the in-phase series were situated at the frontier between fGn and fBm, while the anti-phase series were clearly characterized as fBm by both methods. Because ARFIMA modeling works on stationary series, we applied the method on the original in-phase series and on differentiated anti-phase series. LRD was detected in only 3% of the in-phase series and in 72% of the anti-phase series. For the latter, the mean estimated d was -0.18 (± 0.15), confirming the classification of anti-phase series as anti-persistent motions. Accordingly, simulated DRP series could not be characterized as $1/f^{\beta}$ noise and presented contrasted correlation structures for in-phase and anti-phase.

Second, we addressed the *phase attractive map* developed by deGuzman and Kelso (1991) for multifrequency bimanual coordination. The model obeys the following equation:

$$\phi_{n+1} = \phi_n + \Omega - \frac{K}{2\pi}(1 + A\cos 2\pi\phi_n)\sin 2\pi\phi_n \quad (4)$$

where ϕ_n (in grade) represents DRP at the n^{th} cycle, and Ω the ratio between the eigenfrequencies of the two effectors. The performed coordinations being isofrequential in our study, Ω was neglected in simulations. A was assumed to represent task-induced constraints and K the intrinsic coupling strength between the two oscillators. deGuzman and Kelso (1991) showed that the model is bi-stable for $1<A<10$ and $0<K<1$. We added a stochastic term with variance Q, representing the random perturbations that affect relative phase at each iteration.

We simulated 100 in-phase and 100 anti-phase series, using A = 2, K = 0.15, and Q = 0.0002. These parameters allowed to reproduce the experimental variability: converted into degrees, the mean SDs were 6.09 (± 0.16) for in-phase series and 10.14 (± 0.71) for anti-phase series. $^{low}PSD_{we}$ and DFA gave consistent results, with mean β = 0.21 (± 0.18) and mean α = 0.56 (± 0.08) for in-phase series, and mean β = 0.94 (± 0.15) and mean α = 0.70 (± 0.09) for anti-phase series. These results contrast with our experimental results and suggest that the in-phase series were white noise, whereas the anti-phase series were likely to be $1/f^{\beta}$ noise. However, ARFIMA modeling detected LRD in only 17% of the in-phase and 6% of the anti-phase series, indicating that neither in-phase nor anti-phase series possessed genuine LRD.

Finally, we addressed the *multiple timer model* proposed by Ivry and Richardson (2002), which is a direct extension of the Wing and Kristofferson (1973) model for unimanual tapping. A central timekeeper is assumed to be associated with each effector. Such a timer is conceived as a threshold/activation mechanism: an activation level increases linearly with constant speed a, affected at each iteration by a noise term ε with variance q, until the reaching of a threshold level T that determines a particular time event. In coordination, it is assumed that a *gating process* integrates the thresholds (T_1 and T_2) and the activation speeds (a_1 and a_2) of the two effectors, so that the inter-event intervals are given by:

$$C = \frac{(T_1 + T_2)}{(a_1 + \sqrt{q}\varepsilon_1) + (a_2 + \sqrt{q}\varepsilon_2)} \quad (5)$$

The successive cognitive events generated by this integrated timer trigger the motor responses of both limbs and reset the activation processes. Finally, the response of each effector is affected by random motor noise with variance Q. Note that the original *multiple timer model* only accounts for in-phase coordination.

We simulated 100 series using a = 1.5, q = 0.1, T = 1000, and Q = 7. These parameters allowed us to reproduce the experimental variability, with a mean SD of 5.22 (± 0.21). $^{low}PSD_{we}$ and DFA gave consistent results [mean β = 0.00 (± 0.22), mean α = 0.48 (± 0.08)], indicating that the simulated DRP series were white noise. These results were confirmed by ARFIMA modeling, with only 17% detection of LRD.

None of the three tested models was able to account for the $1/f^{\beta}$ noise in DRP series. Different kinds of mechanisms have been proposed to generate series with $1/f^{\beta}$ structure. One type of model is based on the aggregation of multiple short-range or random processes working at different time scales. For example, the aggregation of autoregressive (Granger, 1980), moving-average (Wing et al., 2004), or white noise (Hausdorff & Peng, 1996) processes have been proposed to produce $1/f^{\beta}$- like series. However, Wagenmakers et al. (2004) emphasized that this type of mechanism could *mimic* $1/f^{\beta}$ series without generating genuine LRD. Another type of model is based on the assumption of the non-stationarity in the system and associates a random non-stationary process with a simple auto-regressive process. For example, West and Scaffetta (2003) showed that performing a random walk on a Markov chain generates $1/f^{\beta}$ series. In the same vein, Wagenmakers et al. (2004) proposed an activation-threshold model, with a non-stationary threshold and an autoregressive activation process that generated LRD.

The objective, however, should not be to imitate the $1/f^{\beta}$ outcome, but rather to give a meaningful account of it in bimanual coordination. The challenge is thus to account for a feature that is a typical outcome of complex systems by using specific mechanisms in a necessarily simplified model. To overcome the limits of models in accounting for $1/f^{\beta}$ noise in DRP series, and to exploit the alternative bimanual coordination models by using systematic experimentation-modeling interaction, further investigation should address the evolution/alteration of $1/f^{\beta}$ properties with different task-specific and system-specific constraints.

CONCLUSION

We clearly demonstrated that DRP series in bimanual coordination possess a $1/f^{\beta}$ correlation structure. This type of statistical long-range memory has been evidenced in a number of natural phenomena and human behavioral outcomes. It appears that the fluctuations inherent to our perceptual-motor system are more likely to be correlated than to be random noise reflecting, as such, an aspect of the stability properties in bimanual coordination. Therefore, the models that aim to reproduce these stability properties and account for the processes that they entail should consider $1/f^{\beta}$ noise in relative phase as a crucial criterion.

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Transition part

In the present section, let us briefly consider some unpublished results of this study to continue with the two lines of thought raised in the discussion of the article: (i) $1/f^{\beta}$ noise as *an alternative framework for thinking about stability*; (ii) $1/f^{\beta}$ noise as *an original window for examining the relationships between the component level and the collective level in coordination.*

First, as mentioned repeatedly, $1/f^{\beta}$ noise has commonly been associated with 'normally' functioning systems and considered as an indicator of health. So, if indeed "*the statistical specificity of $1/f^{\beta}$ noise appears to be the expression of a functional variability that guarantees the simultaneous maintenance and adaptability of ''stable'' coordination [...]*" as argued in the article, then one could expect that fractal exponents should be related to a known indicator of stability in coordination dynamics.

To test this hypothesis, we proceeded as follows[5]:

(1) We tested for inter-individual differences[6] in the mean Hurst exponents and the mean standard deviations computed over the series performed by each participant, all experimental conditions taken together.

(2) We verified that the mean individual Hurst exponents and standard deviations computed over all trials were not significantly correlated.

(3) We tested for correlation between the individual Hurst exponents and the individual transition frequencies (f_c) determined through the incremental protocol in the study, and between the individual standard deviations and transition frequencies.

According to the current theory of bimanual coordination dynamics, one could expect that the amplitude of variability of relative phase series in anti-phase condition performed near the transition frequency would be inversely correlated with transition frequencies. In the assumption that the series' correlation properties were functionally meaningful in terms of

[5] The experiment that supported the above article was only partly exploited for publication. It actually comprised two additional experimental conditions where participants performed in-phase and anti-phase coordination at a 'quasi-critical' oscillation frequency. As for the experimental conditions presented in the article, participants performed two trials in both conditions, and the quasi-critical oscillation frequency was determined in percentage of the individual transition frequencies (87.5% of f_c), so that anti-phase coordination was difficult but possible to maintain during the trial durations. The unpublished results we present in the present section include these additional data.

[6] To test for inter-individual differences we used a one-way ANCOVA, with repeated measures on the 'Participant' factor, using the highest interaction as error term (see Madison, 2004, for a similar analysis). In this analysis, the different trials are supposed to provide estimates of the fractal exponent and the standard deviation characterizing each participant. In order to control possible effects of dispersion of the analyzed variables due to the different experimental conditions, we used the mean value of the variable for all participants and per experimental condition as covariant.

adaptability/stability, one would rather expect a correlation between individual Hurst exponents and transition frequencies.

Results showed a significant effect of the 'Participant' factor on mean Hurst exponent ($F_{13,97} = 2.96$, $p<0.05$) and mean standard deviation ($F_{13,97} = 4.47$, $p<0.005$). This result indicates that the level of long-range correlation and the magnitude of variability represent an individual characteristics, over coordination modes and frequency. (Note that a similar result was previously reported for Hurst exponents by Madison (2004) in a unimanual tapping experiment). There was no significant correlation between the mean individual Hurst exponents and standard deviations of series ($r_{12} = 0.06$). Finally, individual transition frequencies were significantly correlated with mean individual Hurst exponents ($r_{12} = -0.58$, $p<0.05$), but not with mean individual standard deviations ($r_{12} = -0.10$) (see Figure 5).

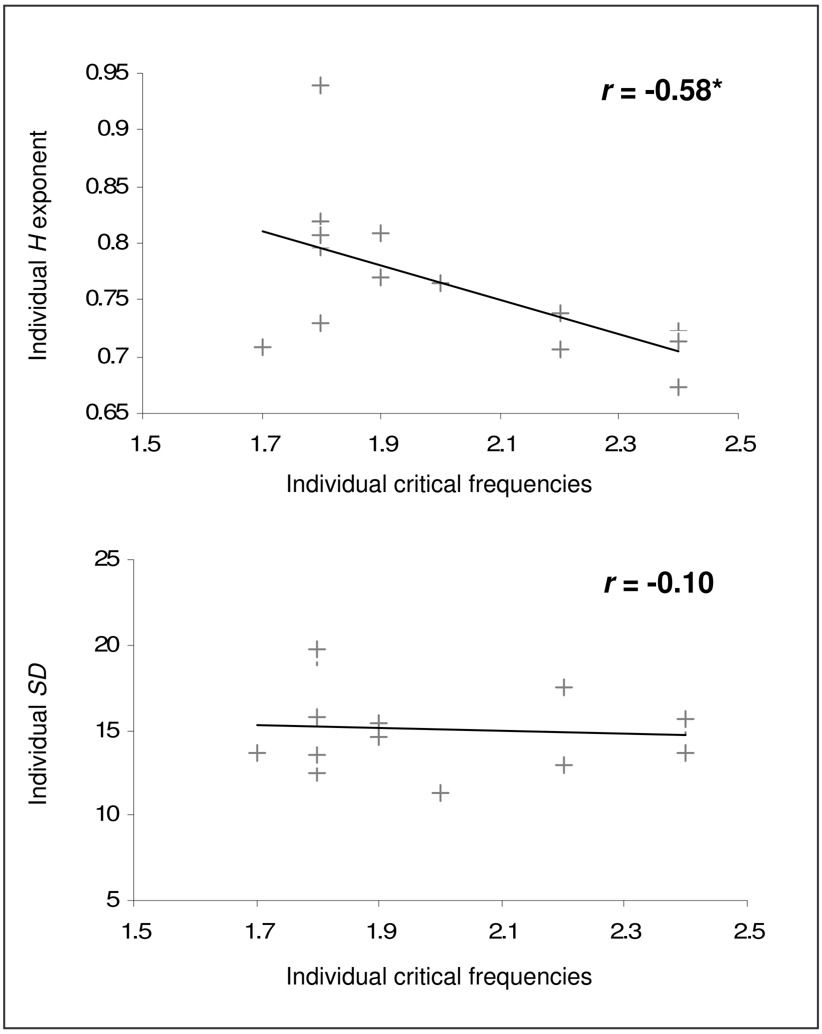

Figure 5. Graphical representation of correlations between the individual critical frequencies and the mean individual Hurst exponents of relative phase series (upper graph), and between the individual critical frequencies and the mean individual standard deviations of relative phase series (bottom graph). Only the individual fractal exponents are significantly correlated with transition frequencies.

This result suggests that long-range correlations may be a better predictor of the system's instability than the magnitude of variability in relative phase: indeed, the stronger were the persistent long-range correlations in relative phase, the more unstable was coordination, and the earlier occured phase transition. The intensity of long-range correlation in relative phase could be related to the 'difficulty' to couple effectors which, in turn, could be related to the intensity of correlations in the effectors' dynamics. Our personal experience in running simulations of the HKB coupled oscillators model tends to corroborate this idea. We introduced noise at the oscillator level of the model and observed that for identical model parameters, including the amplitude of noise introduced and the coupling parameters, simulated anti-phase coordination was stable in the case where variability was white noise while it was unstable in the case where variability contained long-range correlations. For similar amplitude of variability, coupling needed to be much stronger to yield stable relative phase when fluctuations contained long-range correlation.

Although these ad hoc results cannot be taken for sufficient support for any conclusion, they provide strong indication for current, empirical observations suggesting that the long-range correlation structure, in particular $1/f^{\beta}$ noise, reflects the system's functional properties in terms of stability and adaptability. This hypothesis remains to be tested using protocols designed specifically for this purpose. In the bimanual coordination paradigm, such protocol could consist of a perturbation study allowing to assess the covariance between relaxation times and fractal exponents. In practice, however, this kind of study presents some serious limitations due to the methodological requirements (length of series, stationarity) of fractal analysis.

Regarding the second line of thought: *$1/f^{\beta}$ noise as an original window for examining the relationships between the component level and the collective level in coordination*, we performed some complementary analyses of the data collected in the experiment, focusing on series of oscillation periods produced by the two hands during bimanual coordination. Figure 6 shows two individual examples of the power spectra of relative phase series (collective level) and the power spectra of the corresponding right and left hand periods (component level). Obviously, series of periods are featured by qualitatively different correlation structures in the two cases: they contain $1/f^{\beta}$ noise in example A, but anti-persistent fractional Gaussian noise (*i.e.*, negative correlations), as shown by the positive slope in the log-log power spectra, in example B. Despite this qualitative discrepancy in serial correlation at the component level, the relative phase series contain $1/f^{\beta}$ noise in both cases.

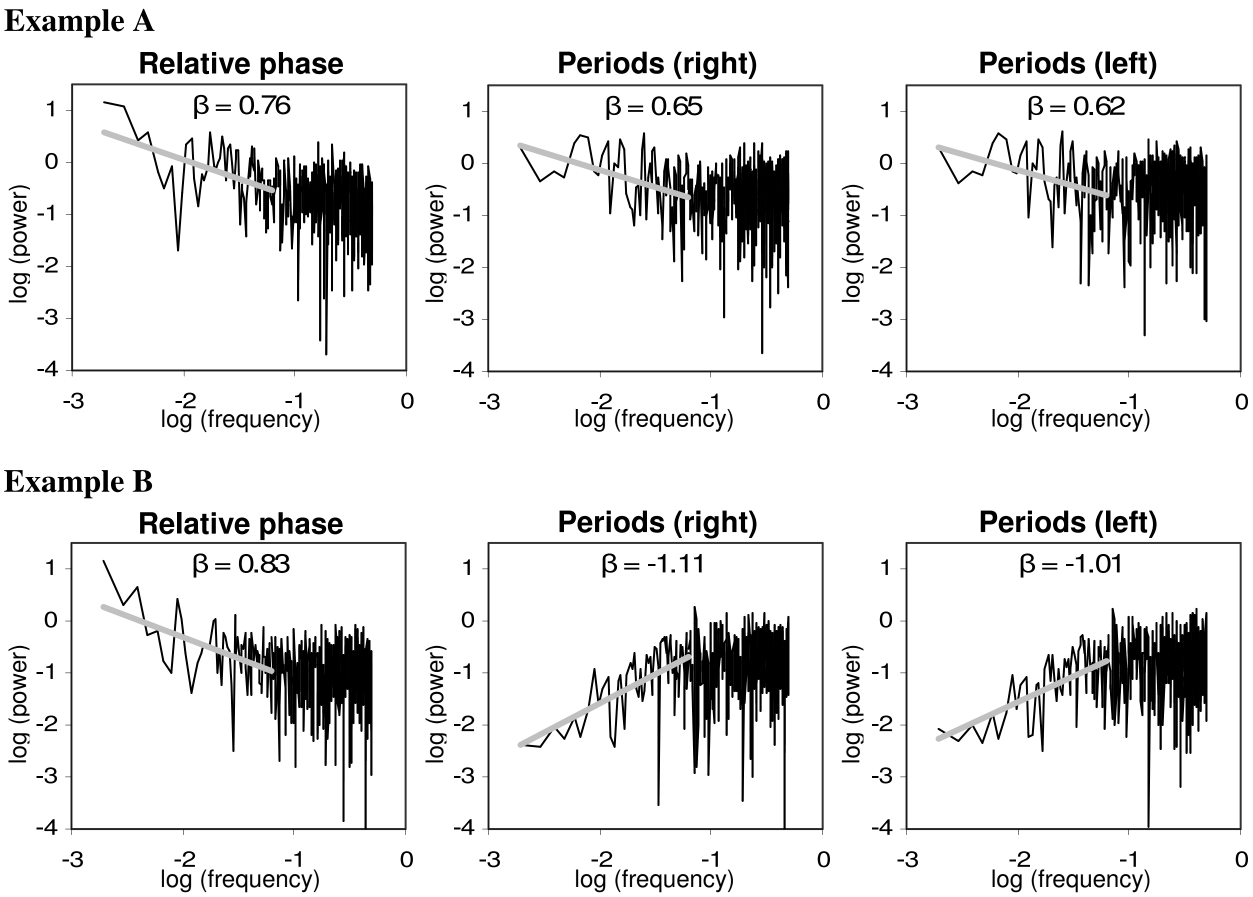

Figure 6. *The upper and bottom graphs show the power spectra of series of relative phase, right hand periods, and left hand periods collected during the performance of bimanual coordination by two participants. The relative phase series contain $1/f^{\beta}$ noise in both performances whereas the periods contain alternatively $1/f^{\beta}$ noise (upper graphs) or anti-persistent noise (bottom graphs).*

This preliminary result opens two ways for further investigation. The first one refers to the theory of metastability as a universal explanation of $1/f^{\beta}$ noise. Kello et al. (2007) contrasted two alternative accounts of $1/f^{\beta}$ noise: metastability and a multi-level account, *i.e.*, the mixture of independent processes at multiple time scales, and formulated strong predictions related to the two hypotheses. According to the authors, metastability predicts pervasive $1/f^{\beta}$ noise, meaning that it should be found at each level of observation of a system. In addition, they predicted that the $1/f^{\beta}$ fluctuations observed at the different levels should be distinct, uncorrelated and separately perturbable. They verified these predictions by analyzing simultaneously series of reaction times and key-contact durations in an experiment requiring repeated key-press responses, and concluded that the metastable basis of cognitive function accounted for $1/f^{\beta}$ noise. Our present result, in contrast, does not verify these predictions since $1/f^{\beta}$ noise is not systematically present at the two observation levels. Would that mean that bimanual coordination dynamics is not metastable? Or that metastability cannot be considered a *universal* explanation of the pervasiveness of $1/f^{\beta}$ noise? Or that Kello and collaborators' predictions have been too loosely formulated?

In other, more domain-specific respects, we know from unimanual timing studies that the external pacing of rhythmic taps induces a qualitative change in the correlation structure of produced periods: while periods contain $1/f^{\beta}$ noise in self-paced conditions, they contain anti-persistent noise in synchronization (*e.g.* Chen et al., 1997, 2001; Ding et al., 2002; Torre & Delignières, 2008). In the present study as in most bimanual coordination studies in literature, a metronome has been used to impose the tempo of movements. Then, obtaining similar results in bimanual coordination than in unimanual synchronized movements leads to question about the actual influence of the metronome on the stability properties of coordination dynamics (Fink, Foo, Jirsa, & Kelso, 2000), notably with regard to serial correlation properties. Especially, one may question whether/how a single coupling function formalizing the coordination of components allows to account for similar correlation structures in relative phase given qualitatively different correlation properties in the componential dynamics.

Apparently, the different ways we outlined to deal with our present results are keeping with two contrasted perspectives on $1/f^{\beta}$ noise in literature (Beltz & Kello, 2006; Kello, Anderson, Holden, & Van Orden, in press; Kello, Beltz, Holden, & Van Orden, 2007; Torre & Wagenmakers, in press): on the one hand, a *nomothetic* perspective, dealing with universal theories to account for the ubiquity of $1/f^{\beta}$ noise; on the other hand, a *mechanistic* perspective, dealing with the issue of how domain-specific models could account for the finding of $1/f^{\beta}$ noise. Therefore, before going on with investigating the role of timing control in bimanual coordination under the perspective of long-range correlation and $1/f^{\beta}$ noise, we need to discern the relevance of the two perspectives on $1/f^{\beta}$ noise, and to clarify which perspective we take for further studies. This is the purpose of Chapter 4.

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4 $1/f^{\beta}$ Noise: Psychomythics or Psychomodeling?

Theories and Models for $1/f^{\beta}$ Noise in Human Movement Science

Theories and Models for $1/f^{\beta}$ Noise in Human Movement Science

Kjerstin Torre[1] and Eric-Jan Wagenmakers[2]
[1] *EA 2991, University Montpellier 1, France*
[2] *University of Amsterdam, The Netherlands*

Abstract. Human motor behavior is often characterized by long-range, slowly decaying serial correlations or $1/f^{\beta}$ noise. Despite its prevalence, the contribution of the $1/f^{\beta}$ phenomenon on human movement research has been rather modest and unclear. The goal of this paper is to outline a research agenda in which the study of $1/f^{\beta}$ noise can contribute to scientific progress. In the first section of this article we discuss two popular perspectives on $1/f^{\beta}$ noise: the nomothetic perspective that seeks general explanations, and the mechanistic perspective that seeks domain-specific models. We believe that if $1/f^{\beta}$ noise is to have an impact on the field of movement science, researchers should develop and test domain-specific mechanistic models of human motor behavior. In the second section we illustrate our claim by showing how a mechanistic model of $1/f^{\beta}$ noise can be successfully integrated with currently established models for rhythmic self-paced, synchronized, and bimanual tapping. This model synthesis results in a unified account of the observed long-range serial correlations across a range of different tasks.

Send correspondence to:
Kjerstin Torre
Faculty of Sport Sciences – University Montpellier 1
EA 2991, “Motor Efficiency and Deficiency”
700 Avenue du Pic Saint Loup
34090 Montpellier – France
Email: ktorre@univ-montp1.fr

1 Introduction

In the field of human movement science, researchers often quantify performance in terms of its accuracy and its consistency. For instance, consider a task in which a participant first listens to the beat of a metronome. When the metronome stops, the participant sets out to reproduce the metronome’s rhythm by tapping a finger. In this so-called continuation tapping task, performance may be measured by the average inter-beat interval of the participant relative to that of the metronome. Performance may also be measured by the variability of the participant’s inter-beat intervals; high variability is then associated with low consistency and poor performance.

Despite the fact that accuracy and consistency are important global indicators of successful performance, neither of the two measures directly relates to the trial-to-trial dynamics of the system under investigation. In fact, the standard statistical analysis of consistency tacitly assumes that trial-to-trial dynamics are absent and that consecutive behaviors are unrelated. This tacit assumption is clearly false, however, as previous research has convincingly demonstrated the presence of a strong correlation between consecutive behaviors. Moreover, the nature of this correlation has been used to constrain theories of human movement production. In particular, several models for finger tapping were developed to accommodate the negative lag 1 correlation observed in consecutive taps (*e.g.*, the Wing and Kristofferson tapping model and its extensions; Wing and Kristofferson, 1973; Vorberg & Wing, 1996; Vorberg & Schulze, 2002).

Thus, it has long been known that human movement production leads to robust serial correlations that can inform us about the trial-to-trial dynamics of the underlying system. It is only recently, however, that researchers have started to examine more closely the specific kind of robust serial correlations observed in human movement production. Much of this recent work suggests that the serial correlations may be part of a special class known as $1/f^{\beta}$ noise. This particular class of serial correlations occurs throughout many widely different systems and signals the presence of fractal features such as *self-similarity* and *scale-invariance*. In addition, the presence of $1/f^{\beta}$ noise implies that the serial correlations decay so slowly that the generating system is called *persistent* or *long-range dependent*. In the field of human movement science, $1/f^{\beta}$ noise has been found in human force production (Gilden, 2001; Wing, Daffertshofer, & Pressing, 2004), unimanual rhythmic movement (*e.g.*, Chen, Ding, & Kelso, 1997, 2001; Delignières, Lemoine, & Torre, 2004; Lemoine, Torre, & Deligniéres, 2006; Gilden, Thornton, & Mallon, 1995; Ding, Chen, & Kelso, 2002; Yoshinaga, Miyazima, & Mitake, 2000; Yulmetyev, Emelyanova, Hänggi, Gafarov, & Prokhorov, 2002; but see Pressing & Jolley-Rogers, 1997), and the dynamics of bimanual coordination (Torre, Delignières, & Lemoine, 2007). In his keynote lecture at the 2007 European Workshop On Movement Science, Jeffrey Hausdorff argued for the diagnostic value of $1/f^{\beta}$ noise in the evaluation of human walking patterns (Hausdorff, 2007; see also Ashkenazy, Hausdorff, Ivanov, & Stanley, 2002; Hausdorff et al., 1996; West & Scafetta, 2003; 2004).

In human movement science, the phenomenon of $1/f^{\beta}$ noise can be approached from at least two different perspectives. The *nomothetic* perspective seeks general principles that explain the existence of $1/f^{\beta}$ noise across a range of different systems and behaviors. Proponents of the nomothetic perspective often explain the presence of $1/f^{\beta}$ noise by referring to the dynamic, self-organizing characteristics of the human nervous system (*e.g.*, Van Orden, Holden, & Turvey, 2003). The *mechanistic* perspective seeks domain-specific explanations for the existence of $1/f^{\beta}$ noise. Proponents of the mechanistic perspective explain the presence of $1/f^{\beta}$ noise by concrete modeling of the underlying processes that supposedly give rise to the serial correlations in the system under study (*e.g.*, Ashkenazy, Hausdorff, Ivanov, & Stanley, 2002; Delignières, Torre, & Lemoine, 2008; Wagenmakers, Farrell, and Ratcliff, 2004).

The primary goal of this article is to discuss the strengths and limitations of the nomothetic and mechanistic perspectives on $1/f^{\beta}$ noise. Note that we believe that both approaches have merit, and a direct comparison between the two is made difficult by the fact that the strengths of the nomothetic perspective corresponds to the limitations of the mechanistic perspective, and vice versa. Thus, the two approaches are best seen as complementary. Nevertheless, we hope to demonstrate that the choice of perspective influences the research agenda in important ways.

While it is true that nomothetic accounts have famously contributed to the understanding of what $1/f^{\beta}$ noise tells about a system's underlying dynamics, our focus here is on the possible contribution of the mechanistic perspective. We hope to convince the reader that the mechanistic perspective on $1/f^{\beta}$ noise can be useful for theories of human movement production.

In the first part of this article we outline the nomothetic and the mechanistic perspectives on the phenomenon of $1/f^{\beta}$ noise. We argue that domain-specific models of the data-generating process are useful not only to explain the observed behavior but also to bridge the gap between the observed data and the existing theories of $1/f^{\beta}$ noise (cf. Gisiger, 2001; Jensen, 1998; Wagenmakers, Farrell, & Ratcliff, 2005).

In the second part of this article we exemplify our line of reasoning by outlining a mechanistic model for $1/f^{\beta}$ noise in self-paced tapping, and by showing how this model can be extended to synchronization tapping and bimanual tapping. This work illustrates how $1/f^{\beta}$ noise can drive theoretical progress and result in a unified framework for the modeling of tapping tasks that are superficially quite different.

Signature of a $1/f^{\beta}$ Noise Process

In order to make this article self-contained, we first briefly discuss the signature of a $1/f^{\beta}$ process and how it differs from more mundane processes. For concreteness, consider a task in which the participant has to estimate a one-second time interval and do this repeatedly for 400 uninterrupted trials without any feedback. Figure 1, panel A, shows an example data set (Wagenmakers, Grünwald, & Steyvers, 2006). From the raw data, it is immediately evident that this data set displays considerable positive trial-to-trial correlation; that is, if the participant's estimate is relatively high at trial *n*, it is likely to still be

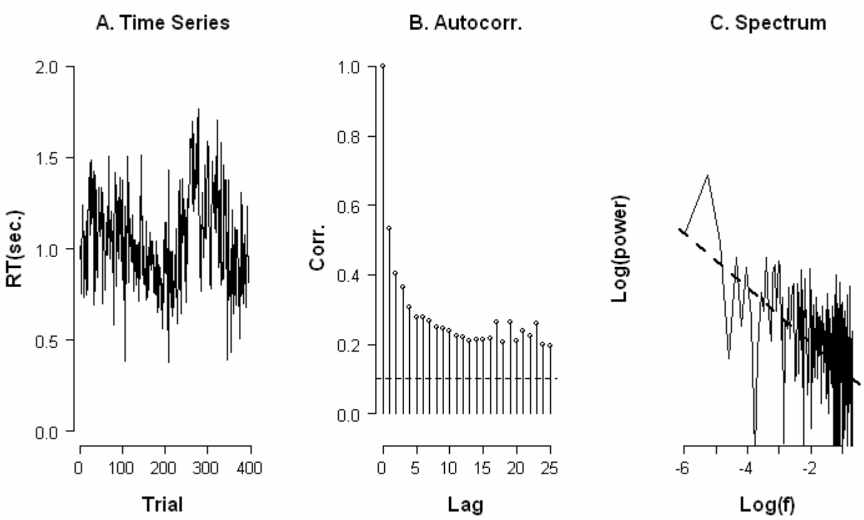


Figure 1. Example of a 1/*f* process. Left panel: 400 consecutive attempts at estimating a one-second time interval without any feedback (Wagenmakers, Grünwald, & Steyvers, 2006); Center panel: The autocorrelation function for the time series from the left panel; Right panel: The log-log power spectrum for the time series from the left panel. See text for details.

relatively high at trial *n+1* (*i.e.*, lag-1 *autocorrelation*). Figure 1, panel B plots all autocorrelations up to lag 25. This autocorrelation function shows that the correlation between different estimation attempts decreases with lag, but at a relatively slow rate. Moreover, a substantial autocorrelation remains even after more than 20 intervening estimation attempts. Finally, Figure 1, panel C shows the log-log power spectrum. In order to obtain this plot, the raw data from panel A are first decomposed into their constituent sine and cosine waves. Next, the frequency of the waves is plotted against their squared amplitude. A visual impression of the power spectrum shows that the low-frequency waves have the highest amplitude, which suggests that an increase of the measurement scale leads to the detection of more and more low frequency waves with high amplitude.

The features of the time series in Figure 1 are characteristic of a $1/f^{\beta}$ process (for details see Beran, 1994; Rangarajan & Ding, 2003). First, a $1/f^{\beta}$ process has an autocorrelation that decays so slowly that its sum does not converge to a finite number. Specifically, the correlation C with *k* intervening trials is given by a power function, $C(k) = |k|^{-\gamma}$, with γ between 0 and 1. This means that the process is *long-range dependent*. Second, the log-log power spectrum of a $1/f^{\beta}$ process is linear with slope -β, where β is usually taken to range from .5 to 1.5. Note that a sequence of independent normal deviates shows a slope of 0, whereas the cumulative sum of normal deviates (*i.e.*, a random walk) shows a slope of -2. A third feature of a $1/f^{\beta}$ process is that it is *self-similar*: the statistical properties of the time series are the same regardless of the scale of measurement, and hence the process lacks a characteristic time scale (*e.g.*, Maylor, Chater, & Brown, 2001).

The features that characterize the $1/f^{\beta}$ process are special. Traditional ARMA time series models, for instance, decompose the time series X_t as $X_t = \sum_{r=1}^{p} \phi_r X_{t-r} + \varepsilon_t + \sum_{r=1}^{q} \theta_r \varepsilon_{t-r}$, where ε is white noise and *p* and *q* indicate the order of the autoregressive and moving average components, respectively. In contrast to the $1/f^{\beta}$ process, an ARMA process has a characteristic time scale, a log-log power spectrum that levels off at the low frequencies, and an autocorrelation function that decays relatively quickly (*i.e.*, ARMA models are short-range dependent). Thus, at least in theory, the $1/f^{\beta}$ process is quite different from the standard type of time series process (Thornton & Gilden, 2005; Wagenmakers et al., 2004).

The $1/f^{\beta}$ process is special, not just because of its unique features, but also because the origins of the process are presently not well understood. In particular, it is not obvious how to construct a general model that produces perfect $1/f^{\beta}$ noise – most models require either detailed knowledge of the specific application, account for only a very specific range of values for β (*i.e.*, β = 1), or break down as the number of observations increases.

The air of mystique that surrounds $1/f^{\beta}$ noise – a special process whose precise origin is unknown – becomes even more intense when one considers that $1/f^{\beta}$ noise is found almost everywhere: examples of $1/f^{\beta}$ noise include electric current in transistors, water levels in the river Nile, the size of tree rings, brain activity as recorded by magnetoencephalogram, the stock market, music, and speech (*e.g.*, Handel & Chung, 1993; Hosking, 1984; Hurst, 1951; Novikov, Novikov, Shannahoff-Khalsa, Schwartz, & Wright, 1997; Voss & Clarke, 1975; Wolf, 1978).[1]

In cognitive psychology, evidence for long-range dependence was recently found in a range of tasks such as mental rotation, lexical decision, speeded visual search, estimation of distance, estimation of rotation, estimation of force, estimation of time, simple reaction time, speech production, and word naming (Gilden, 1997, 2001; Gilden, Thornton, & Mallon, 1995; Gilden & Hancock, 2007; Kello, Anderson, Holden, & Van Orden, in press; Kello, Beltz, Holden, & Van Orden, in press; Van Orden et al., 2003; but see Farrell, Wagenmakers, and Ratcliff, 2006). Long-range dependence has also been reported in day-to-day fluctuations in self-esteem (Delignières, Fortes, & Ninot, 2004), in the temporal dynamics of tics in Gilles de la Tourette syndrome (Peterson & Leckman, 1998), and in day-to-day fluctuations in selfmood of bipolar patients (Gottschalk, Bauer, & Whybrow, 1995).

Despite its special features, mysterious origin, and ubiquity throughout a range of different systems, the phenomenon of $1/f^{\beta}$ noise has often been ignored, both in experimental psychology and in human movement science, perhaps because of the belief that the phenomenon is inconsequential and erratic. However, Gilden (2001) has showed that, at least in certain experimental tasks, $1/f^{\beta}$ noise account for a large proportion of the observed variance, a proportion that is substantially larger than that caused by standard experimental manipulations. In addition, the intensity of $1/f^{\beta}$ noise (*i.e.*, the slope β) is known to change systematically as a function of certain experimental manipulations (*e.g.*, Chen, Ding, & Kelso, 2001; Hausdorff et al., 1996; Jordan, Challis, & Newell, 2006; 2007; Madison, 2001; 2004). Therefore, it appears as if – at least in domains where the phenomenon is well-established, such as in human movement science – the phenomenon of $1/f^{\beta}$

[1] A comprehensive bibliography of $1/f$ noise is maintained by Wentian Li at http://www.nslij-genetics.org/wli/1fnoise/.

noise deserves more attention than it has previously received.

2 Two Complementary Perspectives on $1/f^{\beta}$ Noise in Psychological Research

As mentioned in the introduction, the phenomenon of $1/f^{\beta}$ noise has been approached from either a nomothetic or a mechanistic perspective. In order to appreciate the differences in the interpretation of $1/f^{\beta}$ noise, it is important to clearly distinguish between these two perspectives without falling into caricatured oppositions. Both perspectives have value, and they are perhaps best seen as complementary rather than competing. In order to avoid any ambiguity from the outset, let us briefly clarify the distinction between the two perspectives. The nomothetic perspective focuses on the ubiquity of $1/f^{\beta}$ noise, and searches for general principles that account for its occurrence. The mechanistic perspective departs from the idea that, depending on the behavior under study, different causal mechanisms may be responsible for $1/f^{\beta}$ noise. In the mechanistic perspective, whatever causal mechanism one prefers, that mechanism needs to be modeled in enough detail to allow a quantitative test to data. Note that the distinction between the nomothetic and the mechanistic perspectives of $1/f^{\beta}$ noise is not the same as the distinction between *verbal* and *mathematical* accounts; indeed, nomothetic accounts are usually based on a substantive amount of mathematical formalization.

We acknowledge that the distinction between nomothetic and mechanistic perspectives on $1/f$ noise be not be one that is all-or-none – hybrid perspectives do exists, and there may be considerable shades of grey in between our black and white distinction. Nevertheless, we feel the paradigmatic distinction that we draw has face validity, as it maps on to different research agendas that we will outline in more detail below.

2.1 The Nomothetic Perspective on $1/f^{\beta}$ Noise

Researchers with a nomothetic perspective on $1/f^{\beta}$ noise promote a general explanation of $1/f^{\beta}$ noise. These researchers often stress the fact that the $1/f^{\beta}$ phenomenon is ubiquitous (*e.g.*, Gilden, 2001; Kello et al., in press; Van Orden et al., 2003), arguing that it is futile to try to explain $1/f^{\beta}$ noise using models that apply only in a limited domain, or, in other words, that "$1/f$ scaling is too pervasive to be idiosyncratic" (Kello et al., in press). The nomothetic tradition focuses, first, on empirically demonstrating the presence of $1/f^{\beta}$ noise, and, second, on explaining the presence of $1/f^{\beta}$ noise by referring to the behavior of complex systems, multiple interacting sub-systems, emergent dynamics, metastability, structure at different time scales, and self-organized criticality. Occasionally, the proponents of the nomothetic account argue that the framework of cognitive psychology should be abandoned in favor of the framework of nonlinear dynamical systems theory (*e.g.*, Van Orden et al., 2003).

The main attraction and an obvious strength of the nomothetic perspective is that it proposes general explanations of $1/f^{\beta}$ noise. Such a general explanation could potentially solve the mystery of $1/f^{\beta}$ noise by revealing what it is that the very different systems that display $1/f^{\beta}$ noise have in common. Here we discuss two general explanations of $1/f^{\beta}$ noise; *self-organized criticality* and *aggregation of short-range processes with different time scales*.

2.1.1 Self-organized criticality. In order to explain the ubiquitous presence of $1/f^{\beta}$ noise, the physicist Per Bak and his colleagues developed the concept of self-organized criticality (SOC; *e.g.*, Bak, 1996; Bak, Tang, & Wiesenfeld, 1987, Paczuski, Maslov, & Bak, 1996; but see Jensen, 1998, and Jensen, Christensen, & Fogedby, 1989; for a recent review with respect to biological systems, see Gisiger, 2001; see also Sornette, 2000). Systems with SOC can display $1/f^{\beta}$ noise, albeit only under specific conditions and only for specific dependent variables. The concept of SOC is aptly illustrated by the behavior of a particular pile of sand (*e.g.*, Jensen, 1998; Wagenmakers et al., 2005). This pile of sand is constrained by two orthogonal walls, so that it is bunched up in a corner. At random positions along the walls, new grains of sand are continually dropped onto the pile. At some point, the local slope of the sand pile exceeds a certain threshold and grains of sand are transported downhill until the local slope is again below threshold. This mechanism can cause avalanches of different sizes; when several adjacent slopes are near their local threshold, a single added grain of sand can lead to a chain reaction that brings about a cascade of avalanches.

The above pile of sand is said to self-organize to reach a critical state. Once in this state, small perturbations (*i.e.*, single grains of sand) may have large consequences (*i.e.*, a cascade of avalanches). Similar models have been proposed for evolution (*e.g.*, Bak & Sneppen, 1993; but see Davidsen & Lüthje, 2001), forest fires (*e.g.*, Malamud, Morein, & Turcotte, 1998), earthquakes (*e.g.*, Davidsen & Paczuski, 2002; Davidsen & Schuster, 2000, 2002), and populations of neurons (da Silva, Papa, & de Souza, 1998; Usher, Stemmler, & Olami, 1995). The above models all assume that the system of interest is gradually pushed toward a threshold, and that there are dominant interactions between many of the system's individual units. Hence, Jensen (1998) termed these kinds of models "*slowly driven, interaction-dominated threshold* systems" (p. 126).

Proponents of the nomothetic account often point to the advantages of SOC for neural networks: A neural network that is in a state of criticality is able to quickly reorganize and swiftly adapt to new situations (Alstrøm & Stassinopoulos, 1995; Bak & Chialvo, 2001; Chialvo & Bak, 1999; Linkenkaer-Hansen, Nikouline, Palva, & Ilmoniemi, 2001). Thus, it is argued, the presence of $1/f^{\beta}$ noise in human cognition or human motor behavior may signal SOC as the underlying design principle. This design principle is beneficial because it allows the system to adjust to changes in environmental demands.

The theory of SOC is elegant and attractive. Its main weakness, as was hinted at above, is that SOC systems generate $1/f^{\beta}$ noise only under specific conditions and only for specific dependent variables. For instance, the total mass of the self-organizing pile of sand shows $1/f^{\beta}$ noise across a wide range of frequencies (Jensen, 1998, pp. 30–42), but this only happens when the new grains are added along the two orthogonal walls. Surprisingly, the pile of sand does not show $1/f^{\beta}$ noise when the grains of sand are added to random positions on the interior of the pile (see Jensen, 1998, p. 42). Also, certain piles do not generate $1/f^{\beta}$ noise when they are made up of sand, but do generate $1/f^{\beta}$ noise when they are made up of rice (for details, see Jensen, 1998). The dramatic impact of such design details highlights the need for specific models of the underlying process. For the proponents of the nomothetic account, the lack of robustness with which SOC systems generate $1/f^{\beta}$ noise may take away some of considerable appeal.

2.1.2 Aggregation of short-range processes with different time scales. It has been known for a long time that $1/f^{\beta}$ noise can be produced by summing component short-range processes with different characteristic time scales (see, *e.g.*, Gardner, 1978; Jensen, 1998, p. 9; Van der Ziel, 1950; Wagenmakers et al., 2004, pp. 603-605). For example, consider a time series $X(t) = Y_1(t) + Y_2(t) + Y_3(t)$, where X denotes the observed output of the system as a whole, and the Ys indicate the output of individual subcomponents. The Ys may or may not be observed. Suppose that all Y are switching series, that is, they retain their value on the previous trial with probability $p = \exp(-1/\tau)$. When Y_1 is a quickly changing process with $\tau = 1$, Y_2 is an intermediate process with $\tau = 10$, and Y_3 is a slowly changing process with $\tau = 100$, the composite series $X(t) = Y_1(t) + Y_2(t) + Y_3(t)$ has structure at three different time scales – for a time series of about 1000 observations, this yields almost perfect $1/f^{\beta}$ noise (Wagenmakers et al., 2004, Figure 12).

Several models for $1/f^{\beta}$ noise in complex natural systems have exploited the above principle of aggregation. In biology, the model by Hausdorff and Peng (1996) assumes that heart rate fluctuations are subject both to relatively quick adjustments (*e.g.*, beat-by-beat) via the autonomic nervous system, and to relatively slow adjustments (*e.g.*, circadian rhythms) via hormonal systems. Ivanov, Nunes Amaral, Goldberger, and Stanley (1998) proposed a model for the regulation of heart rate through homeostasis that is based on similar principles. In cognitive psychology, Ward (2002) has promoted the principle of aggregation by distinguishing between fast fluctuating preconscious processes, slowly fluctuating conscious processes, and unconscious processes that operate on an intermediate time scale. In the movement sciences, Pressing (1999) has applied the principle of aggregation to explain the presence of $1/f^{\beta}$ noise in synchronous tapping.

The main benefits of explaining $1/f^{\beta}$ noise by aggregation of component processes is that such an explanation is conceptually transparent (*i.e.*, it demystifies the $1/f^{\beta}$ phenomenon), and focuses attention on the latent processes that influence the system's behavior. One main drawback of explaining $1/f^{\beta}$ noise by aggregation is that, as the length of the time series increases, more and more short-range processes need to be invoked to keep the spectrum from flattening at the low frequencies. In the limit of many samples, the aggregation approach is thus not very parsimonious. In addition, it could be argued that, in order to generate $1/f^{\beta}$ noise, the time scales of the component processes need to coordinate in just the right way – for instance, the above time series $X(t) = Y_1(t) + Y_2(t) + Y_3(t)$ only shows $1/f^{\beta}$ noise if the time scales for the Ys are sufficiently different.

This latter concern was addressed by Granger (1980; see also Robinson, 1978), who showed that "blind" aggregation of short-range processes can also produce $1/f^{\beta}$ noise. For instance, assume that the observed behavior X_t of a given system is just the sum of infinitely many component series, $X_t = \sum_{i=1}^{\infty} Y_t^{(i)}$. Further assume that each individual Y components is a first-order autoregressive process, $Y_t^{(k)} = \phi^{(k)} Y_{t-1}^{(k)} + \varepsilon_t$, and let $\phi^{(k)}$ be sampled from a beta distribution with sufficient mass near 1. Then the observed behavior X_t shows $1/f^{\beta}$ noise. This explanation of $1/f^{\beta}$ noise is popular in the fields of economics and finance (see, *e.g.*, Baillie, 1996), but it can easily be extended to human cognition and motor control; one only needs to make the plausible assumption that the observed behavior is jointly determined by many independent groups of neurons, each with their own different autoregressive decay parameter (cf. Chen, Ding, & Kelso, 2001; Ding, Chen, & Kelso, 2002).

2.1.3 Limitations of the nomothetic perspective. The nomothetic perspective is valuable in that it tries to formulate general explanations for a ubiquitous phenomenon. That is, the ubiquitous finding of $1/f^{\beta}$

noise in human coordination may be accounted for by the general hypothesis that the human nervous system displays self-organized criticality, just as sand piles and forest fires do. The counterpart of this level of generality is that it is to some extent accompanied by a detachment from the singularity of the phenomenon of interest. For instance, the skeptical researcher may wonder what exactly we can learn from the presence of $1/f^{\beta}$ noise in, say, human motor coordination, other than that human coordination shares certain statistical similarities with sand piles and forest fires. The mere fact that $1/f^{\beta}$ noise occurs throughout nature does not make the phenomenon psychologically meaningful (*e.g.*, Uttal, 2003).

2.2 The Mechanistic Perspective on $1/f^{\beta}$ Noise

Proponents of the mechanistic perspective explain the presence of $1/f^{\beta}$ noise by concrete modeling of underlying processes. These researchers point out that their purpose is to account for the workings of a particular psycho-physiological system, not solely to account for the $1/f^{\beta}$ noise the system may display. From this perspective, $1/f^{\beta}$ noise is just another finding that provides a useful constraint for modeling.

The need for concrete models, the proponents point out, is further motivated by some of the following concerns. First, concrete models produce concrete questions and concrete answers, as they are closely related to the phenomenon of interest. Using concrete models, the importance of discovering $1/f^{\beta}$ noise for, say, human motor coordination becomes much clearer. Second, concrete models are experimentally testable and falsifiable. Third, the majority of empirical studies do not find that experimental manipulations cause a discrete transition from pure $1/f^{\beta}$ noise (*i.e.*, $\beta = 1$) to uncorrelated white noise (*i.e.*, $\beta = 0$); instead, experimental manipulations often lead to a gradual shift of the exponent, such that the intensity of the long-range dependence might change from, say, $\beta = 0.8$ to $\beta = 0.4$. The explanation of this pattern of results requires a domain-specific model that takes into account the singularity of the observed behavior. Finally, there is no a priori reason why long-range and short-range dependence should be mutually exclusive, and the observed serial correlation are likely the result of both. In these cases, statistical models are needed to separate the long-range from the short-range components.

Here we discuss two accounts of $1/f^{\beta}$ noise that have been implemented as concrete models for specific tasks – the hopping model and the shifting strategy model.

2.2.1 The hopping model. The hopping model was developed to account for the long-range correlations observed in stride intervals in human gait (Ashkenazy et al., 2002; West & Scafetta, 2003). The model builds on previous theories of human gait dynamics. A *central patterns generator*, regrouping firing neuron centers, has been assumed to be responsible for the gait pattern, and the stride frequency in particular (Collins & Richmond, 1994; Hausdorff et al., 1995). The dynamics of gait cycles has been modeled by a forced Van der Pol oscillator (Guckenheimer & Holmes, 2002), so that the firing intensity of the neural centers was assumed to determine the eigenfrequency of the oscillator through its linear stiffness parameter (West & Scafetta, 2003). The hopping model was specifically designed for modeling the firing intensities delivered by the central pattern generator (Ashkenazy et al., 2002; West & Scafetta, 2003; 2005). The neural centers that compose the central patterns generator were assumed to deliver impulses of particular frequencies, which are mutually correlated. These impulses are modeled by the nodes δi of a Markov chain that obeys a first-order autoregressive process:

$$\delta i = \phi\, \delta i\text{-}1 + \varepsilon i, \qquad (1)$$

where $0 < \phi < 1$ is a constant, and ε_i is a white noise process. These neural centers were moreover assumed to be randomly activated. This assumption is implemented as a random walk along the Markov chain, with jump sizes obeying a Gaussian distribution of width ρ (see Figure 2). These random jumps, that gave the hopping model its name, thus generate a new series of values δ_n. These values correspond to the successively activated neural centers, and are injected into the Van der Pol oscillator. The hopping model accounts for the long-

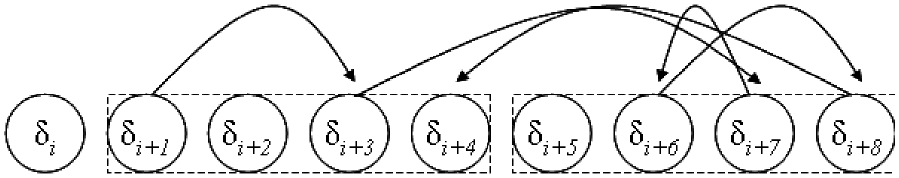


Figure 2. Illustration of the hopping model. The dashed boxes represent correlated zones whose size r is related to auto-regressive parameter determining the Markov chain by $r = -1/\log\phi$. The random walk activates successively the variable linear stiffness parameters δ_{i+1}, δ_{i+3}, δ_{i+7}, δ_{i+6}, δ_{i+8}, and δ_{i+4}, that are associated with the different neural centers.

range correlation evidenced in the stride interval series (West & Scafetta, 2003).

Although the processes that are engaged in the hopping model are quite simple – serial correlation basically arises from the combination of a random process and an autoregressive process – the model has been shown to generate genuine long-range correlation with systematic variations in intensity according to variations in parameters ϕ and ρ (Delignières et al., 2008). Regarding the theoretical interpretations of the hopping model, Ashkenazy et al. (2002) hypothesized that the range ρ of the random walk steps increases with neural maturation, accounting for developmental changes in the serial correlation of gait dynamics. Further studies showed that the parameterization of the external forcing function also explained the changes in serial correlation in normal, stressed, and externally paced gait conditions (West & Scafetta, 2003; 2004). Finally, $1/f^{\beta}$ noise has recently been found in the periods of unimanual self-paced oscillations. The dynamics of unimanual oscillations have commonly been modeled by a hybrid limit-cycle oscillator (Kay, Saltzman, Kelso, & Schöner, 1987). Delignières et al. (2008) proposed to inject the hopping model at the level of the oscillator's stiffness parameter, to account for the evidenced $1/f^{\beta}$ noise. According to the authors, the Markov chain could in that case be interpreted in terms of a chain of possible 'states' of the system, with neighboring states determined by similar factors and mutually correlated.

2.2.2 Shifting strategy model. Some models assume that processes show discrete transitions from one mode of operation (*i.e.*, a specific mean or variance) to the next. These so-called regime switching models have been extensively studied in the field of econometrics and finance, and were shown to closely mimic long-range correlation (Diebold & Inoue, 2000; Guégan, 2005; Gourieroux & Jasiak, 2001; Smith, 2005). The behavior of many natural systems, including human performance, often presents such form of nonstationary (*e.g.* Gilden & Wilson, 1995a; 1995b). Local nonstationarities, *i.e.*, changes in mean or variance that occur on relatively short time scales, are moreover typical for $1/f^{\beta}$ fluctuations. Examining serial correlations thus might reveal local nonstationarities as an integral part of persistent long-range correlation structures (for details on the relationships between nonstationarity and long-range correlation see Beran, Feng, Franke, Hess, & Ocker, 2003).

In order to account for $1/f^{\beta}$ noise in temporal estimation tasks, Wagenmakers et al. (2004) proposed a *shifting strategy* model. This model is an extension of the regime switching models and the classical activation-threshold models. First, it is assumed that, over the course of the temporal estimation task, participants repeatedly change strategies. During the time that they are in use, the different strategies are associated with particular threshold levels that determine the criterion amount of temporal information that has to be accumulated for a response. The threshold thus presents plateau-like variations in time. Second, the speed with which the accumulation process approaches the current threshold is assumed to vary between the successive estimations. The successive time intervals are given by the ratio between the threshold and the activation speed.

In the following section of this article we propose the shifting strategy model as a unifying mechanistic account of the specific correlation structures evidenced in absolute and relative timing series, for different rhythmic movement tasks. For that reason we are going to detail the formal aspects of the model at that time. Nevertheless, we can already notice this model appears consistent with the nonstationarity observed in temporal estimation data (Madison, 2001), and that it was shown to generate long-range correlation (Wagenmakers et al., 2004).

2.2.3 Limitations of the mechanistic perspective. One possible pitfall of the mechanistic modeling perspective is that one may mistakenly believe that a good quantitative model fit equals qualitative or theoretical insight. It has often been pointed out that a good fit to the data is a necessary, but not a sufficient criterion for a model's usefulness (Roberts & Pashler, 2000). A consideration of a model's usefulness involves, for instance, also a consideration of the theoretical foundations of the model, a consideration of the extent to which the model points to new research directions, and a consideration of the generalizability of the model. When these aspects of a model start to play an important role, mechanistic models may potentially benefit by borrowing ideas that have been developed from within the nomothetic framework, although this is largely unexplored territory.

3 Mechanistic Account of $1/f^{\beta}$ Noise in Absolute and Relative Timing

In order to illustrate the mechanistic perspective on $1/f^{\beta}$ noise, we now discuss the modeling of serial correlations in absolute and relative timing in self-paced tapping, synchronization tapping, and bimanual tapping.

Historically, models for absolute and relative timing have not been designed to account for serial correlations or $1/f^{\beta}$ noise – the explanation of the negative lag one autocorrelation within the Wing and Kristofferson absolute timing framework being the exception that confirms the rule. Nevertheless, empirical research has found $1/f^{\beta}$ noise to be present both in timing tasks such as unimanual tapping (*e.g.*, Chen, Repp, & Patel, 2002; Gilden, 2001; Gilden, Thornton, & Mallon, 1995; Lemoine et al., 2006; Yamada, 1995), and in relative timing that requires bimanual coordination (Torre et al., 2007).

Thus, although the current models for absolute and relative timing capture several characteristic features of human performance, they have thus far ignored the observed patterns of serial correlation, and, in particular, the presence of $1/f^{\beta}$ noise. Here we show that the current models can be extended to account for $1/f^{\beta}$ noise, and that this extension is consistent and straightforward. In particular, we let the shifting strategy model guide the behavior of an internal timekeeper. In self-paced tapping, this model extends the Wing and Kristofferson model (Wing & Kristofferson, 1973); in synchronization tapping, it extends Vorberg and Wing's linear phase correction model (Vorberg & Wing, 1996); and in bimanual tapping, it extends Ivry's multiple timer model (Ivry & Richardson, 2002). For each model, we compare the pattern of serial correlations that it generates to the pattern of serial correlations generated by human participants (see Appendix A for details).

3.1 Self-Paced Tapping

In self-paced tapping, participants are required to reproduce the rhythm of a metronome after it has stopped. Time interval series produced in self-paced tapping are often described by the Wing and Kristofferson model (Wing & Kristofferson, 1973). This model assumes a timekeeper that triggers successive motor responses (*i.e.*, taps) by generating regularly spaced cognitive events. The execution of each tap is affected by a motor delay, so that the inter-response intervals IRI_n are given by

$$IRI_n = C_n + M_{n+1} - M_n, \qquad (2)$$

where C_n is the time that it takes the timekeeper to generate a cognitive event and M_n is the motor delay. The model does not elaborate on the specific role of the timekeeper; the assumption is that C_n and M_n are uncorrelated white noise processes.

Gilden et al. (1995) were the first to report that IRI series show $1/f^{\beta}$ noise. Figure 3 presents the typical shape of their power spectra. The characteristic shape of the spectrum is wedge-shaped with a negative slope, close to –1, in the low-frequency region, and a positive slope in the high-frequency region. The negative low frequency slope is though to reflect the contribution of a $1/f^{\beta}$ component, whereas the positive high frequency slope is thought to reflect a differenced white noise component (Delignières et al., 2004; Delignières et al., 2008). According to the Wing and Kristofferson model, the differenced white noise in the IRI series originates from the difference between motor delays on successive trials (cf. Equation 2). This led Gilden et al. (1995) to suggest that the timekeeper is not a source of white noise, but rather a source of $1/f^{\beta}$ noise (Gilden et al., 1995; Gilden, 2001, Delignières et al., 2004; Delignières et al., 2008). The origin of the $1/f^{\beta}$ noise was not modeled.

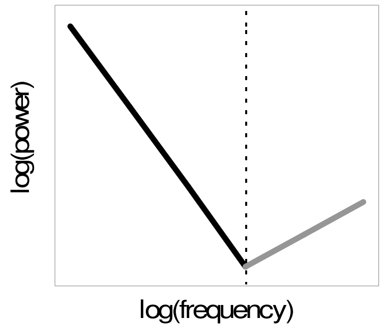


Figure 3. Shape of the typical log-log power spectrum of inter-response interval series in self-paced tapping.

3.1.1 Incorporating the Shifting Strategy model into the Wing and Kristofferson framework. In order to account for the long-range correlations in temporal estimation tasks, Wagenmakers et al. (2004) suggested a *shifting strategy model*. This model is an extension of the classical activation-threshold model (*e.g.*, Ivry, 1996; Schöner, 2002) in which an activation process grows linearly in time until it reaches a particular threshold level. This threshold crossing determines a cognitive event that triggers the motor response and resets the activation process (see Durstewitz, 2004, for neural plausibility). The shifting strategy model extends the activation-threshold mechanism in two ways.

First, as can be seen in Figure 4a, the threshold level is not constant, but it is assumed to be affected by a sequence of "cognitive states" that causes plateau-like deviations of variable amplitudes and durations from its baseline level. For each of the successively adopted cognitive states, the amplitude T' of the threshold deviation is sampled from a uniform distribution of range R; this deviation is maintained for a duration d_n that is uniformly sampled from an interval $[d_{min} ; d_{max}]$ of possible state durations. For each iteration, the current threshold is then given by

$$T_n = T_0 + T'_n, \qquad (3)$$

Second, the speed of the linear activation process is assumed to vary in an auto-regressive way around the baseline speed a_0:

$$a_n = a_0 + \varphi(a_{n-1} - a_0) + \lambda\, \varepsilon_n, \qquad (4)$$

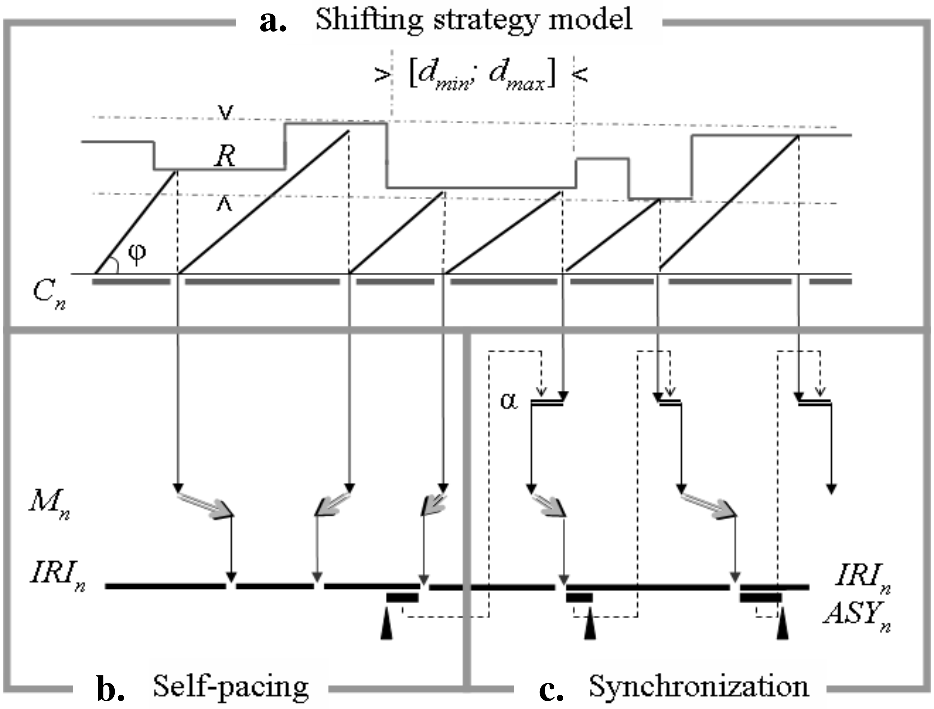
where φ is the auto-regressive parameter, and ε_i a centered white noise with unit variance. As in the classical activation-threshold mechanism, the activation process is reset after it crosses threshold.

Thus, the time it takes the timekeeper to generate a discrete cognitive event is determined by the ratio

$$C_n = T_n / a_n. \qquad (5)$$

The Wing and Kristofferson model can be easily combined with the shifting strategy model; all that is needed is to feed the timekeeper periods C_n (determined by Equation 5) into the Wing and Kristofferson model (*i.e.*, Equation 2). Figure 4b presents the complete model.

Figure 4. Illustration of (a) the shifting strategy model, and its the incorporation into (b) the Wing and Kristofferson model for self-paced tapping, and (c) the Vorberg and Wing model for synchronization tapping.



3.1.2 Model performance for real data. The Wing and Kristofferson shifting strategy model provides a satisfactory account of self-paced tapping data. The mean of IRI series collected in a self-paced tapping experiment was 481 ms, with a mean standard deviation of 33 ms (data from Delignières et al., 2008; see Appendix A for details). The mean of simulated IRI series was 501 ms, with an average standard deviation of 36 ms. As shown in Figure 5, the model also reproduced the characteristic wedge-shaped log-log power spectra[2]; thus, the model captures both the $1/f^{\beta}$ component and the differenced white noise component (Delignières et al., 2008). Specifically, the mean slopes for experimental and simulated series were –0.49 and –0.49 in the low-frequency region of the spectrum, and 0.41 and 0.39 in the high-frequency region.

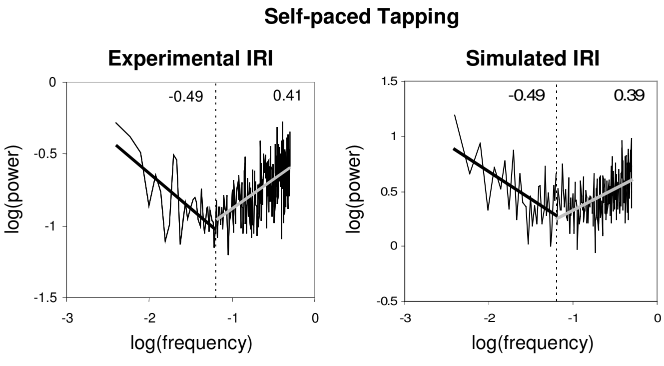


Figure 5. Average log-log power spectra of experimental (left) and simulated (right) inter-tap interval series, in unimanual self-paced tapping. Parameters used for simulation were $T_0 = 1000$, $a_0 = 2$, $R = 40$, $\varphi = 0.4$, and $\lambda = 0.08$ for the shifting strategy model. The standard deviation of the motor noise M was set to 20. We randomly chose 12 of the 100 simulated series for the computation of the average spectrum (recall that the experiment featured 12 participants).

3.1.3 Discussion. The extended Wing and Kristofferson model provided a satisfactory account of the serial correlation pattern that is typical for self-paced tapping. The model is specific in the sense that the long-range correlations are thought to result from a shifting strategy process at the level of the timekeeper (see Delignières et al., 2008, for details).

It is likely that extensions of the Wing and Kristofferson model other than the shifting strategy model could also have produced $1/f^{\beta}$ noise. We prefer the shifting strategy extension because it is concrete, simple, and conceptually close to common physical representations of time (Schöner, 2002). Simulations of the shifting strategy model showed that variations in the values of the model parameters (R for the threshold deviations, and φ for the activation process) cause systematic variations in the intensity of long-range correlations. For extreme values, the long-range correlations are extinguished altogether (Delignières et al., 2008). The mechanisms of the model map on to factors that are likely to

[2] Throughout this article, we quantify the serial correlations using the $^{low}PSD_{we}$ method (Eke et al., 2000; see Appendix B for details).

influence the functioning of the timekeeper (*i.e.*, available attentional resources, number of cognitive strategies used, or the length of the target intervals), and this mapping allows the model to be further tested in a qualitative fashion.

The central assumption of the present model for self-paced tapping is that the source of $1/f^{\beta}$ noise is at the level of the timekeeper. If this assumption holds, $1/f^{\beta}$ noise would also have to be present in any other paradigm that involves the timekeeper. To illustrate this point we now turn to the paradigm of synchronization tapping.

3.2 Synchronization Tapping

In the synchronization tapping paradigm, participants have to synchronize their taps with an external pacing signal that prescribes a regular tempo. In addition to an IRI series, the synchronization tapping paradigm also yields a series of asynchronies (ASY), which are defined as the time intervals between the participant's taps and the metronome's pacing signals.

In contrast to the IRI series in self-paced tapping, IRI series in synchronized tapping do not show $1/f^{\beta}$ noise – instead, these series show anti-persistent noise (*i.e.*, negative correlations). The ASY series, however, do show $1/f^{\beta}$ noise (Chen, Ding, & Kelso, 1997, 2001, 2002; Ding, Chen, & Kelso, 2002; Torre & Delignières, 2008). The IRI series and the ASY series actually contain similar information, since the IRI series correspond to the differentiation of asynchronies:

$$IRI_n = ASY_{n+1} - ASY_n + \tau, \qquad (6)$$

where τ is the constant period of the metronome. Thus, the anti-persistent noise in IRI series is the direct consequence of the presence of $1/f^{\beta}$ noise in the ASY series.[3]

One of the most used formal accounts of synchronization tapping is the linear phase correction model (Vorberg & Wing, 1996; see Repp, 2005, for a review). This model contains a timekeeper that is active in both self-paced tapping and synchronization tapping. Just as the Wing and Kristofferson model, the linear phase correction model can be extended to account for $1/f^{\beta}$ noise by assuming that the timekeeper follows a shifting strategy process.

3.2.1 Incorporating the shifting strategy model into the linear phase correction framework. Vorberg and Wing's (1996) linear phase correction model assumes that the Wing and Kristofferson framework for self-paced tapping also applies to synchronization tapping. The Vorberg and Wing model accounts for synchronization tapping by a local correction of asynchronies, meaning that the timekeeper periods are supposed to be unaffected by the synchronization process (Semjen, Schulze, & Vorberg, 2000; Semjen, Vorberg, & Schulze, 1998; Vorberg & Wing, 1996; Vorberg & Schulze, 2002). In the model's initial and simplest formulation, each asynchrony between tap and metronome signal is corrected at the following tap by a first-order autoregressive or AR(1) process:

$$A_{n+1} = (1 - \alpha)A_n + K_n - \tau, \qquad (7)$$

where τ represents the period prescribed by the metronome. In this equation, K_n represents the inter-response intervals predicted by the original Wing and Kristofferson continuation model (IRI_n in equation 2), that is, the intervals that would have been produced in the limit case where there is no effective synchronization process ($\alpha = 0$). The serial correlation in synchronization series thus result from the two-level architecture of the Wing and Kristofferson model, including the properties given to the timekeeper periods, plus the error correction processes.

Just as the original Wing and Kristofferson model, the Vorberg and Wing model conceptualizes the timekeeper as a source of uncorrelated white noise. This means that the model does not account for the correlation structures that have been observed in experimental ASY and IRI series (Torre & Delignières., 2008). Here we propose to extend the Vorberg and Wing model by incorporating the shifting strategy process at the timekeeper level. Figure 4c provides a graphical illustration of the shifting strategy model for synchronization tapping.

Note that Vorberg and Schulze (2002) further proposed a more complex version of the linear phase correction model, including a second-order autoregressive term and a feedback delay on the perceived asynchronies. However, our present purpose was not to determine the best fitting and most complete model for synchronization tapping, but rather to show that, by letting the timekeeper process generate $1/f^{\beta}$ noise, we can produce a consistent account of the serial correlations observed in both self-paced and synchronization tapping.

3.2.2 *Model performance for real data.* The Vorberg and Wing shifting strategy model provides a satisfactory account of synchronized tapping data. The mean of IRI series collected in a synchronization tapping experiment was 499 ms, with a mean standard deviation of 32 ms, and the mean of experimental asynchronies was -62 ms, with a mean standard deviation of 35ms. (see Appendix A for details on the experimental procedure). The mean of simulated IRI series was 500 ms, with an average

[3] Differencing a $1/f^{\beta}$ time series creates a $1/f^{\beta-2}$ time series, so that a persistent time series with a slope of –1.1, say, becomes an anti-persistent time series with a slope of + 0.9.

standard deviation of 26 ms, and the mean of simulated asynchronies was 2 ms, with a mean standard deviation of 26 ms. Figure 6 presents the average power spectra of experimental and simulated IRI and asynchrony series. For experimental series, the mean spectral slopes were 1.11 for IRI series, and –0.69 for ASY series. For the simulated series, the mean slopes were 1.38 for IRI series, and –0.95 for ASY series.

3.2.3 Discussion. The extended linear phase correction model provided a satisfactory account of the serial correlation pattern that is typical for synchronization tapping. As was the case for self-paced tapping, the model assumes that the long-range correlations result from a shifting strategy process at the level of the timekeeper.

Our modeling efforts show that the hypothesis of a simple autoregressive correction in synchronization is consistent with the finding of $1/f^{\beta}$ noise in asynchronies, contrary to what was previously assumed (Chen, Ding, & Kelso, 1997; Pressing & Jolley-Rogers, 1997). The extended linear phase correction model should be easily testable within the synchronization tapping paradigm, in particular with respect to experimental factors – such as use of an auditory versus a visual metronome, or tapping on a

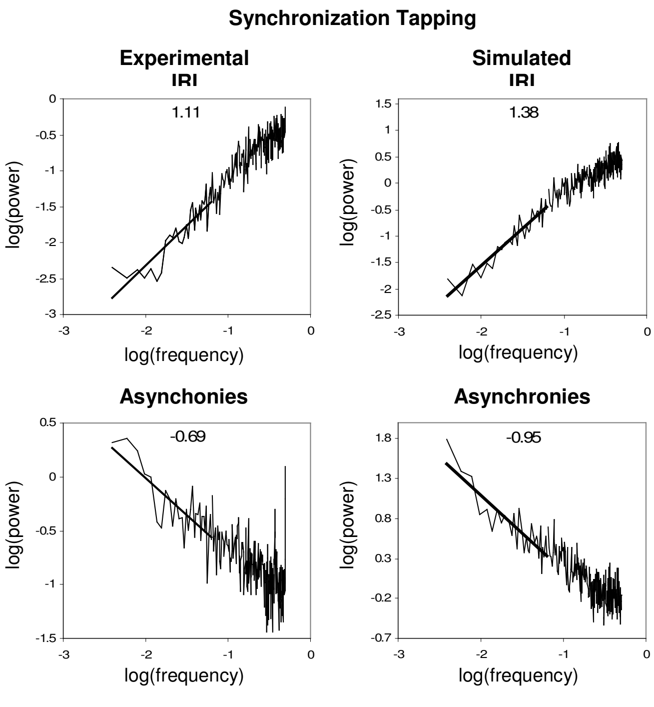


Figure 6. Average log-log power spectra of experimental (left) and simulated (right) inter-tap interval series (top) and asynchronies (bottom), in unimanual synchronization tapping. Parameters used for simulation were $T_0 = 1000$, $a_0 = 2$, $R = 40$, $\varphi = 0.4$, and $\lambda = 0.05$ for the shifting strategy model. The standard deviation of the motor noise M was 12 and the auto-regressive parameter α was set to 0.5. We randomly chose 12 of the 100 simulated series for the computation of the average spectrum.

contact surface versus air tapping – that are likely to influence the accuracy of the feedback on the produced asynchronies.

In the two *absolute timing* paradigms modeled so far, the timekeeper was assumed to function independently of the motor noise and the error correction process. That is, no constraint has been imposed on the shifting strategy component that generates the $1/f^{\beta}$ noise, and as a result the relationship between the correlations in the timekeeper series and the produced time interval series is relatively straightforward. We now consider a more complicated *relative timing* paradigm, in which two timers have been assumed to interact – the paradigm of bimanual tapping.

3.3 Bimanual Tapping

In the bimanual tapping paradigm, participants have to produce a constant phase relationship between the movements of the two hands. Here we consider in-phase coordination, meaning that the right and the left taps have to coincide. In addition to the IRI series of the two hands (*i.e.*, the component level), the bimanual coordination paradigm yields series of relative phase that describes the collective dynamics of the hand movements. The series of relative phase is defined as the time interval between the corresponding right and left taps, normalized by the completed IRI of the dominant hand.

The relative phase series has been the focus of most of the modeling work in bimanual coordination. Recently, this relative phase series was shown to display $1/f^{\beta}$ noise (Torre et al., 2007), a finding that speaks against the common prediction of a relative phase series that is white noise (*e.g.*, Schöner, Haken, & Kelso, 1986). Within an experimental paradigm influenced by dynamical systems theory (Haken, Kelso, & Bunz, 1985), one could interpret the finding of $1/f^{\beta}$ noise as evidence for self-organized criticality. Considering a two-level analysis of the collective dynamics and the component dynamics, the structure of correlations in the relative timing pattern is determined by the congruence of the correlation structures of the two within-hand timing patterns. Thus, the correlation structure in the relative phase series can be assumed to be (at least partly) caused by processes determining the within-hand temporal patterns (Riley, Santana, & Turvey, 2001).

Current perspectives on the organization of bimanual coordination highlight the role of feed-forward timing (Ridderikhoff, Peper, & Beek, 2005), and support the hypothesis that similar timekeeping processes could be at work in bimanual coordination and unimanual timing tasks (Helmuth & Ivry, 1996; Ivry & Richardson, 2002; Ivry, Richardson, & Helmuth, 2002; Semjen, 2002; Semjen & Ivry, 2001; Semjen & Summers, 2002). Ivry and collaborators (Helmuth & Ivry, 1996; Ivry & Richardson, 2002; Ivry & Richardson, & Helmuth, 2002) extended the

Wing and Kristofferson model for unimanual tapping to the paradigm of bimanual tapping. In Ivry et al.'s *multiple timer model* for bimanual tapping, a timer is responsible for the within-hand timing pattern of each hand, and a *gating process* implements a discrete temporal coupling that performs a sort of averaging of the two timers. A recent analysis of the serial correlation in the within-hand IRI series, however, showed that the absolute timing patterns in unimanual and bimanual tapping tasks are qualitatively similar (Torre & Delignières, in press). This finding suggests that by combining the *shifting strategy* model and the multiple timer model, one might be able to account for the serial correlations in both IRI and relative phase series that are observed in bimanual tapping.

3.3.1 Incorporating the shifting strategy model into the multiple timer framework. In the original multiple timer model, the timers associated with each hand were conceived as noisy activation-threshold mechanisms (Ivry & Richardson, 2002). The two timers produced white noise, and, in contrast to the unimanual case, they had no direct access to the effectors. A gating process ensures the temporal coordination by adding the thresholds (T_i and T_j) and the activation processes (a_i and a_j) of the two timers. When the integrated activation process reaches the normalized threshold, this marks the event in time that, in the case of in-phase coordination, triggers the taps of the two hands simultaneously. The time intervals prescribed by the gating process to the two hands are given by:

$$C = \frac{(T_i + T_j)}{(a_i + \sqrt{q}\varepsilon_i) + (a_j + \sqrt{q}\varepsilon_j)} \qquad (8)$$

where ε is a white noise with variance q. According to the Wing and Kristofferson model, the execution of the taps was assumed to be affected by white noise. This original multiple timer model obviously does not account for long-range correlations, whether in the IRI series or in the relative phase series (Torre et al., 2007).

In order to account for the correlation properties of IRI and relative phase series in bimanual tapping, we propose to extend the multiple timer model in two ways. First, in order to account for the specific correlation structure in the effectors' IRI series, we replaced the noisy activation-threshold mechanisms that govern the behavior of the two timers by shifting strategy processes. However, since the multiple timer model assumes that the two timing processes merge into a single common signal for the two hands, the correlations in the relative phase series are only determined by the motor noise that affects the taps. That is, the model in its original configuration only generates white noise in the relative phase series, whatever the correlation properties of the time intervals generated by the two timers.

Thus, the goal of the second extension is to couple the two timing processes that account for correlations in the relative phase series, without altering the correlation properties of the IRI series. Therefore we proposed a continuous coupling of the thresholds, on the basis of their difference at each time:

$$T_{i,n} = T_{i,n} - \theta(T_{i,n} - T_{j,n})$$

$$T_{j,n} = T_{j,n} - \theta(T_{j,n} - T_{i,n}), \qquad (9)$$

where θ is the coupling parameter ($0 < \theta < 0.5$). This coupling increased the congruence between the correlation structures of the two timekeeper series. Moreover, in order to prevent the divergence that would automatically occur between two noisy series, that is, to make stationary the delays between right and left taps, the onset of the activation process associated with the first tapping effector is assumed to await the completion of the tap of the second effector:

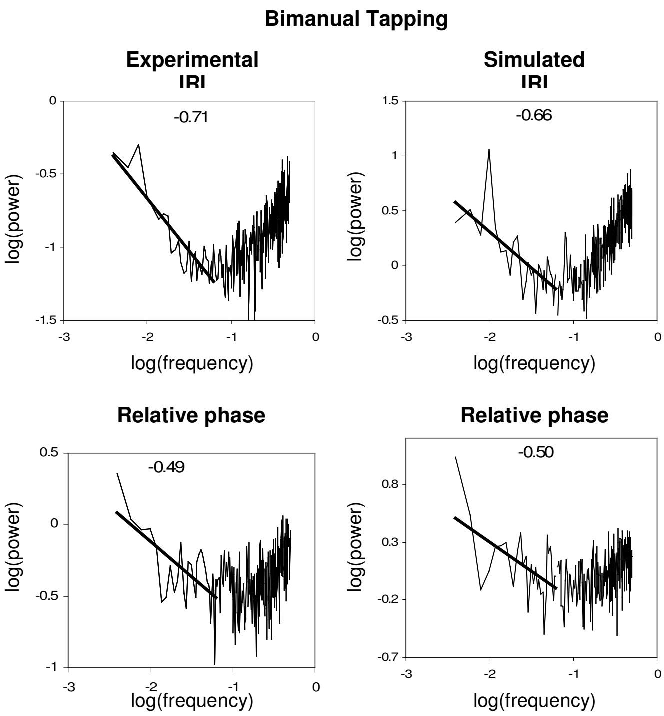


Figure 7. Average log-log power spectra of experimental (left) and simulated (right) inter-tap interval series (top) of the right hand, and relative phase series (bottom) in bimanual in-phase tapping. For the two coupled timers, the parameters used for simulation were $T_0 = 1000$, $a_0 = 2$, $R = 60$, $\varphi = 0.4$, $\lambda = 0.05$ for the shifting strategy model, and the standard deviation of the motor noise M was 12. The coupling parameter γ was set to 0.25. We randomly chose 12 of the 100 simulated series for the computation of the average spectrum.

$C_{i,n} = [T_{i,n} - \theta(T_{i,n} - T_{j,n})] / a_{i,n} + \delta_{j,i}$

for $\delta_{j,i} \geq 0$

$C_{j,n} = [T_{j,n} - \theta(T_{j,n} - T_{i,n})] / a_{j,n} + \delta_{i,j}$

for $\delta_{i,j} \geq 0$ (10)

In this equation, a_n is the current activation speed of each timer as defined in the shifting strategy model (see Equation 4). δ is the delay between the two effective taps:

$$\delta_{j,i} = (C_{j,n} + M_{j,n}) - (C_{i,n} + M_{i,n}), \quad (11)$$

where M_n represents the random motor delay affecting each tap.

3.3.2 Model performance for real data. The model provides a satisfactory account of bimanual tapping data. The mean of IRI series collected in a bimanual tapping experiment was 470 ms, with a mean standard deviation of 30 ms, the two hands taken together. The mean of experimental relative phase series was -12°, with a mean standard deviation of 53° (see Appendix A for details on the experimental procedure). The mean of simulated IRI series was 511 ms, with an average standard deviation of 30 ms, and the mean of simulated relative phase series was 0°, with a mean standard deviation of 22°. Figure 7 presents the average power spectra of experimental and simulated IRI and relative phase series. For experimental series, the mean spectral slopes were –0.71 for IRI series, and –0.49 for the relative phase series. For the simulated series, the mean slopes were –0.66 for IRI series, and –0.50 for the relative phase series.

3.3.3 Discussion. The extended multiple timer model provided a satisfactory account of the serial correlation pattern that is typical for bimanual tapping, both for the within-hand IRI series and for the relative phase series. The nature of the extension was twofold. First, just as in the case of unimanual tapping, we assumed that the two timers are the sources of $1/f^{\beta}$ noise, and we modeled their behavior through the shifting strategy model. Second, we determined the conditions of coupling the two shifting strategy models under which the correlation structures of both the IRI and the relative phase series matched the experimental correlations.

This development led us to formulate the coordination in bimanual tapping as a parallel organization with continuous interaction between the two coupled timers (as for example the HKB coupled oscillator model; Haken, Kelso, & Bunz, 1985), instead of the sequential organization assumed in the original multiple timer model, where a gating process locked the two timing process. Our present model first assumed a coupling of the thresholds of the two timers. This seems consistent with the above consideration that the evolution of the threshold in the shifting strategy model was related to factors as cognitive strategies or the target intervals to produce, since these factors can be assumed common to the two timers. Second, it was assumed that the onset of the activation process associated to the earliest of the two tapping effectors awaits the effective tap of the second effector. This effectively makes stationary the delays between the two taps. Moreover, this functioning could be easily extended to anti-phase tapping, by assuming that the activation processes of the two timers grow two times faster. In that case, the between-hand intervals would be regulated instead of the within-hand intervals, as suggested in earlier studies (*e.g.* Semjen, 2002; Semjen & Ivry, 2001).

Further experimental testing of this model appears more constraining than for the self-paced or synchronization models, since each factor would simultaneously influence the correlation structures of the absolute and the relative time interval series. Consider the example of directing attention on one of the two effectors. When one assumes that the autoregressive variations of the activation process in the shifting strategy model maps on to attentional fluctuations, one can expect that directed attention would increase the discrepancy between the correlation structures produced by the two timers. We computed the spectral coherence between the right and left IRI series collected in the present bimanual tapping experiment. The analysis showed very high coherence coefficients, with a mean r^2 of about 0.94 (SD = 0.08). In the condition of directed attention, this coherence coefficient should decrease. Moreover, since the correlation structure in the relative phase series is directly related to the coherence between the correlation structures of the two IRI series, the persistent correlations in the relative phase should increase in the same time.

4 Concluding Comments

The aim of this paper was to show how a mechanistic perspective on $1/f^{\beta}$ noise can advance theories of human movement production. We argued that domain-specific models are useful to establish a clear link between $1/f^{\beta}$ noise on the one hand and the substantive psychological and/or biological phenomenon on the other. We supported our claim by applying the shifting strategy model (*e.g.*, Wagenmakers et al., 2004) to three standard tasks in the field of human rhythmic movement production: self-paced tapping, synchronization tapping, and bimanual tapping. In all cases, we extended current models by assuming that the timekeeper undergoes a shifting strategy process.

It should be clearly acknowledged that nomothetic accounts of $1/f^{\beta}$ noise – those that seek general explanations and refer to concepts such as complex systems, emergent dynamics, metastability, and self-organized criticality – that such accounts certainly point to universal principles that produce $1/f^{\beta}$ noise in number of systems and phenomena which superficially have little in common. However, the general focus of the nomothetic accounts can sometimes make it difficult to respect and account for the idiosyncrasies of $1/f^{\beta}$ noise processes in specific applications.

A challenge for the nomothetic account is to handle the fact that $1/f^{\beta}$ noise is not observed always and everywhere. (*e.g.*, series of asynchronies from the present experiment on synchronization tapping contained $1/f^{\beta}$ noise, whareas periods did not). The intensity of $1/f^{\beta}$ noise is often sensitive to particular experimental manipulations, and may even be absent altogether (*e.g.*, Chen, Ding, & Kelso, 2001; Hausdorff et al., 1996; Jordan, Challis, & Newell, 2006; 2007; Madison, 2001; 2004). Clearly, it is unlikely that, as a result of these experimental manipulations, the human brain has ceased to be complex, multileveled, or metastable. This challenge for the nomothetic account may be more apparent then real; perhaps concrete modeling of the phenomena under consideration will confirm that the nomothetic concepts such as SOC are sensitive to the same manipulations that influence the intensity of $1/f^{\beta}$ noise in an experiment.

It may be possible to achieve a resolution between the nomothetic and mechanistic perspectives on $1/f^{\beta}$ noise by arguing that these perspectives operate on different levels of explanation. As we have hinted at throughout this article, the development of concrete, domain-specific models is not fundamentally at odds with the claim that $1/f^{\beta}$ noise originates from some universal principle. For instance, both in the hopping model and in the shifting strategy model, $1/f^{\beta}$ noise comes about through the combination of a simple autoregressive process and a regime switching process – that is, in both cases $1/f^{\beta}$ noise originates through the aggregation of processes that operate on different time scales.

In sum, the two perspectives on $1/f^{\beta}$ noise would gain in being considered complementary, the strength of the one defining the limitation of the other one. Nevertheless, the impact of $1/f^{\beta}$ noise on human movement science depends to a large extent on which perspective one adopts to account for the phenomenon. Domain-specific mechanistic models can not pretend to uncover the universal principles that account for the ubiquity of $1/f^{\beta}$ noise, as nomothetic accounts can do. In contrast, mechanistic accounts offer the advantages of specific, experimentally testable and thus falsifiable models of human behavior. Regardless of which perspective on $1/f^{\beta}$ noise one prefers, it is clear that current models of human behavior have not been designed to account for serial long-range correlations. But the $1/f^{\beta}$ noise phenomenon is there, and its presence constitutes a constraint that should be taken into account. Our article shows how one can model the presence and intensity of $1/f^{\beta}$ noise in human movement science in a way that is generalizable, testable, and quantitatively precise.

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Appendix A: Experimental Procedures

Twelve participants (mean age 29 ± 7 yrs), performed series of 600 taps (trial durations about 5 minutes) with their dominant hand index finger(s) on a flat pressure sensor disposed on a table. Three experimental conditions were randomly assigned.

(1) In the self-paced tapping condition, the target time interval was 500 ms. This interval was presented in a continuation paradigm: a 30-s video displayed the task to perform at the required tempo, and the participants had to reproduce the tempo as accurately and regularly as possible over the task duration, immediately after watching the video. We analyzed the produced inter-response interval (IRI) series.

(2) In the synchronization tapping condition, a PC-driven metronome delivered acoustic signals with a constant period of 500 ms, and the participants were instructed to synchronize their taps with the metronome. We analyzed the IRI series and the series of asynchronies, defined as the differences between the times of the effective taps and the signals of the metronome.

(3) In the bimanual tapping condition, participants had to perform in-phase coordination, *i.e.*, simultaneous taps, without an external pacing signal. Participants were instructed to be as accurate and regular as possible in the coordination of the taps and in the tempo of movements. We analyzed the absolute timing patterns of the hands with the IRI series, and the relative timing pattern with the relative phase series.

Appendix B: Data Analyses

We analyzed experimental and simulated time series of 512 points. The power spectra were computed using the $^{low}PSD_{we}$, initially developed by Eke et al. (2000). The method includes three preprocessing operations before performing the Fast Fourier Transform. First the mean of the series is subtracted from each value. Second, a parabolic window is applied to taper the series. Third, a linear detrending is performed on the entire series. Finally, in order to obtain a more accurate estimate of the spectral slope, Eke et al. perform a linear regression only on the low-frequency region of the log-log power spectrum. The low frequencies were defined as $f < 1/8$ of the maximal frequency that composes the signal. We also used this same boundary frequency to divide the spectrum in a low-frequency and a high-frequency region in case separate slope estimates were required.

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Transition part

So far in this document, we presented a methodological contribution for the detection and characterization of long-range correlation which led us to show that variability in relative phase performed in bimanual coordination exhibits $1/f^{\beta}$ correlations. Here, we contrasted different ways to deal with the finding of $1/f^{\beta}$ noise from a theoretical perspective, and we argued for the relevance and usefulness of domain-specific, mechanistic models to account for $1/f^{\beta}$ noise. On the one hand, in the nomothetic approach, the accent is on the pervasiveness of $1/f^{\beta}$ noise which is considered a property that in itself has to be accounted for. On the other hand, we consider $1/f^{\beta}$ noise as one among many other features of a behavior to which models have to live up. However, the two approaches are complementary rather than competing; the main issue may thus be to go for the perspective that offers the most pertinent entry point to answer a given scientific question. As a matter of fact, current models for timing and bimanual coordination have not been developed with regard to serial long-range correlation. In this respect, by giving up the idea of a universal account of $1/f^{\beta}$ noise, the mechanistic modeling approach allows to bridge the gap between long-range correlation or $1/f^{\beta}$ noise and current theoretical frameworks and related models for timing and coordination.

The aim of this thesis was to inquire into the role of timing processes in bimanual coordination using information provided by the characterization of serial correlation. Therefore, we stayed with a mechanistic approach to examine the distinctive correlation properties which feature the unimanual (or uncoupled components') timing, their theoretical implications, and plausible models to account for the empirical properties.

At this stage, we actually considered four unimanual timing situations which appeared particularly relevant to the bimanual coordination literature: (1) self-paced tapping, assumed to involve an event-based form of timing control (2) self-paced oscillations, assumed to involve a form of timing control known as emergent, (3) synchronization tapping, and (4) synchronization oscillations.

In a first study, we addressed the self-paced tapping and self-paced oscillation situations jointly (Delignières, Torre, & Lemoine, 2008. See Appendix 1, p. 124). This study followed from the theoretical distinction between event-based and emergent timing. Event-based timing is conceived as prescribed by events produced by a central clock, and seems to be used in discontinuous movement tasks (*e.g.*, finger tapping). Emergent or dynamical timing refers to the exploitation of the dynamical properties of effectors, and is typically used in continuous tasks (*e.g.*, circle drawing). The analysis of period series showed that both timing control processes possess fractal properties, characterized by self-similarity and long-range dependence. However, as shown in a previous study (Delignières et al., 2004) the two forms of timing control entailed clearly distinct short-range correlation properties. The aim of this article was to present two models that produce period series presenting the statistical properties evidenced in discrete and continuous rhythmic tasks. The first one (the *shifting-strategy* model) is an adaptation of the classical activation/threshold models, including a plateau-like evolution of the threshold over time. The second one (the *hopping* model) is a hybrid limit-cycle model, including a time-dependent linear stiffness parameter. Both models reproduced satisfactorily the spectral signatures of event-based and dynamical timing processes, respectively. They also produced auto-correlation functions similar to those experimentally observed. Using ARFIMA modeling we showed that these simulated series possessed fractal properties.

On the basis of this study, we further addressed the cases of unimanual synchronization tapping and synchronization oscillations. For sake of conciseness, we chose to focus in chapter 5 only on the synchronization tapping study which is a good illustration of the mechanistic perspective of 1/*f* noise, starting from a domain-specific model and extending it for taking additional constraints into account. (Experimental data used for this study are available at http://www.edm.univ-montp1.fr/fr/recherche-membre.php?membre=55)

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5 GIVING UP THE IDEA OF A UNIVERSAL ACCOUNT OF $1/f^{\beta}$ NOISE TO DIALOGUE WITH CURRENT THEORY OF RHYTHMIC MOVEMENT TIMING

Unraveling the finding of $1/f^{\beta}$ noise in self-paced and synchronized tapping: A unifying mechanistic model

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Unraveling the finding of $1/f^{\beta}$ noise in self-paced and synchronized tapping: A unifying mechanistic model

Kjerstin Torre & Didier Delignières

EA 2991, University Montpellier 1, France

Abstract

$1/f^{\beta}$ noise has been revealed in both self-paced and synchronized tapping sequences, without being consistently taken into consideration for the modeling of underlying timing mechanisms. In this study we characterize variability, short-range, and long-range correlation properties of asynchronies and inter-tap intervals collected in a synchronization tapping experiment, attesting statistically the presence of $1/f^{\beta}$ noise in asynchronies. We verify that the linear phase correction model of synchronization tapping in its original formulation cannot account for the empirical long-range correlation properties. On the basis of previous accounts of $1/f^{\beta}$ noise in the literature on self-paced tapping, we propose an extension of the original synchronization model by modeling the timekeeping process as a source of $1/f^{\beta}$ fluctuations. Simulations show that this '$1/f$-AR synchronization model' accounts for the statistical properties of empirical series, including long-range correlations, and provides an unifying mechanistic account of $1/f^{\beta}$ noise in self-paced and synchronization tapping. This account opens the original synchronization framework to further investigations of timing mechanisms with regard to the serial correlation properties in performed time intervals.

Key-words: linear phase correction, long-range correlation, self-pacing, synchronization, tapping, timing

Send correspondence to:
Kjerstin Torre
Faculty of Sport Sciences – University Montpellier 1
EA 2991, "Motor Efficiency and Deficiency"
700 Avenue du Pic Saint Loup
34090 Montpellier – France
Email: ktorre@univ-montp1.fr

1. Introduction

Biological rhythms are the outcome of complex systems with multiple interacting processes. Analyzing the variability of rhythms generated by diverse functions in the human organism has revealed the presence of persistent long-range correlation, or $1/f^{\beta}$ noise. This recurrent result was evidenced notably in heart rate (*e.g.* Peng, Havlin, Stanley and Goldberger 1995; Peng et al. 1999; West et al. 1999), respiratory cycles (Fadel, Barman, Phillips, Gerard and Gebber 2004; Peng et al. 2002), stride intervals in gait (Hausdorff, Peng, Ladin, Wei and Goldberger 1995, Hausdorff et al. 1996; West and Scafetta 2003), brain activity (*e.g.* Bédard, Kröger and Destexhe 2006; Novikov, Novikov, Shannahoff-Khalsa, Schwartz and Wright 1997), or in the production of rhythmic movements (Gilden, Thornton, and Mallon 1995; Gilden 2001; Delignières, Lemoine, and Torre 2004; Delignières, Torre and Lemoine 2008; Madison 2004; Pressing and Jolley-Rogers 1997; Yamada, 1995). $1/f^{\beta}$ noise denotes a very specific kind of variability defined by two main features: First, series of successive measures present persistent long-range correlation. Taking rhythmic movement as an example, the current period is positively correlated with a large set of previous values, and the series is said to possess long-term memory as opposed to the short-term dependence typical of ARMA processes. Second, series of successive measures are self-similar: their statistical properties remain unchanged whatever the scale of measurement, so that the process has no characteristic time scale. These features are notably revealed by a the auto-correlation function of series that exhibits a slow power-law decay, and the power spectrum in bi-logarithmic coordinates that shows a linear shape with a slope close to -1.

In the present article we focus on timing mechanisms, and the finding of $1/f^{\beta}$ noise in rhythmic self-paced and externally paced finger tapping. Self-paced tapping requires an internal regulation of timing, which has been supposed to involve a central timekeeping process (Wing and Kristofferson 1973). Numerous studies have shown that series of inter-tap intervals (ITI) in self-paced tapping contained $1/f^{\beta}$ noise (Chen, Repp, and Patel 2002; Delignières et al. 2004; Gilden et al. 1995; Gilden 2001; Madison 2004, Yamada 1995), which was assumed to represent the variability inherent to the internal timekeeping process (Gilden et al. 1995; Gilden 2001; Delignières et al. 2004; 2008). In this view, Delignières et al. (2008) proposed to provide the timekeeper with $1/f$ properties using the so-called *shifting strategy model*, and showed that a combination of this fractal timekeeper and the original Wing and Kristofferson (1973)'s model allowed to account for the statistical properties typically observed in self-paced tapping.

In externally-paced tapping, the goal is to maintain a constant phase relationship between the taps and the metronome signals. In particular, synchronization consists in tapping in-phase with the metronome. Analyses focus on series of asynchronies (ASYN), defined as the time intervals between the taps and the metronome. In this condition, $1/f^{\beta}$ noise has been evidenced in ASYN series; ITI series, in contrast, present anti-persistent (negative) correlations (Chen et al. 1997, 2001; Chen et al. 2002; Ding, Chen and Kelso 2002; Pressing and Jolley-Rogers 1997). These two results are consistent, as ITI series correspond to the differentiation of ASYN series, plus the constant period (τ) imposed by the metronome:

$$ITI_n = ASYN_{n+1} - ASYN_n + \tau \qquad (1)$$

Vorberg and collaborators (Vorberg and Schulze 2002; Vorberg and Wing 1996) proposed a linear phase correction model for synchronization tapping, as an extension of the Wing and Kristofferson model. According to its simplest formulation, asynchronies are locally corrected by a first-order auto-regressive process, without modifying the functioning of the timekeeper:

$$ASYN_{n+1} = (1 - \alpha)ASYN_n + K_n - \tau \qquad (2)$$

In this model τ represents the constant time intervals imposed by the metronome, and K_n the inter-tap intervals predicted by the Wing and Kristofferson model for self-paced tapping, as limit case without any effective correction process ($\alpha = 0$):

$$K_n = C_n + M_{n+1} - M_n \qquad (3)$$

Here, C_n are the time intervals prescribed by the central timekeeper, delimited by the consecutive discrete events (n and n+1), and M_n represents the motor delays that affect each tap associated to a central event. C_n and M_n are defined as two independent white noise processes (Wing and Kristofferson 1973; Vorberg and Wing 1996; Vorberg and Schulze 2002).

Despite of an important amount of experimental studies and analytical works based on this model, the way the assumed timing processes involved in synchronization tapping could account for the observed $1/f^{\beta}$ noise has been largely disregarded so far (for an exception, see Pressing and Jolley-Rogers 1997). One reason for this might be a still hesitant and doubtful approach of the $1/f^{\beta}$ phenomenon, and what it represents. For instance, following the idea that $1/f^{\beta}$ noise represents simple background noise inherent to the timing system, one might indeed question 'why modeling the errors of a clock'? Another, more insidious reason may be an unclear vision of how to deal with $1/f^{\beta}$ noise, which seems actually related to the existence of controversial perspectives on the $1/f^{\beta}$ phenomenon (see Kello, Beltz, Holden and Van Orden 2007; Torre and Wagenmakers, in press; Van Orden, Holden and Turvey 2003; Wagenmakers, Farrel and Ratcliff 2005). On the one hand, a *nomothetic* perspective seeks generic principles accounting for $1/f^{\beta}$ noise, given its ubiquity. In this view, $1/f^{\beta}$ noise has often been associated with concepts such as complex systems, emergent dynamics, metastability, and self-organized criticality (Van Orden et al. 2003; Kello et al. 2007). On the other hand, a *mechanistic* perspective seeks domain-specific and experimentally testable (and thus falsifiable) models of the processes that generate $1/f^{\beta}$ noise (Delignières et al. 2008; West and Scafetta 2003). Beyond the theoretical issue raised by these contrasted perspectives, they also affect the way to deal with $1/f^{\beta}$ noise in research, and the actual impact on the modeling of psycho-biological phenomena (Torre and Wagenmakers, in press).

Also with regard to our present issue, following a nomothetic or a mechanistic perspective to account for the finding of $1/f^{\beta}$ noise in series of inter-tap intervals produced in self-paced tapping and series of asynchronies in synchronization tapping would bring about divergent conclusions. In the nomothetic perspective it would be assumed that both the way the system globally organizes to perform self-paced taps and the way it (re)organizes to synchronize the taps with an external stimulus answered some generic form of organization that caused $1/f^{\beta}$ noise in the resulting inter-tap intervals and asynchronies, respectively. Following this perspective, modeling synchronization through a simple auto-regressive correction process appears clearly inconsistent with the finding of $1/f^{\beta}$ noise in asynchronies, since simple auto-regressive mechanisms can only produce short-range dependence in series, featuring an exponential decay in the auto-correlation function and a plateau in the low-frequency region of the power spectrum, as opposed to the typical signatures of $1/f^{\beta}$ noise we outlined above (see Thornton and Gilden 2005; Wagenmakers, Farrel and Ratcliff 2004).

The second perspective consists in assuming that there may be specific processes and/or localizable entities in the system that generate $1/f^{\beta}$ noise, and which are common to the performance of self-paced and synchronization tapping. In this case, following the results of previous studies on self-paced tapping, the internal timekeeping process can be considered the source of $1/f^{\beta}$ correlation (Delignières et al. 2004, 2008; Gilden et al. 1995). This approach implies a specific mechanistic model, where the correlation properties in ASYN series would result from the combination of correlations generated by the timekeeping process, the auto-regressive synchronization process, and motor variability (Equations 2 and 3).

The aim of this article was to examine how the established linear phase correction framework for synchronization tapping could be reassessed for proposing an experimentally testable and plausible model that accounts for the experimentally observed long-range correlations. Therefore we organized the article as follows: In the first part we characterize the serial correlation properties of asynchronies and inter-tap interval series collected in a synchronization tapping experiment. Experimental results are briefly discussed. In the second part, we show that the synchronization tapping model in its original formulation (Vorberg and Schulze 2002) does not allow accounting for the empirical long-range correlation. In the third part we propose to extend the original synchronization model by considering the internal timekeeping process as a source of $1/f^{\beta}$ noise, and modeling it using the shifting strategy model. We show that this $1/f$ synchronization model accounts for the empirical serial correlation. Notably, as the auto-regressive correction parameter varies, the model accounts for the typical correlation structures of inter-tap intervals in self-paced tapping, and asynchronies in synchronization tapping. We discuss the implications and the theoretical meaning of this model.

2 Synchronization tapping data

2.1 Method

Data used here were obtained as part of a larger study addressing the timing mechanism engaged in unimanual and bimanual tasks (Torre and Delignières, in press); here we only present the part that is of interest for our present concern.

Twelve participants (8 male and 4 female, mean age 29 ± 7.2) took part in the experiment. Ten participants declared themselves right-handed, and two left-handed. None of them had extensive practice in music. They declared no particular competence involving specific coordination between the upper limbs, and no neurological injury or recent upper limb injury. They signed an informed consent form, and were not paid for their participation.

The task consisted in tapping in synchrony with an auditory signal delivered at a constant frequency of 2 Hz. Participants performed series of about 600 taps, corresponding to 5 minutes trials. Participants were seated comfortably, their forearm, hand palm and other fingers resting on the table so that only the index finger of the dominant hand moved. They were instructed to keep the taps on the beep and to minimize the contact duration on the surface.

The auditory signals were generated by a PC-driven metronome. The taps were performed on a flat rectangular (4cm × 4cm) pressure sensor fixed on a table and adjusted to the participants' comfort. The pressure data and metronome sequences were recorded with a sampling frequency of 300 Hz, using LabJack U12 device.

2.2 Data analysis

We analyzed ITI and ASYN series. The times of the taps (t_T) and the auditory signals (t_M) were identified as the reaching of a threshold at each signal onset. ASYN were defined as the difference t_T - t_M between the tap and the corresponding auditory signal.

For examining the short-range correlation properties of ITI and ASYN, we computed the auto-correlation functions (ACF) of series from lag 1 to lag 20. For assessing the long-range correlation structure, we first examined the series spectral properties using $^{low}PSD_{we}$ (Eke et al. 2000), an improved version of the classical spectral analysis, including some preprocessing operations before the application of the Fast Fourier Transform (for details, see Eke et al. 2000). The spectral exponent β was estimated by the negative of the linear regression slope in the log-log power spectrum. As proposed by Eke et al. (2000) we excluded in the fitting of β the high-frequency power estimates (f > 1/8 of maximal frequency) for obtaining more accurate estimates. Spectral analysis allows classifying series as fractional Gaussian noise (fGn), *i.e.* stationary series (β <1), or fractional Brownian motion (fBm), *i.e.* non-stationary series (β > 1; Eke et al. 2000).

So far, the presence of $1/f^{\beta}$ noise in ASYN series has been reported on the basis of the linear regression slope in log-log power spectra. However, combinations of different short-range processes are known to be likely to mimic the characteristic $1/f^{\beta}$ power spectra (see Thornton and Gilden 2005; Wagenmakers, Farrell and Ratcliff 2004), and the presence of 'genuine' long-range correlation in ASYN series has not been attested statistically. In the present study we used ARFIMA/ARMA modeling (*Auto-regressive Fractionally Integrated Moving Average*, Wagenmakers, Farrel and Ratcliff 2004; Torre, Delignières and Lemoine 2007) in addition to spectral analysis, in order to evaluate the statistical evidence for the presence of genuine long-range correlation in experimental and simulated ASYN series. This method consists in fitting 18 models to the studied series: nine are ARMA (*p*,*q*) models, *p* and *q* varying systematically from 0 to 2, and the other nine are the corresponding ARFIMA (*p*,*d*,*q*) models, where *d* is the fractional integration parameter. The best model is selected using a goodness-of-fit statistic that is based on a trade-off between accuracy and parsimony. We used the Bayes Information Criterion (BIC) that was proven to give the best results in the detection of long-range dependence (Torre et al. 2007). The ARFIMA/ARMA procedure provides two complementary criteria. The first one is the percentage of series that are better fitted by an ARFIMA model. The second is based on a transformation of the raw BIC values into weights (*i.e.* the probability that this model is the best over the set of candidate models; see Wagenmakers and Farrell 2004). We then computed the sum of the weights captured by the nine ARFIMA models, considering that the weights of all tested model sum to one.

		Experimental data	Original synchronization model	1/*f* synchronization model
SD (ms)	ASYN	35	33	31
	ITI	32	35	34
auto-correlation lag1	ASYN	0.39 (0.21)	0.45 (0.04)	0.39 (0.04)
	ITI	-0.33 (0.13)	-0.38 (0.04)	-0.37 (0.04)
asymptotic	ASYN	**0.12 (0.10)**	**-0.01 (0.04)**	**0.08 (0.04)**
	ITI	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
spectral index (β)	ASYN	**0.69 (0.48)**	**0.20 (0.36)**	**0.69 (0.29)**
	ITI	**-1.11 (0.43)**	**-1.86 (0.31)**	**-1.27 (0.29)**
% ARFIMA	ASYN	**83.3%**	**65%**	**94%**
Sum of weights		**0.94**	**0.76**	**0.94**

Table 1. Results obtained from experimental series, and from simulated series generated by the original synchronization model (white noise assumption), and by the proposed fractal model. Experimental ASYN series contain $1/f^{\beta}$ noise, as indicated in bold by the positive asymptote value in the auto-correlation function, the spectral index, and the results of ARFIMA modeling. Accordingly, experimental ITI series contain anti-persistent noise. While the two simulations gave *SD* and lag 1 auto-correlations similar to experimental results, only the 1/*f* synchronization model allowed to reproduce the empirical long-range correlation properties assessed by the spectral index and ARFIMA modeling.

2.3 Statistical properties of experimental series

The results for experimental series are summarized in Table 1 (left column). The mean experimental ASYN was -62 ms, with a mean within-trial standard deviation of 35 ms. The mean ITI was 499 ms, with a mean within-trial standard deviation of 32 ms.

The average autocorrelation functions of ASYN and ITI series are presented in Figure 1 (upper panel). Mean lag 1 auto-correlation of ASYN series was 0.39 (*SD* = 0.21), and the ACF exhibited an asymptote approaching 0.12 (*SD* = 0.10, mean over lag 10 to lag 20). The mean lag 1 auto-correlation of ITI series was -0.33 (*SD* = 0.13), and the ACF exhibited an asymptote approaching 0.00 (*SD* = 0.00, mean over lag 10 to lag 20).

Regarding the long-range correlation structure, the mean log-log power spectra of ASYN and ITI series are presented in Figure 1 (upper panel). The mean spectral indexes were β = 0.69 (*SD* = 0.48) for ASYN, and β = -1.11 (*SD* = 0.43) for ITI. Importantly, the mean power spectrum for ASYN did not present any plateau in the low-frequency region, but a linear negative slope along the whole range of frequencies.

ARFIMA/ARMA modeling provided statistical evidence for the presence of long-range correlation in 10 out of 12 (83.3%) experimental ASYN series, with a mean sum of ARFIMA weights captured by ARFIMA models of 0.94.

2.4 Discussion

Our series presented Gaussian statistics consistent with those reported in similar experiments, especially reproducing the typical anticipation tendency (negative mean asynchronies) that has usually been reported in synchronization tapping (for a review see Repp 2005). Regarding short-term correlation, our data notably reproduced the characteristic negative lag 1 auto-correlation that is expected in ITI series, in both self-paced and synchronization conditions (Vorberg and Wing 1996).

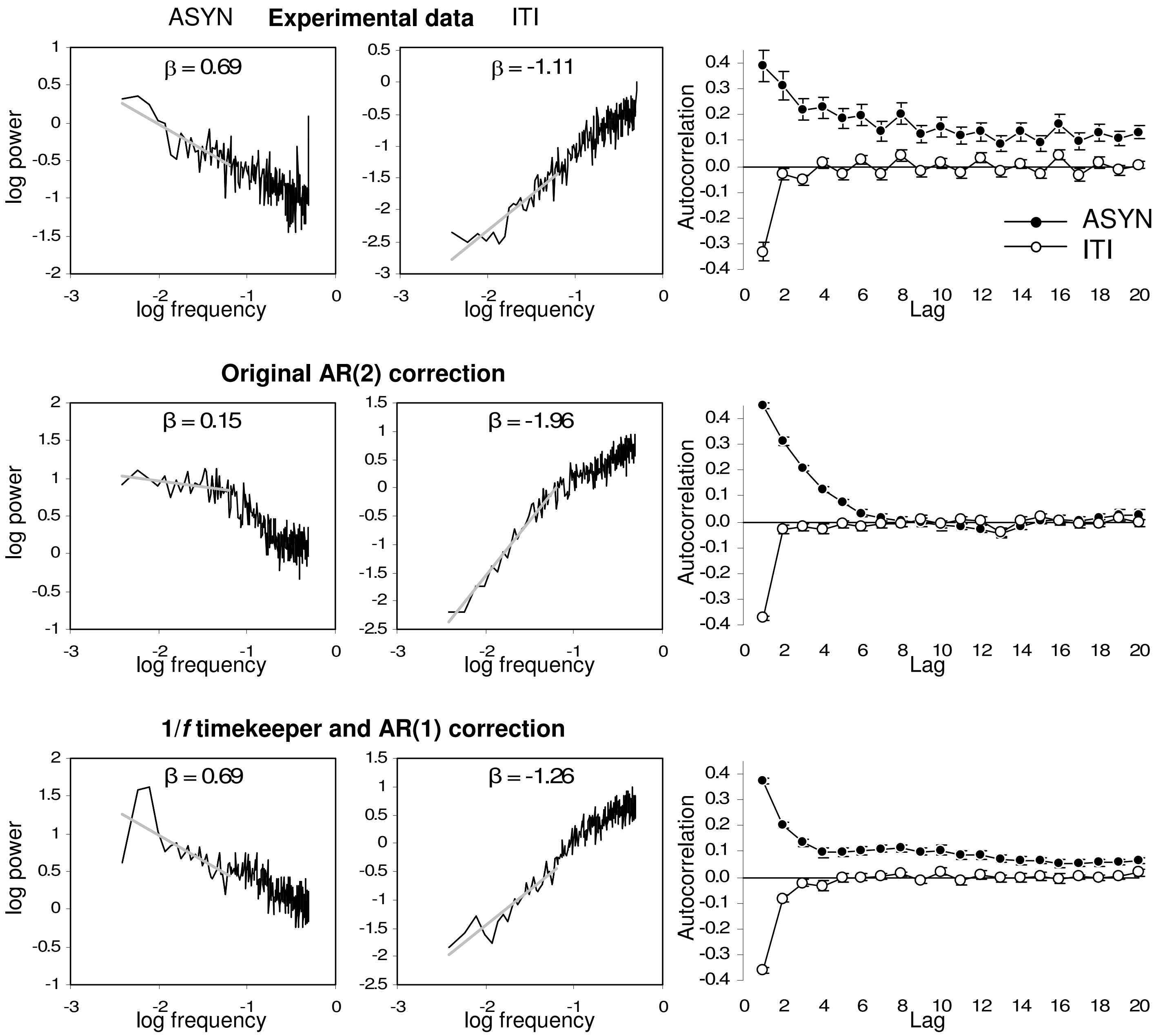

Figure 1. Average log-log power spectra and auto-correlation functions of asynchronies and inter-tap intervals collected experimentally, simulated by the original synchronization model, and simulated by the 1/*f* synchronization model. For the auto-correlation functions, error bars represent standard errors. The graphs for simulated series were determined from 12 over 100 simulated series chosen randomly.

Regarding long-range correlation, our results showed that ITI and ASYN series could be characterized as fGn (*i.e.* stationary series). This is not so surprising, since the metronome imposes a constant timing reference that allows controlling the mean time intervals over the trial and then to avoid drifts, such as those often reported in self-paced ITI (Ogden and Collier 1999).

Our analyses gave consistent statistical evidence for the presence of genuine long-range correlation in ASYN series. Mean spectral indexes indicated that series were $1/f^{\beta}$ noise ($0.5 < \beta < 1.5$), and ARFIMA/ARMA modeling tended to attest this result statistically. Although the percentage of series recognized as ARFIMA processes remained slightly lower than the indicative threshold of 90% recommended by Torre et al. (2007), this result could be considered with some confidence, given the low number of analyzed series and their relative 'shortness' (the performance of ARFIMA/ARMA modeling as other methods for analyzing long-range correlation are known to be sensitive to the length of series, see Delignières, Ramdani, Lemoine, Torre , Fortes and Ninot 2006; Torre et al. 2007). Moreover, this result was reinforced by the mean sum of weights captured by ARFIMA models (0.94) that was close to the maximal obtainable sum.

The analysis of experimental ITI series gave a completely different pattern of results, consistent with the fact that those series are mathematically the differentiation of ASYN series. ITI series were anti-persistent fGn ($\beta < 0$), and the auto-correlation function presented the expected shape for this kind of processes, with a negative value at lag 1, and an extinction of correlation at higher lags.

On the basis of this characterization of serial correlations in experimental ASYN and ITI data, the aim of the following sections was to test the capability of two candidate models in accounting for the evidenced statistical features in experimental synchronization tapping series, in terms of variability, short-range, and long-range correlation. First, we focus on the model proposed by Vorberg and Schulze (2002) in its original formulation. Second, we propose an amended version of the model, considering the central timekeeper as generating $1/f^{\beta}$ noise.

3 Simulation 1: The original synchronization model

Vorberg and Schulze (2002) conducted an analytical study of the stochastic behavior of the above presented AR(1) synchronization model (Equation 1; Vorberg and Wing 1996). This framework is based on the *linear phase correction assumption*: in the case where the metronome delivers constant periods, participants are supposed to sufficiently know the requested periods, so that correction of periods at the central timekeeper level can be neglected, and local phase corrections are sufficient to keep synchronized (Semjen, Schulze and Vorberg 2000; Semjen, Vorberg and Schulze 1998; Vorberg and Wing 1996; Vorberg and Schulze 2002). Vorberg and Schulze (2002) further analyzed an extended version of the initial model including a second-order auto-regressive correction (AR(2)) on asynchronies, so that corrections of the last and the next-to-last asynchronies were combined. The expression of asynchronies (Equation 2) becomes

$$ASYN_{n+1} = (1-\alpha)ASYN_n - \gamma ASYN_{n-1} + K_n - \tau \qquad (4)$$

Note that according to Equation 3, the error correction is based on the 'real' asynchronies between the effective tap and the metronome. Considering that real asynchronies are actually perceived with some feedback delay, Vorberg and Schulze (2002) further proposed to apply the auto-regressive correction on perceived asynchronies by including two distinct delay terms into the model, one depending on the physical properties of the metronome and the other related to the internal registration of the produced motor responses. However, for conciseness with regard to the present focus on serial correlation, and because the inclusion of the delay terms influences the mean asynchrony without modifying the correlation properties substantially (Flach 2005), we limited ourselves to examine the AR(2) synchronization model (Equation 4).

Vorberg and Schulze (2002) showed analytically that this model, based on the assumption that both the timekeeper (*C*) and the motor components (*M*) were sources of uncorrelated white noise, was able to account for the Gaussian properties and typical short-range correlation in synchronization tapping series. The consistency of this model with regard to the typical long-range correlation properties has not been evaluated. Because the only sources of serial correlation in this model are the term of differenced white noise which affects each produced ITI and the AR(2) correction of asynchronies, one may predict, nevertheless, that the synchronization model in its original formulation would be confined to account for short-range correlations up to the second lag and would provide no account for long-range correlation properties.

We verified this assumption by simulating 100 series (512 points) obeying Vorberg and Schulze's AR(2) synchronization model (Equation 4). The timekeeper periods C_n and motor delays M_n were considered white noise processes with standard deviations σ_C and σ_M, respectively. We estimated the parameters θ, γ, σ_C and σ_M on the basis of the auto-covariance function of experimental synchronization series averaged over participants, following the equations proposed by Vorberg and Schulze (2002, p.75, Eq.17). Accordingly, we set $\alpha = 0.5$, $\gamma = -0.1$, $\sigma_C = 26$, and $\sigma_M = 10$.

The main results are summarized in Table 1 (middle column). Simulations allowed the reproduction of the experimental variability of ASYN series, with a mean within-series standard deviation of 33 ms. The mean of simulated ITI was 500 ms, with a mean within-series standard-deviation of 35 ms.

The average auto-correlation functions of simulated ASYN and ITI series are presented in Figure 1 (middle panel). The mean lag 1 auto-correlation of ASYN series was 0.45 (*SD* = 0.04), and the ACF exhibited an asymptote approaching -0.01 (*SD* = 0.04, mean over lag 10 to lag 20), that contrasted with the persistent, slightly positive correlation obtained in experimental asynchronies. The mean lag 1 auto-correlation of IRI series was -0.38 (*SD* = 0.04), and the ACF exhibited an asymptote approaching 0.00 (*SD* = 0.00, mean over lag 10 to lag 20).

Regarding long-range correlation, the average power spectra of simulated asynchronies and inter-response interval series are presented in Figure 1 (middle panel). The power spectrum of ASYN series presented an obvious inflexion between a mean slope of -0.20 (*SD* = 0.36) in the low-frequency region (frequencies lower than $1/8^{th}$ of the maximal frequency), and a mean slope of -0.72 (*SD* = 0.15) in the high-frequency region. Such a spectrum is typical of pure auto-regressive series (see Pressing and Jolley-Rogers 1997; Wagenmakers, Farrell and Ratcliff 2004). Accordingly ARFIMA/ARMA modeling detected long-range correlation in only 65% of ASYN simulated series, and the mean sum of weights captured by ARFIMA models was 0.76. For simulated ITI series, the mean spectral index was β = -1.86 (*SD* = 0.31).

In sum, the Vorberg and Schulze (2002)'s AR(2) model gave a satisfying account for experimental variability, as well as for the lag 1 autocorrelation of ASYN and ITI series. However, assuming that the timekeeper variability was white noise, this model was not able to account for the long-range correlation structure evidenced in experimental ASYN and ITI series. Recently, Delignières et al. (2008) proposed to re-assess the Wing and Kristofferson model for self-paced tapping by providing the timekeeper periods (C_n) with $1/f^{\beta}$ variability. In the following section, we propose to extend Vorberg and collaborators' synchronization model by including the timekeeper as a source of $1/f^{\beta}$ noise. We test for the capability of this extended model to account for the empirical correlations observed in synchronization tapping.

4 Simulation 2: 1/*f*-AR synchronization model

4.1 A fractal model for timekeeper periods

The central timekeeper of the Wing and Kristofferson model for self-paced tapping is assumed to generate regularly spaced cognitive events, and the fluctuations in the resultant successive time intervals are assumed to follow a random distribution. Such a timer can be represented by a threshold/activation mechanism (Ivry 1996; Schöner 2002), where an activation process (a) increases linearly over time until the reaching of a given threshold level (T) that determines a particular 'event'. Such an event triggers the motor response and simultaneously resets the activation process. Thus, the iteration of this model generates regular inter-event intervals whose duration is entirely determined by the ratio between threshold level and activation growth rates. For constant threshold and activation growth rate, the process produces periodic events. Assuming that the threshold and the activation growth rate fluctuate randomly around their baseline levels T_0 and a_0, the resulting inter-event interval series would be uncorrelated white noise.

In order to allow the Wing and Kristofferson model to account for $1/f^{\beta}$ properties previously evidenced in ITI series produced in self-paced tapping, Delignières et al. (2008) proposed to model the timekeeper using an amended version of this classical threshold/activation mechanism: the *shifting strategy model*, initially developed by Wagenmakers et al. (2004). This model rests on the assumptions that (i) the threshold presents non-stationarity over time, characterized by a plateau-like evolution around the baseline level T_0, and (ii) the growth rate of activation varies over successive iterations according to an auto-regressive mechanism around a baseline rate a_0.

The amplitudes T'_n of the threshold deviations from the baseline level are sampled from a uniform distribution of range R. Each deviation T'_n is maintained for a duration d_n sampled uniformly from a range [d_{min} ; d_{max}] of possible state durations. For each iteration, the current threshold is then given by

$$T_n = T_0 + T'_n, \qquad (5)$$

and the current activation rate by

$$a_n = a_0 + \varphi(a_{n-1} - a_0) + \mu\varepsilon_n, \qquad (6)$$

where φ is the auto-regressive parameter, and ε_n a centered white noise with unit variance. In agreement with the Wing and Kristofferson model, the motor responses are then triggered by an internal timer generating inter-event intervals C_n given by

$$C_n = T_n / a_n, \qquad (7)$$

Incorporating the shifting strategy model at the timekeeper level into the original Wing and Kristofferson model (Equation 2) was shown to give a satisfying account for both the short- and long-range correlation properties of ITI series (notably the negative lag 1 autocorrelation and $1/f^{\beta}$ noise) in self-paced tapping (Delignières et al. 2008).

4.2 A fractal model for synchronization tapping

We propose to account for the empirical properties of synchronization tapping by incorporating the shifting strategy model at the timekeeper level of the AR(2) synchronization model (Vorberg and Schulze 2002).

As for the simulation of the original synchronization model, we used the estimations of timekeeper and motor delay standard deviations σ_C = 26 and σ_M = 10. For simulating the timekeeper variability, the parameters of the shifting strategy model were set in order to provide C_n with reproducible $1/f^{\beta}$ structure, centered around 500 (corresponding to the experimental 2 Hz tapping frequency) and with the requested variance σ_C. We used linear activation parameters a_0 = 2, φ = 0.3, μ = 0.09, and threshold parameters T_0 = 1000, R = 30, d_{min} = 1, d_{max} = 100. We checked that these parameters produced the expected timekeeper series by simulating 100 series of 512 points: The mean of simulated C_n was 501 ms, with a mean standard deviation of 25 ms. Series were $1/f^{\beta}$ noise, with a mean spectral index β of about 0.68, and ARFIMA/ARMA modeling provided evidence for long-range correlation in 89% of C_n series with a mean sum of weights captured by ARFIMA models of about 0.91. On this basis, the synchronization parameters α and γ were set in order to provide the best fit (in terms of Gaussian statistics and serial correlation properties) for our experimental ASYN and ITI series. The best solution was obtained for $\alpha = 0.8$ and $\gamma = 0$, and we performed 100 simulations (512 points) using these parameters.

Simulation results are summarized in Table 1 (right column) for comparison with experimental results. The mean within-series standard deviation of simulated ASYN series was 30 ms. The mean of simulated ITI was 500 ms, with a mean within-series standard deviation of 33 ms.

The average auto-correlation functions of simulated ASYN and ITI series are presented in Figure 1 (bottom panel). The mean lag 1 auto-correlation of ASYN series was 0.39 ($SD = 0.04$), and the ACF exhibited an asymptote approaching 0.08 (SD = 0.04, mean over lag10 to lag20). The mean lag 1 auto-correlation of ITI series was -0.37 (SD = 0.04), and the ACF exhibited an asymptote approaching 0.00 (SD = 0.00, mean over lag10 to lag20).

Regarding long-range correlation, the average power spectra of simulated ASYN and IRI series are presented in Figure 1 (bottom panel). The mean spectral index was $\beta = 0.69$ ($SD = 0.29$) for ASYN, and $\beta = -1.27$ ($SD = 0.29$) for ITI. ARFIMA/ARMA modeling provided statistical evidence for the presence of long-range correlation in 94% of ASYN series, with a mean sum of weights captured by ARFIMA models of 0.94.

In sum, simulation results showed that this '$1/f$ - AR synchronization model' accounted for both the empirical short- and long-range correlations, reproducing the negative lag 1 auto-correlation in ITI series, the global shapes of auto-correlation functions of ASYN and ITI, and the spectral indexes showing that ASYN and ITI series contained $1/f^{\beta}$ noise and anti-persistent noise, respectively[1]. ARFIMA/ARMA modeling attested statistically for the presence of long-range correlation in simulated ASYN series. Note that we retained an AR(1) correction process for the $1/f$ - synchronization model. This result is consistent with the observations of Pressing and Jolley-Rogers (1997) showing that in this range of prescribed periods, a first-order process was sufficient.

5 General discussion

Following the current synchronization tapping framework, the correlation structure in produced asynchronies results from the association of three sources of variability: the timekeeping process, the synchronization process, and the motor implementation. The original formulation of Vorberg and collaborators' synchronization model assumes that the variabilities inherent to motor implementation and to the timekeeping process are both uncorrelated white noises, and that synchronization is achieved by an auto-regressive (short-range) mechanism. Accordingly, this model accounts for short-range correlations, but not for the empirical long-range correlations.

Two alternative approaches to this issue were possible. On the one hand, one could have assumed that the correlation structure of asynchronies arises from the processes that are specific to synchronization tasks. In this perspective, for instance, Chen et al. (1997) argued that the finding of $1/f^{\beta}$ noise in asynchronies opposed Vorberg and collaborators' formalization of synchronization processes as auto-regressive error correction. Similarly, Thaut and collaborators (Thaut and Miller 1994; Thaut, Miller and Schauer 1998) argued that the finding of a negative lag 1 auto-correlation in inter-tap intervals but a positive lag 1 auto-correlation in asynchronies indicated different synchronization strategies for phase and period corrections, and opposed a simple auto-regressive correction.

In contrast, in the present study we assume that serial correlation properties including the finding of $1/f^{\beta}$ noise in asynchronies do not arise from synchronization processes themselves, but from the internal timekeeping processes which is common to self-paced and synchronized rhythmic tapping. Therefore, we incorporated the shifting strategy model which has previously been used for modeling self-paced tapping, at the timekeeper level into Vorberg and collaborators' (Vorberg and Wing 1996; Vorberg and Schulze 2002) original synchronization model. We showed that provided that the timekeeper is considered a source of $1/f^{\beta}$ noise, the auto-regressive synchronization process assumed in Vorberg and collaborators' framework remains

[1] As it becomes obvious in Table 1, autocorrelations and spectral indexes exhibited higher variability in experimental than in simulated series. With regard to this discrepancy one might wonder whether the number of empirical series was sufficient and what could be the meaning of simulating such variable results. However, the main purpose of the present study was to perform a qualitative characterization of empirical correlation properties and to propose a model that accounted for these properties with respect to the average empirical behavior. Despite the large inter-individual variability in autocorrelations and spectral indexes the present results were qualitatively consistent, allowing notably to discriminate unambiguously between $1/f^{\beta}$ noise in ASYN series and anti-persistent noise in ITI series, as well as between the characteristic autocorrelation functions. The comparatively low variability observed for simulated series is related to the fact that simulations were performed using a single set of parameters, aiming to fit the average empirical behavior.

consistent with the empirical correlation properties. In this regard, the present '$1/f$-AR synchronization model' provides an unifying account for the typical long-range correlation properties evidenced in self-paced and synchronized tapping.

Note that the finding of $1/f^{\beta}$ correlations in self-paced tapping has sometimes been considered spurious, as it was assumed to be an experimental artefact due to the collection of very long tapping sequences, rather than being inherent to the timekeeping process (Pressing and Jolley-Rogers 1997). Factors such as fatigue and boredom were supposed to induce artifactual non-stationarity, participants being unable to maintain a constant timing reference over the whole trial, so that the observed long-range correlations in self-paced tapping were attributed to mechanisms that are extraneous to the actual timing processes. In the case of synchronization, however, the external pacing provides a continual reference that prevents such artifactual drifts. Consequently, if the assumption of an artifactual origin of long-range correlations was true, such dependencies should not appear in asynchronies. The present results show, on the contrary, that the origin of $1/f^{\beta}$ correlation is not trivial, and our model suggests that the timekeeping process is the source of $1/f^{\beta}$ noise in both self-paced and synchronisation conditions.

5.1 Implications and experimental testing of the '1/f-AR synchronization model'

We argue that the $1/f^{\beta}$ noise in asynchronies arises from the variability inherent to the timekeeping process. Taking account of the long-range correlation caused by the timing mechanisms might probably allow to explain a source of residuals of the fit processes of different synchronization models proposed in literature. Let us consider the implications of the present '$1/f$-AR synchronization model' for the estimation of model parameters based on experimental series, in the current synchronization framework.

The asynchrony at time n is given by the difference between the summation of completed inter-tap intervals and the sum of completed metronome periods:

$$ASYN_n = \sum_{1}^{n-1} ITI_n - (n-1)\tau + \varepsilon_n \qquad (5)$$

From the moment that noise affects the timekeeper periods, 'correction' is needed for preventing a divergence between the tapping sequence and the metronome sequence, and obtaining stationary ASYN series. The auto-regressive correction of asynchronies assumed in Vorberg and collaborators' synchronization framework allows stationarizing the differences between the two sequences, without modifying the properties of timekeeper periods. However, consider the 'virtual' asynchronies that would be obtained without any effective synchronization process. According to the nature of variability at the timekeeper level, the virtual asynchronies have different correlation properties: Assuming that the timekeeper variability is white noise, the virtual asynchronies would be Brownian motion, corresponding to the integration of white noise. Assuming that the timekeeper variability is $1/f^{\beta}$ noise, in contrast, virtual asynchronies would be persistent fractional Brownian motion, corresponding to the integration of $1/f^{\beta}$ noise. Consequently, the nonstationarity of asynchronies should be stronger in the case of a $1/f^{\beta}$ timekeeper (see Figure 2), and a stronger correction should be needed with a $1/f^{\beta}$ timekeeper than with a white noise timekeeper for obtaining similar variance and correlation properties in ASYN series.

The estimation of auto-regressive parameters (α and γ) have usually been performed on the basis of the covariance function of asynchronies (Pressing and Jolley-Rogers 1997; Vorberg and Schulze 2002; Vorberg and Wing 1996). As we showed here, co-variation in the successive asynchronies does not only arise from the error correction process, but also from the initial persistent long-range correlations induced by the timekeeper. Thus, one implication of the present model is that analytical approaches yielding parameter estimation should in some way take the non-random structure of series into account to prevent underestimation of the strength of

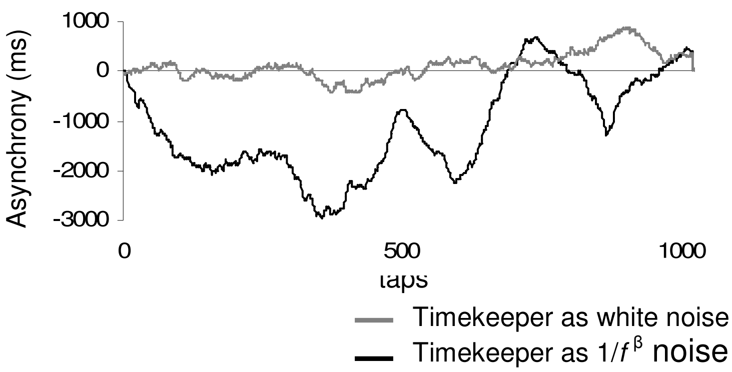

Figure 2. Evolution of asynchronies without any correction ($\alpha = 0$), simulated with the timekeeper generating $1/f^{\beta}$ noise (black trait), and the timekeeper generating white noise (gray trait). The curve structure results from the addition of persistent fractional Brownian motion and white noise, minus a linear function in the first case, and from the addition of Brownian motion plus white noise, minus a linear function in the second case. 'Virtual' asynchronies diverge more when the timekeeper variability is $1/f^{\beta}$ noise.

correction. For instance, in the present study the estimation of auto-regressive parameters on the basis of experimental series gave α = 0.5 and γ = -0.1; however we needed to use α = 0.8 in the '1/*f* -AR synchronization model' to approach the statistical properties of empirical asynchronies.

Similarly, estimations of the relative contributions of timekeeper variance and motor variance have usually been based on the negative lag 1 auto-correlation in ITI series. However, because ITIs were shown to be anti-persistent noise (*i.e.* negatively correlated) corresponding to the differentiation of $1/f^{\beta}$ noise in asynchronies, the negative lag 1 auto-correlation does not only come from the differenced white noise corresponding to motor variability, but also from the long-range correlation induced by the timekeeper. Accordingly, estimations based on the assumption that the timekeeper variability is white noise might over-estimate the ratio between motor and timekeeper variances.

Following the original linear phase correction model (Vorberg and Schulze 2002; Vorberg and Wing 1996) we assumed that synchronization was achieved through a local correction of asynchronies that did not modify the timekeeper periods. Because of this independence between the timekeeping process and the error correction processes, as a function of the strength of correction starting from the limit case of self-paced tapping (α = 0) and increasing progressively, the long-range correlation structure in asynchronies turns gradually from persistent Brownian motion (*i.e.* non-stationary series) into the $1/f^{\beta}$ noise of empirical asynchronies (here for α = 0.8; see Figure 3). At the same time, the correlation structure of inter-tap-intervals naturally turns from $1/f^{\beta}$ noise into anti-persistent noise. Figure 3 makes obvious that the '1/*f* -AR synchronization model' allows accounting for the whole range of correlation properties, in particular the typical correlations of self-paced and synchronization tapping, by modulating the auto-regressive parameter only. As such, this model – and the underlying assumption that the timekeeper causes the $1/f^{\beta}$ fluctuations – provides a unifying account of the finding of $1/f^{\beta}$ noise in inter-tap intervals in self-paced tapping and in asynchronies in synchronized tapping.

In this regard, the model offers perspectives for further empirical testing. Notably, the model predicts an increase of persistent correlations in asynchronies in conditions where synchronization is known to be less efficient and error correction can be assumed to be less accurate. This prediction seems indeed

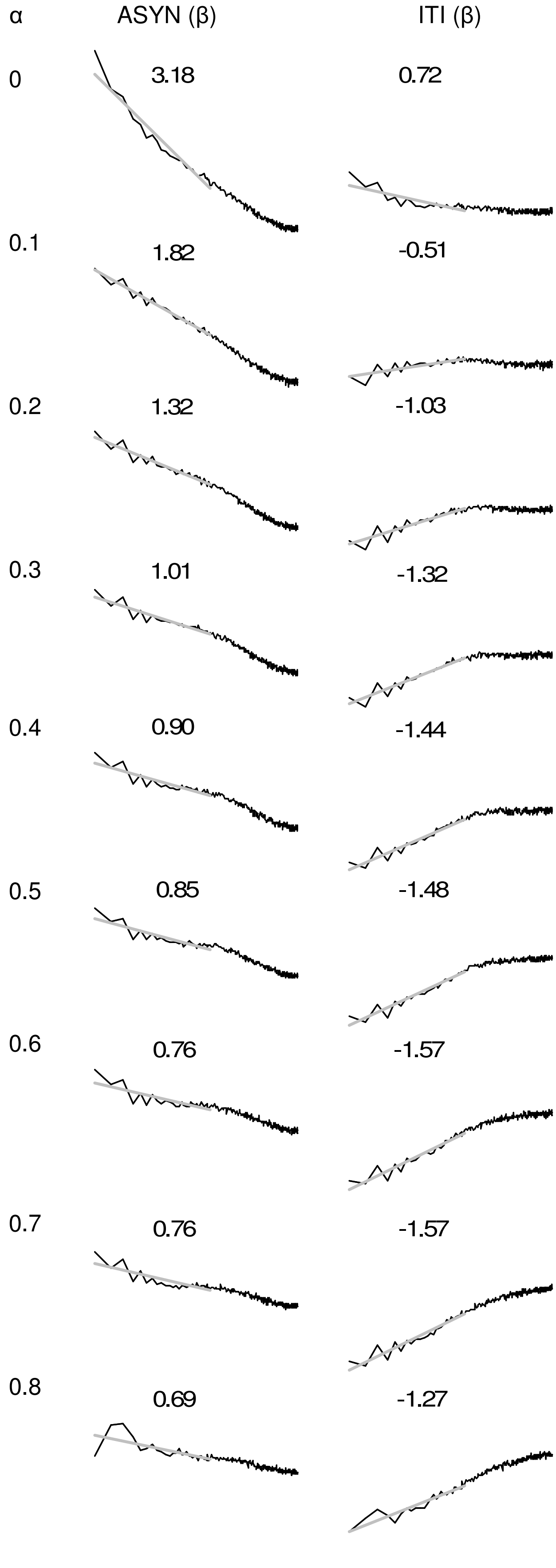

Figure 3. Average power spectra (N = 100) of ASYN series and the corresponding ITIs simulated by the 1/*f* -AR(1) synchronization model. All parameters except the auto-regressive parameter α are constant ($a_0 = 2$, $\varphi = 0.3$, $\varepsilon = 0.09$, $T_0 = 1000$, $T'_{max} = 4$, $d_{min} = 1$, $d_{max} = 100$). α varies from 0 to 0.8 by steps of 0.1, so that the top graphs show the results for self-paced tapping, as a limit case. This graph illustrates the fact that the model provides a unifying account of the finding of $1/f^{\beta}$ noise in self-paced and synchronized tapping since the empirical correlation properties showed in the two situations can be obtained by simply adjusting the synchronization parameter α.

consistent with results obtained by Chen et al. (2002), showing stronger persistent correlations with use of a visual metronome than with an auditory metronome. As well, this prediction could account for the results of Chen et al. (2001), evidencing stronger persistent correlations in syncopation than in synchronization tapping.

5.2 A mechanistic account of $1/f^{\beta}$ noise in synchronization tapping: Theoretical meanings

The present '$1/f$-AR synchronization model' is a mechanistic model for $1/f^{\beta}$ noise that clearly assumes that the timekeeping process is the source of the observed long-range correlations. It might by discussed in two respects.

Firstly, different theories have alternatively accounted for the timing of rhythmic movements in terms of central time-keeping processes that are responsible for the representation of the required time intervals, or in terms of temporal properties of movements that emerge from a complex (self-organizing) system. In particular, the information processing approach essentially deals with synchronization in terms of 'error correction', while the dynamic systems approach formalizes synchronization in terms of 'coupling' between an oscillator and an external stimulus (see Pressing 1999; Repp 2005, for a review). Even though the linear phase correction model we investigated in the present study is typically in line with the first approach, we did not wish to argue in favor of one or another theoretical perspective. Our approach actually relies on previous studies of the neural basis and the variability in performance of rhythmic movement timing (Delignières et al. 2004, 2008; Robertson et al. 1999; Ivry, Spencer, Zelaznik and Diedrichsen 2002; Spencer and Ivry 2005; Spencer, Zelaznik, Diedrichsen and Ivry 2003; Zelaznik, Spencer and Doffin 2000; Zelaznik, Spencer and Ivry 2002; see also Huys, Jirsa, Studenka, Rheaume and Zelaznik, in press) that have supported the distinction between two modes of temporal control: *event-based* and *emergent* timing, associated with the performance of discontinuous movements like tapping, and continuous movements like oscillations or circle drawing, respectively. While emergent timing is assumed to arise from the continuous regulation of non-temporal parameters (as oscillator stiffness) that determine movement frequency without needing any explicit representation of time, event-based timing is thought to involve an internal, explicit representation of temporal goals that is prescribed to the effectors independently of the motor execution itself.

Secondly, the assumption that the timekeeping process is the source of the observed long-range correlations might be discussed with regard to current controversial perspectives on the $1/f^{\beta}$ phenomenon (see Van Orden, Holden and Turvey 2003; Wagenmakers et al. 2005). The nomothetic perspective aims at uncovering the universal principles that explain the ubiquitous occurrence of $1/f^{\beta}$ noise. Following this perspective, $1/f^{\beta}$ noise has notably been related to self-organized criticality (Bak, Tang and Wiesenfeld 1987) and considered as emerging from the coordinated interactions between the functional elements of a complex system (*e.g.* Kello, Beltz, Holden and Van Orden 2007). Accordingly, nomothetic accounts are hardly compatible with the idea of identifying specific processes or entities in a particular system that may cause $1/f^{\beta}$ noise. Here, in contrast, we adopt a mechanistic perspective that aims at identifying the specific sources of $1/f^{\beta}$ noise in a specific phenomenon instead of providing a universal account of $1/f^{\beta}$ noise. One main reason for going for a mechanistic approach, we argue, is that domain-specific models may be more directly 'useful' for research since they allow to bridge the gap between accounts of $1/f^{\beta}$ noise and currently existing theories and models of a given object of research. Also, because such mechanistic models allow to establish relationships between model parameters and different experimental factors, they may predict specific variations in the correlation structure that are experimentally testable. In this perspective, our present model represents a way of summarizing the functioning of the system by a few simple mechanisms that are psychologically and biologically plausible. Such a mechanistic approach does not oppose theories of self-organizing complex systems. It just proposes a simplified view of the system that constitutes a relevant entry point for unraveling the finding of $1/f^{\beta}$ noise in synchronized and self-paced tapping.

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Transition part

In parallel to the above study of synchronization tapping, we also addressed synchronized oscillations (Torre & Delignières, submitted. See Appendix 2, p. 143). In this article we analyze serial dependencies in periods and asynchronies collected during oscillations performed in synchrony with a metronome. Results show that asynchronies present 1/*f* fluctuations, and periods series anti-persistent dependence. The analysis of limit cycle dynamics reveals a specific asymmetry induced by synchronization. We propose a hybrid limit cycle model including a cycle-dependent stiffness parameter provided with fractal properties, and a parametric driving function based on velocity. This model accounts for most experimentally evidenced statistical features, including serial dependence and limit cycle dynamics.

In Chapter 6, in order to assess the role of timing processes in coordination, we integrate the theoretical considerations and modeling efforts developed in the unimanual timing studies into an analysis of absolute and relative timing in bimanual coordination, using different task conditions. (Experimental data used for this study are available at http://www.edm.univ-montp1.fr/fr/recherche-membre.php?membre=55)

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6 THE BIMANUAL COORDINATION PARADIGM UNDER THE PERSPECTIVE OF SERIAL CORRELATION

Distinct ways of timing movements in bimanual coordination tasks: Contribution of serial correlation analysis and implications for modeling

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Distinct ways of timing movements in bimanual coordination tasks: Contribution of serial correlation analysis and implications for modeling

Kjerstin Torre and Didier Delignières

EA 2991, University Montpellier 1, France

Abstract
Bimanual coordination dynamics have been conceived as the outcome of a global coordinative system, and coordination stability properties and theories of underlying processes have often been generalized over various bimanual tasks. In unimanual timing tasks it has been shown that different timing processes are involved according to tasks, yielding distinctive correlation properties in the within-hand temporal patterns. In this study we compare unimanual with bimanual, tapping with oscillation, and self-paced with externally paced tasks, and we analyze the correlation properties of temporal patterns at both the component level and the coordinative level. Results show that the distinctive signatures of event-based *versus* emergent, and self-paced *versus* synchronization timing control known from unimanual tasks persist in the corresponding bimanual coordination tasks. Accordingly, we argue that these different timing processes, and related temporal patterns at the component level, constitute a task-dependent background on which coordination builds. One direct implication of these results is that the bimanual coordination paradigm should be considered multifaceted and not governed by some unitary generic principle. We discuss the need to assess the relationship between temporal patterns at the component level and the collective level, and to integrate serial (long-range) correlation properties into bimanual coordination models. Finally, we test whether the architectures of current bimanual coordination models can account for the experimentally observed serial correlations.

Send correspondence to:
Kjerstin Torre
Faculty of Sport Sciences – University Montpellier 1
EA 2991, "Motor Efficiency and Deficiency"
700 Avenue du Pic Saint Loup
34090 Montpellier – France
Email: ktorre@univ-montp1.fr

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1 Introduction

In the bimanual coordination paradigm participants are generally asked to produce regular rhythmic movements with the two hands, fingers, or forearms by maintaining a constant phase relationship between the two movements. Different theories of bimanual coordination have been proposed, based on different (sometimes emblematic) ways of doing experiments, analyzing data, and modeling. For instance, commonly used task modalities include continuous (*e.g.*, oscillatory) or discontinuous (*e.g.*, tapping) movements, and the presence or absence of an external pacing signal for prescribing the rhythm of movements. While the different accounts of coordination would ideally come together to form a coherent and comprehensive picture, they have often been neither directly opposable nor easily comparable because of methodological differences.

Bimanual coordination tasks basically have two interrelated goals: an absolute timing goal defined by the maintenance of a regular rhythm by the two limbs and a relative timing goal defined by the maintenance of a stable phase relationship between the limbs. Despite that, the dynamics at the relative timing (coordinative) level have mostly been studied separately from the within-limb (component) dynamics, and some authors have stressed the need to establish links between these two levels (Peper, Ridderikhoff, Daffertshofer, & Beek, 2004; Riley, Santana, & Turvey, 2001; Schöner, 2002).

Absolute timing governs the production of movements with stable and reproducible temporal patterns, without interaction with other timed movements and without an external prescription of time intervals. Absolute timing has typically been studied by analyzing time intervals produced in unimanual self-paced movement tasks. Comparative studies based on variability analysis (Robertson et al., 1999; Zelaznik, Spencer & Doffin, 2000; Zelaznik, Spencer & Ivry, 2002) and neurophysiological investigation (Ivry, Spencer, Zelaznik & Diedrichsen, 2002; Spencer & Ivry, 2005; Spencer, Zelaznik, Diedrichsen & Ivry, 2003) have shown that two distinct forms of timing control, namely, *event-based* timing and *emergent* timing, are engaged according to the discontinuous/continuous character of movements. Event-based timing (also referred to as *explicit timing*, Robertson et al., 1999; Zelaznik et al., 2002) implies an explicit representation of the temporal pattern by an internal timekeeper. This central representation determines cognitive events that trigger motor responses. Event-based timing was nicely captured by the well-known Wing and Kristofferson's (1973) model. This kind of timing mode is involved in discontinuous rhythmic movements such as finger tapping. In emergent timing (also referred to as *implicit timing*, Robertson et al., 1999; Zelaznik et al., 2002) the temporal regularity is assumed to come from a continuous modulation of the dynamical (and not explicitly temporal) properties of the effector system, so that no explicit representation of time is needed. Timing emerges from the dynamical, limit-cycle properties of effector's motion, and especially stiffness plays a major role for determining movement frequency (Delignières, Lemoine & Torre, 2004). Emergent timing is supposed to be involved in continuous movements, like forearm oscillations or circle drawing.

Serial correlation analyses based on the autocorrelation function and the bi-logarithmic power spectra of series of produced movement periods have furthermore provided statistical signatures that distinguish between the two modes of timing control (Delignières et al., 2004; Delignières, Torre, & Lemoine, 2008). Event-based timing is characterized by the presence of differenced white noise in the successive periods, yielding a negative lag 1 autocorrelation and a positive slope in the high-frequency region of power spectra. Emergent timing is characterized by simple white noise affecting the successive periods, yielding a non-negative (slightly positive or zero) lag 1 autocorrelation, and a non-positive (slightly negative or zero) slope in the high-frequency region of power spectra. A deeper analysis of these signatures will be proposed later. These methods allowed evidencing the exploitation of event-based timing in discontinuous tasks and emergent timing in continuous tasks (Delignières et al., 2004, 2008).

Despite these statistical differences, the series of periods in both self-paced tapping and oscillations have been shown to exhibit persistent long-range correlation or $1/f^{\beta}$ noise, revealed by a very slow power-law decay of the auto-correlation function over lags, and a negative slope of between -0.5 and -1.5 in the low-frequency region of log-log power spectra. These results suggest that both event-based and emergent timing generate $1/f^{\beta}$ noise (Delignières et al., 2004, 2008; Gilden, Thornton, & Mallon, 1995; Gilden, 2001).

Relative timing governs the production of stable and reproducible coordination patterns between multiple component movements, or between a movement and external events (sensorimotor synchronization). Unimanual rhythmic movements have been widely studied in synchronization tasks, where participants have to produce movements in synchrony with the signals of a metronome dictating the rhythm. Synchronization has been shown to modify the statistical properties of unimanual tapping (Chen, Ding, & Kelso, 1997; Torre, Lemoine, & Delignières, 2006; Torre & Wagenmakers, in press) and of unimanual oscillations, compared to self-paced movements (Torre, Lemoine, & Delignières, 2006). Notably, the series of periods show anti-persistent correlation (a short period is likely to be followed by a long period, and vice-versa), revealed by a positive slope in the low-frequency region of power spectra, instead of $1/f^{\beta}$ noise. At the same time, the series of asynchronies (*i.e.*, the time intervals between the movement goal and the signal of the metronome) have been shown to present $1/f^{\beta}$-like correlation. Thus, the serial correlation properties of the series of periods in unimanual tasks provide distinctive signatures of the involvement of event-based timing (tapping) *versus* emergent timing (oscillations), and of absolute (self-paced) versus synchronization (externally paced) timing.

Studies of bimanual coordination have focused on relative timing properties and their dependence on factors such as movement frequency (Kelso, 1995), behavioral information (*e.g.* Schöner & Kelso, 1988; Zanone & Kelso, 1992), attention (Monno, Temprado, Zanone, & Laurent, 2002; Temprado, Zanone, Monno, & Laurent, 1999), and perceptual cues (Bingham, 2004). Bimanual coordination theories generally consider coordination dynamics as the outcome of general coordinative principles that regulate the relative timing between components. Very few studies adopting this perspective, to our knowledge, have addressed the relation between the relative timing pattern between limbs and the timing processes and resulting temporal patterns at the component level. Nevertheless, in a task where participants have to perform bimanual coordination paced by a metronome for instance, the levels of timing that are likely to contribute to the coordinated movement patterns include (i) the absolute timing of the movements of each limb, (ii) the synchronization between the movements of the limbs and the metronome, and (iii) the relative timing between the two limbs. Considering a self-paced coordination task for instance, the absolute timing patterns of the limb movements can be highly variable, while the relative timing between limbs remains stable (Schöner, 2002), provided that the two absolute timing patterns have very similar structures of variability. Bimanual coordination does not presuppose any particular temporal pattern in the component movements. Thus, coordination between limbs could not be considered responsible for the finding of any reproducible temporal pattern in the limb movements in bimanual coordination compared with unimanual tasks.

A few recent articles have proposed to extend the distinction between event-based and emergent timing, established in unimanual tasks, to the control of bimanual coordination. Summers, Maeder and Hiraga (2005) and Summers, Maeder, Hiraga, and Alexander (2008) compared the variability of periods of the hands, and of the relative timing between hands, in bimanual coordination of intermittent *versus* continuous movements. Simultaneously, they assessed the attentional cost associated with the two conditions. They showed that the coefficients of variation of periods were not correlated between the intermittent and continuous conditions, suggesting the engagement of different timing processes (Robertson et al., 1999). Moreover, the attentional demand was higher for intermittent than for continuous coordination. Finally, the coefficients of variation of the relative timing series were not correlated between the intermittent and continuous conditions. The authors thus concluded that two different timing modes were operating, event-based in the former case, and emergent in the latter. However, although this conclusion is certainly consistent with the literature, in principle the differences in the variability of periods and relative timing could have been due to distinct coordination processes rather than to distinct timing modes.

Kennerly, Diedrichsen, Hazeltine, Semjen, and Ivry (2002) analyzed bimanual coordination of discontinuous versus continuous movements in split-brain patients and showed a strong temporal coupling in the first case and temporal uncoupling in the second case. Regarding these results, Ivry, Diedrichsen, Spencer, Hazeltine, and Semjen (2004) argued that the representational basis for continuous and discontinuous movement coordination was different: Continuous movement coordination was assumed to need no representation of an event-structure by a timekeeping entity and was considered an emergent coordination with analogy to emergent timing, in contrast to the event-based coordination of discontinuous movements. It seems straightforward, indeed, to assume that coordination of discontinuous movements would be achieved on the basis of a congruence between events rather than by a coupling of parameters that determine movements, given that the within-hand timing of unimanual discontinuous movements is known to be event-based rather than emergent[1]. However, without analyzing the collective level and the component level simultaneously, one cannot determine whether the temporal patterns are the result of coupling processes or whether the within-hand timing plays a significant role in coordination.

Regarding the difference between self-paced and synchronization tasks, unimanual studies have shown that external pacing of oscillations causes an *anchoring* effect, *i.e.*, a reduction of the variability of oscillators in the vicinity of metronome beats (Byblow, Carson, & Goodman, 1994; Carson, 1995). Focusing on the influence of external pacing on bimanual coordination, Fink, Foo, Jirsa, and Kelso (2000) and Jirsa, Fink, Foo, and Kelso (2000) examined simultaneously the limit-cycle dynamics of the two oscillating limbs and the stability properties of coordination in bimanual oscillations with a metronome. In addition to the reduction of spatial variability at the oscillator level, they observed a global stabilization of coordination. The authors assumed that the local anchoring of oscillators could *serve* the global stabilization of coordination (Fink et al., 2000). They accounted for this effect by extending the HKB model (Haken, Kelso, & Bunz, 1985) *ad hoc*, by adding a parametric driving term that couples the position of each oscillator to the auditory signal. Fink et al. (2000) and Jirsa et al. (2000) addressed the collective level and the component level simultaneously. However, while they focused on the changes in the within-cycle dynamics of oscillators, our present study proposes to focus on the qualitative changes in the cycle-to-cycle dynamics due to the

[1] At this point, it might be mentioned that arguing for the involvement of emergent timing in bimanual oscillations and event-based timing in bimanual tapping does not exclude the involvement of any cognitive representation of an event structure in coordination of continuous movements. For instance, Spencer and colleagues (Spencer & Ivry, 2007) recently proposed an event-based account of the stability of bimanual coordination, using a concomitant repeated vocalization task. Actually, some salient, discrete events can easily be defined even in continuous and smooth movements (*e.g.*, the reversal points in oscillation, or a spatial reference point in circling). Thus, certain experimental conditions such as discrete vocalizations during coordination (Spencer & Ivry, 2007; Spencer et al., 2006), attentional focus (Hiraga, Summers, & Temprado, 2005), or haptic information (Kelso, Fink, DeLaplain, & Carson, 2001), might indeed reinforce these discrete events and favor the exploitation of an event structure in continuous movements. Accordingly, finding an event basis of coordination in such experimental conditions does not contradict the hypothesis of emergent processes as the preferential and 'normal' way for coordinating continuous movements; it rather shows the ability to exploit complementary resources for the temporal control of coordination according to given task constraints.

synchronization process. This could provide a complementary perspective for assessing the effect of external pacing on the global stability of coordination and, more generally, the relationship between the statistical properties at the component level and the collective level.

In this study we address the question of whether coordination patterns are the outcome of a coordinative system that regulates the relative timing between limbs, or whether distinct forms of timing control at the component level contribute to coordination. We hypothesize that the distinct timing processes and related temporal patterns at the component level constitute the background on which coordination bases. In others words, we suppose that according to the nature of the task, effectors are engaged in different timing control modes, and that coordination builds on this intrinsic componential dynamics. To pursue this hypothesis, we analyze simultaneously the relative timing series at the coordinative level in bimanual coordination tasks, and the temporal patterns at the within-limb level in bimanual and unimanual tasks. We compare tapping with oscillation, and self-paced with externally paced tasks, which have commonly been implemented in bimanual coordination studies and have been shown to involve different forms of timing control in unimanual movement tasks. We rely on serial correlation analysis of the within-limb temporal patterns for detecting and discriminating between the involved timing processes.

The investigation proceeded in three stages. The first stage aimed at revealing the statistical signatures of event-based *versus* emergent timing and of absolute *versus* synchronization timing that are known from previous unimanual movement studies. These distinctive statistical properties served as qualitative references for comparison with the statistical properties obtained in bimanual coordination tasks.

The second stage aimed at characterizing the within-hand temporal patterns in bimanual coordination. Our hypothesis predicts that the distinctive signatures of timing modes in the unimanual tasks persist in the corresponding bimanual coordination tasks. If coordination was not based on distinct timing processes and related temporal patterns at the component level, we should obtain similar and non-specific temporal patterns at the within-hand level, whatever the movement and pacing conditions. Beyond the theoretical issue, one consequence of verifying our hypothesis would be that bimanual coordination paradigm should be considered multifaceted, and that the methodological diversity of coordination studies should be taken into account when advancing or challenging alternative theories.

In the last stage, we characterize the correlations of relative timing series in the bimanual coordination tasks. Knowing the temporal patterns at the component level, we ask whether they can fully predict the observed relative timing pattern. In this way, we tested whether formalizations of current bimanual coordination models can account for serial correlations in relative phase when the correlation properties at the component level are taken into account.

2 Method

2.1 Participants and apparatus

Twelve participants (8 male and 4 female, mean age 29 ± 7 yrs) took part in the experiment. Ten participants declared themselves right-handed, and two left-handed. They were non-musicians and declared no particular competence involving specific coordination between the upper limbs, and no neurological injury or recent upper limb injury. They gave written informed consent and were not paid for their participation. The local ethics committee approved the experiment.

Participants were seated comfortably, with the elbows slightly flexed and the forearms supported in a horizontal position. The continuous movement tasks consisted in performing smooth and regular forearm oscillations holding one or two wooden joysticks, 15 cm in length, with a single degree of freedom in the frontal plane. The angular displacement of the joysticks was captured by two potentiometers. The discontinuous movement tasks consisted in performing sequences of discrete taps with the index finger(s) on one or two flat pressure sensors. The positions of the joysticks and the pressure sensors were adjusted to the participants'comfort. Data were recorded via a LabJack U12 device, with a sampling frequency of 300 Hz.

2.2 Experimental conditions

The experiment was composed of two sessions during which the participants performed eight tasks, four per session. The experimental conditions crossed three factors: continuous (oscillatory) *versus* discontinuous (tapping) movements, self-paced *versus* synchronized movements, and unimanual *versus* bimanual coordination tasks. The eight tasks were randomly assigned to the participants over the two sessions. Each task consisted of performing a series of 600 taps or oscillation cycles (trial duration about 5 minutes), with a required frequency of 2 Hz (500-ms periods). For the oscillation tasks, participants were instructed to perform the oscillations as smoothly and regularly as possible, with an amplitude of approximately 45° on either side of the vertical axis. No physical stop or other systematic feedback was given in order to maintain this amplitude. For the tapping tasks, participants were instructed to produce taps that were as brief as possible by minimizing the contact duration on the pressure sensor. For the self-paced tasks, the movement was presented by a 30-s video showing in close-up one (or two) hand(s) performing the task at the required frequency.

Participants did not move while watching the video. Then they immediately began the task, following the required frequency as accurately as possible. Note that we did not use the classical synchronization-continuation paradigm in order to avoid possible influences of timing modes favored in the synchronization phase on those engaged in the continuation phase (Jantzen, Steinberg, & Kelso, 2004, 2005; Kolers & Brewster, 1985); such influences could notably occur in the case where oscillations are continued after being first paced by discrete auditory events, favoring an event-based control of timing. For the synchronization tasks, a PC-driven metronome delivered an auditory signal at the required frequency during the whole duration of the trial. The participants were instructed to keep the taps, or the pronation reversals of oscillations, on the beep. For the unimanual tasks, participants performed the movements with their dominant hand. The bimanual coordination tasks were identical to the corresponding unimanual tasks, except that the primary goal was defined in terms of relative phasing between the hands, with a required relative phase of 0° (in-phase). By definition in this mode of coordination the two hands move in mirror symmetry. In self-paced conditions participants were instructed to maintain the frequency initially imposed by the video display. In synchronization conditions they were asked to focus their attention on the between-hand synchronization as a priority, and to keep synchronized with the metronome.

2.3 Data reduction and analysis

We analyzed the time series of three variables: the periods (PER) for all tasks, the asynchronies (ASYN) between the movement and the metronome for the synchronization tasks, and the relative phase (RP) for the bimanual coordination tasks. The series of periods were defined as the within-hand time intervals between the onsets of two successive taps for the tapping tasks, and as the within-hand time intervals between two successive reversal points in maximal pronation for the oscillation tasks. The series of asynchronies were defined as the time intervals between the onsets of one hand's taps and the corresponding signals of the metronome, and as the time intervals between one hand's pronation reversal points and the signals of the metronome, according to the performed movement. The RP series (in degrees) were computed using the point estimate method, and defined as the difference between the timings of the corresponding right and left tapping onsets or pronation reversal points, normalized by the completed period of the dominant hand.

For all variables we retained for analysis the last 512 points of the series, and we examined the autocorrelation functions (ACF) and the power spectra. Regarding the ACF, we focused in particular on the lag 1 autocorrelation which provides information about the very short-range dependence in the series. Regarding the power spectra, the series of 512 points have been shown to be an acceptable length for obtaining accurate results with spectral analysis (Chen et al., 1997; Delignières et al., 2006; Gilden, 2001). The power spectra were computed using ${}^{low}PSD_{we}$, an improvement of the classical spectral analysis including some preprocessing operations before the application of the Fast Fourier Transform (for details, see Eke et al., 2000). For a reliable estimate of the spectral index, Eke et al. proposed to limit the fitting of the regression line of the power distribution to the low-frequency region of the log-log spectra, determined as $f < 1/8$ of the maximal frequency in the signal. We retained the same boundary frequency in order to perform separate estimations of the regression slopes in the low-frequency and the high-frequency regions of the spectra.

2.4 Statistical analysis

For testing the statistical difference from zero of the lag 1 autocorrelation of PER series, and of the high-frequency spectral slopes of PER series, we performed single-means t-tests with zero reference, per variable and experimental condition.

This current study refers to previous evidence that PER series produced in self-paced tapping and self-paced oscillations possess persistent long-range correlation, or $1/f^{\beta}$ noise, arising from the underlying timing processes (Delignières et al., 2004, 2008; Gilden et al., 1995; Gilden, 2001). The presence of $1/f^{\beta}$ noise in experimental series has often been affirmed on the basis of the linear log-log power spectrum with slope $-\beta$, where β is between 0.5 and 1.5. Recent studies, however, have stressed the fact that the linear shape of power spectra is likely to mimic the presence of long-range correlation in series, even if they only contain short-range correlation. These studies showed the need of a statistical test for the presence of genuine long-range correlation (Thornton & Gilden, 2005; Wagenmakers, Farrell, & Ratcliff, 2004). For attesting the presence of long-range correlation in PER series collected in the unimanual and bimanual self-paced conditions, we applied the ARFIMA/ARMA modeling method (*Autoregressive Fractionally Integrated Moving Average*, Wagenmakers et al., 2004; Torre et al., 2007a). Since a recent study also evidenced persistent long-range correlation in RP series in bimanual oscillation tasks (Torre et al., 2007b), we further performed ARFIMA/ARMA modeling on the RP series obtained in bimanual coordination tasks.

This method consists in fitting 18 models to the series, 9 ARMA models and 9 ARFIMA models, the latter differing from the former by inclusion of a fractional integration parameter. The method selects the best model on the basis of a goodness-of-fit criterion (*Bayes Information Criterion*) based on a trade-off between the accuracy of the fit and the parsimony of the model. Two complementary criteria can then be used for attesting the presence of long-range correlation: (i) the number of series that are better fit by an ARFIMA model than an ARMA model, and (ii) the mean sum of weights captured by ARFIMA models for all series (the weight of a model is derived from the value of the goodness-of-fit criterion and represents the probability that this model is the best of the 18 candidate models for a given series; for details see Torre et al., 2007a).

These statistics provided a complete qualitative characterization of the time series collected in the unimanual and corresponding bimanual conditions. In addition to this qualitative approach, we tested for quantitative differences between the unimanual and corresponding bimanual conditions, considering each criterion (lag 1 autocorrelation, high-frequency slope, low-frequency slope) and each experimental condition (self-paced tapping and oscillations, synchronization tapping and oscillations), and using paired t-tests.

Finally, for an assessment of the coordination between the two effectors, we performed a 2 (movement) x 2 (pacing condition) repeated measures ANOVA on the low-frequency slopes of RP series. Furthermore, we computed the spectral coherence between the individual power spectra of the right and left PER series in the four bimanual coordination conditions. As the spectral characteristics in high-frequency regions essentially express how uncorrelated noise contributes to the signal, the analysis focused on the coherence between the low-frequency components to assess coordination between the effectors' timing patterns. This was done by averaging the squared coherence coefficients obtained in the above-defined low-frequency region.

3 Results

3.1 PER and ASYN series in unimanual tasks

The aim of the analysis of PER and ASYN series obtained in unimanual tasks was to provide a reference for qualitative comparison with PER and ASYN series in the corresponding bimanual coordination tasks. We summarize the results for unimanual tasks in Table 1.

The average autocorrelation functions, computed by point-by-point averaging per variable and experimental condition, are shown in Figure 1. In self-paced conditions, the mean lag 1 autocorrelation of PER series was negative ($t_{11} = -3.20$; $p < 0.05$) in tapping, but positive ($t_{11} = 3.72$; $p < 0.05$) in oscillations (see Table 1). For the synchronization conditions, the mean lag 1 autocorrelation of PER series was negative ($t_{11} = -8.93$; $p < 0.05$) in tapping, but not significantly different from zero ($t_{11} = 1.70$; $p > 0.05$) in oscillations. The mean lag 1 autocorrelation of ASYN series was positive in both cases. The autocorrelation functions of both synchronization tapping and synchronization oscillations ASYN series showed a slow power-law like decay (Figure 1).

The average log-log power spectra, computed by point by point averaging per experimental condition and variable, are also represented in Figure 1. Slope values are shown and also reported in Table 1. For self-paced conditions, the mean slope in the high-frequency region of the log-log power spectra of PER series was positive and significantly greater than zero in tapping ($t_{11} = 3.57$; $p < 0.05$), and not significantly different from zero in oscillations ($t_{11} = -0.38$; $p > 0.05$). In the low frequency region, power spectra exhibited negative slopes in both conditions.

Table 1. Descriptive statistics and dynamical signatures for series collected in unimanual tasks (tapping versus oscillation, self-paced versus synchronization). From top to bottom we present (1) the mean value of series, (2) the standard deviation of series, (3), the mean lag 1 auto-correlation, (4) the mean slope in the high-frequency region of the log-log power spectrum, and (5) the mean slope in the low-frequency region of the log-log power spectrum. Between-participant standard deviations in parentheses.

	Tapping			Oscillations		
	Self-paced	Synchronization		Self-paced	Synchronization	
	PER	PER	ASYN	PER	PER	ASYN
Mean (ms)	481 (±31)	499 (±1)	-60 (±36)	493 (±48)	499 (±2)	11 (±59)
SD (ms)	33 (±12)	32 (±8)	34 (±12)	22 (±11)	17 (±5)	42 (±14)
Mean lag1 AC	-0.11 (±0.12)	-0.33 (±0.13)	0.39 (±0.21)	0.18 (±0.17)	0.06 (±0.12)	0.75 (±0.18)
HF slope	0.41 (±0.40)	0.96 (±0.44)	- -	-0.04 (±0.37)	-0.23 (±0.29)	- -
LF slope	-0.49 (±0.52)	1.11 (±0.43)	-0.69 (±0.48)	-0.88 (±0.50)	1.05 (±0.69)	-0.82 (±0.28)

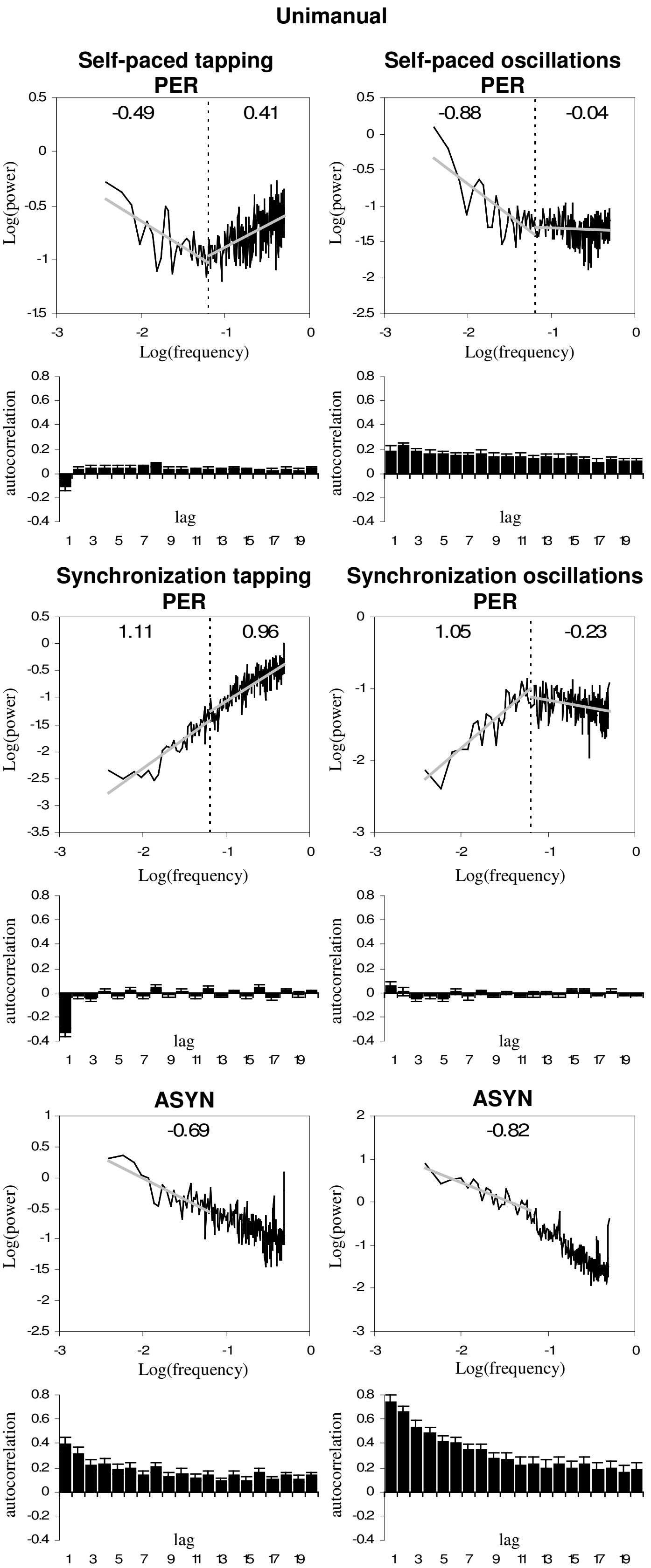

Figure 1. Distinctive signatures of the timing processes involved in unimanual self-paced (upper panels) and synchronization (bottom panels) tasks, in tapping (left) and oscillations (right). Results are presented in the form of average log-log power spectra and average autocorrelation functions of PER series and ASYN series (N = 12). For the autocorrelation functions, errors bars represent standard errors.

For synchronization conditions, the mean slope in the high-frequency region of log-log spectra of PER series was positive ($t_{11} = 7.62$; $p < 0.05$) in tapping, and negative ($t_{11} = -2.81$; $p < 0.05$) in oscillations. The mean slopes in the low-frequency region were positive and close to 1.00 in both conditions. Regarding ASYN series, the log-log power spectra revealed 1/*f*-like shapes in both cases, with negative slopes over the entire range of frequencies.

ARFIMA/ARMA modeling detected long-range correlation in 11 of 12 PER series (92%) in self-paced tapping, with a mean weight of ARFIMA models of 0.95, and in 12 of 12 PER (100%) series in self-paced oscillations, with a mean weight of 0.92. These results confirmed the presence of $1/f^{\beta}$ noise in PER series in unimanual self-paced tapping and oscillations.

3.2 PER and ASYN series in bimanual tasks

Taking the task-specific statistical properties of PER and ASYN series obtained in the unimanual tasks as reference, we tested for the persistence of these properties in the corresponding bimanual coordination tasks. The analyses were performed separately for each hand. For comparison with the results obtained in the unimanual tasks and for greater conciseness, we only present the results obtained for the dominant hand (the same that performed the unimanual tasks) in the bimanual coordination tasks. Results for the non-dominant hand were qualitatively and quantitatively similar. Table 2 summarizes the results obtained in bimanual tasks. PER and ASYN series collected in bimanual tasks presented means and standard deviations similar to those of their unimanual counterparts.

The average autocorrelation functions are represented in Figure 2. These functions are similar to those of their unimanual counterparts (see Table 2 for the values of lag 1 auto-correlation).

The average log-log power spectra obtained in bimanual tasks are also represented in Figure 2. These spectra are qualitatively similar to those obtained in the unimanual tasks. Notably, in the self-paced conditions the mean slope in the high-frequency region of the spectra of PER series was positive ($t_{11} = 6.01$; $p < 0.05$) in tapping, but not significantly different from zero ($t_{11} = 0.25$; $p > 0.05$) in oscillations. The mean slope in the low-frequency region was negative in both cases. In the synchronization conditions, the mean slope in the high-frequency region for PER series was positive ($t_{11} = 11.22$; $p < 0.05$) in tapping, but not significantly different from zero ($t_{11} = -1.74$; $p > 0.05$) in oscillations. Finally, for ASYN series the power spectra showed in both cases 1/*f*-like shapes, with negative slopes over the entire range of frequencies.

ARFIMA/ARMA modeling detected long-range correlation in 10 of 12 PER series (83%) in self-paced tapping, with a mean weight of ARFIMA models of 0.75, and in 9 of 12 PER (75%) series in self-paced

Table 2. Descriptive statistics and dynamical signatures for series collected in bimanual tasks (tapping versus oscillation, self-paced versus synchronization). The presented results are for the dominant hand. From top to bottom we present (1) the mean value of series, (2) the standard deviation of series, (3), the mean lag 1 auto-correlation, (4) the mean slope in the high-frequency region of the log-log power spectrum, and (5) the mean slope in the low-frequency region of the log-log power spectrum. Between-participant standard deviations in parentheses.

	Tapping			Oscillations		
	Self-paced	Synchronization		Self-paced	Synchronization	
	PER	PER	ASYN	PER	PER	ASYN
Mean (ms)	470 (±38)	500 (±0)	-48 (±38)	519 (±65)	499 (±2)	26 (±48)
SD (ms)	30 (±10)	28 (±6)	29 (±8)	22 (±7)	18 (±5)	41 (±13)
Mean lag1 AC	-0.17 (±0.15)	-0.38 (±0.07)	0.35 (±0.19)	0.16 (±0.19)	0.01 (±0.15)	0.79 (±0.07)
HF slope	0.65 (±0.38)	1 (±0.31)	- -	0.02 (±0.31)	-0.09 (±0.18)	- -
LF slope	-0.71 (±0.35)	1.46 (±0.46)	-0.57 (±0.48)	-1.04 (±0.40)	1.07 (±0.61)	-0.83 (±0.29)

oscillations, with a mean weight of 0.81. These results provided strong support for the presence of $1/f^{\beta}$ noise in PER series in bimanual self-paced tapping and oscillations, as in their unimanual counterparts.

Regarding the testing for quantitative differences between the unimanual and corresponding bimanual tasks, t-tests showed no significant differences between unimanual and bimanual lag 1 autocorrelation values in any of the experimental conditions. Likewise, there was no significant difference between unimanual and bimanual high-frequency slopes in any of the experimental conditions, and no significant difference between unimanual and bimanual low-frequency slopes, except for synchronization tapping (1.11 *versus* 1.46, respectively; $t_{11} = -2.29$; $p < 0.05$).

3.3 Coordination in bimanual tasks

Table 3 shows the results obtained with RP series. Mean RP values were close to the expected value of 0°, and variability remained moderate in all conditions.

The average autocorrelation functions are shown in Figure 3. The mean lag 1 autocorrelation of RP series was close to zero in tapping tasks, and slightly positive in oscillation tasks. For all conditions, single means t-tests showed that the mean autocorrelations over the successive lags (N = 19) remained significantly positive, although very slightly in the case of self-paced and synchronization tapping ($t_{19} = 8.77$, $t_{19} = 17.26$, $t_{19} = 5.46$, $t_{19} = 11.21$; $p < 0.05$).

The average log-log power spectra of RP series are also represented in Figure 3. In all cases the power spectra presented negative linear slopes in the low-frequency region. The ANOVA performed on these low-frequency slopes showed only a main effect of the movement factor: the slopes were significantly steeper in bimanual oscillations than in bimanual tapping (-0.72 and -0.36, respectively; $F_{1,11} = 5.56$; $p < 0.05$).

ARFIMA/ARMA modeling detected long-range correlation in 10 of 12 RP series (83%) in bimanual self-paced tapping, with a mean weight of ARFIMA models of 0.89, in 11 of 12 RP series (92%) in bimanual self-paced oscillations, in 7 of 12 RP series (58%) in bimanual synchronization tapping, with a mean weight of ARFIMA models of 0.68, and in 12 of 12 RP series (100%) in bimanual synchronization oscillations, with a mean weight of ARFIMA models of 0.97. These results provided strong indication of the presence of long-range correlation in RP series in self-paced tapping, self-paced oscillations, and synchronization oscillations, while the presence of long-range correlation in synchronization tapping remains uncertain.

Finally, spectral coherence analysis showed high coherences between the individual power spectra of the right and left hand PER series, with mean r^2 coefficients ranging from 0.80 to 0.96 (see Table 3). Figure 4 shows an example of the individual power spectra of the right and left PER series obtained in one trial.

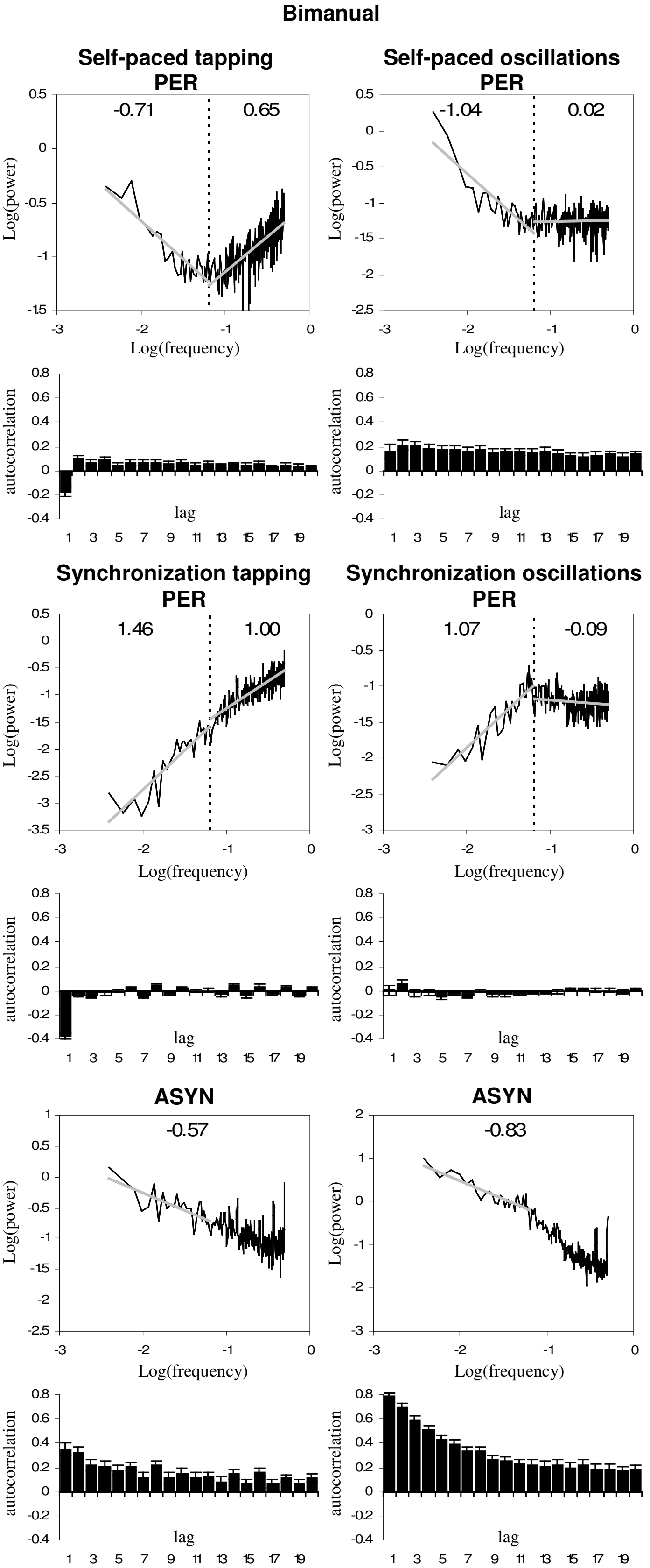

Figure 2. Average log-log power spectra and autocorrelation functions of PER series and ASYN series (N = 12), obtained from the dominant hand in bimanual self-paced (upper panels) and synchronization (bottom panels) tasks, in tapping (left) and oscillations (right). For the autocorrelation functions, errors bars represent standard errors. The distinctive signatures of timing processes were similar to those evidenced in the corresponding unimanual tasks (Figure 1).

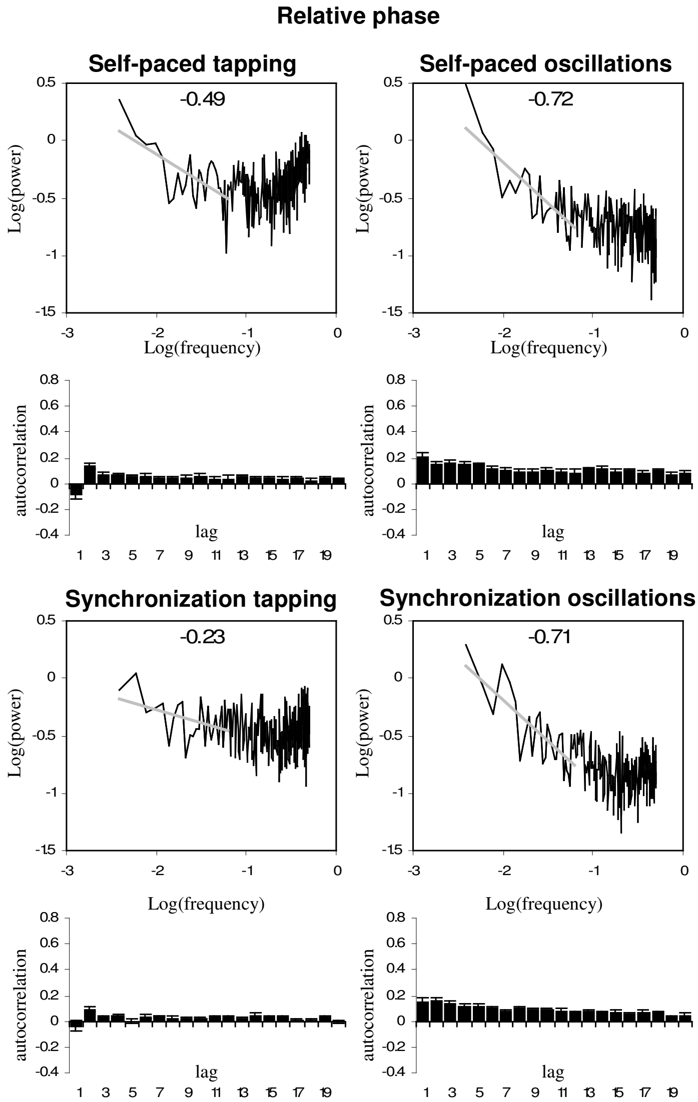

Figure 3. Average log-log power spectra and autocorrelation functions (N = 12) of the relative phase series obtained in the four bimanual coordination conditions. For the autocorrelation functions, errors bars represent standard errors (N = 12).

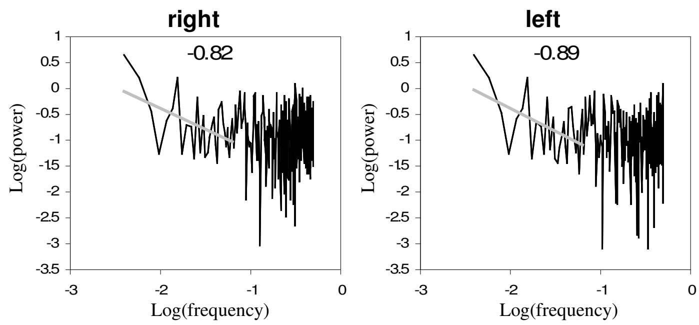

Figure 4. Representative example of the individual power spectra obtained for the right and left hand PER series of one participant, in bimanual self-paced oscillation.

Table 3: Descriptive statistics and dynamical signatures for relative phase series (RP) in bimanual tasks (tapping versus oscillation, self-paced versus synchronization). From top to bottom we present (1) the mean value of RP series, (2) the standard deviation of RP series, (3), the mean lag 1 auto-correlation, (4) the mean slope in the low-frequency region of the log-log power spectrum, and (5) the coefficient of spectral coherence between the two effectors. Between-participant standard deviations in parentheses.

	Tapping		Oscillations	
	Self-paced	Synchronization	Self-paced	Synchronization
RP (deg)	-4 (±6)	-3 (±4)	-5 (±6)	-6 (±5)
SD (deg)	16 (±7)	13 (±4)	9 (±2)	9 (±2)
Mean lag1 AC	-0.08 (±0.15)	-0.03 (±0.14)	0.21 (±0.10)	0.15 (±0.12)
LF slope	-0.49 (±0.57)	-0.23 (±0.34)	-0.72 (±0.40)	-0.71 (±0.33)
Spectral coherence (r^2)	0.94 (±0.08)	0.8 (±0.20)	0.96 (±0.04)	0.95 (±0.05)

4 Discussion

In this study we asked whether temporal patterns in bimanual coordination result from a coordinative system regulating the relative timing between limbs, or if distinct timing processes at the component level are engaged in a specific way in coordination. We compared tapping and oscillation, self-paced and synchronization, unimanual and bimanual task modalities, and analyzed the temporal patterns at the component level and the collective level. Our results showed that the distinctive signatures of timing evidenced in unimanual tasks persisted in the bimanual coordination tasks with the same modalities. We have organized the discussion as follows: In the first part, in order to make this article self-containing, we briefly outline the theoretical assumptions related to the distinctive signatures of timing modes evidenced in unimanual tasks. In the second part, we discuss the persistence of these distinctive signatures in bimanual coordination tasks. Then, we turn to three implications of our results: (i) the need of assessing the relationship between the temporal patterns at the component level and at the collective level for a thorough understanding of bimanual coordination, (ii) the way that current models of bimanual coordination account for the evidenced serial correlation, and (iii) the problems created by the current methodological diversity in research on bimanual coordination.

4.1 Distinctive signatures of timing modes in unimanual tasks

For conciseness, we limit the discussion of the results obtained in the unimanual tasks (Figure 1) to the minimum account that is necessary with regard to our present concerns. We are going to distinguish between event-based and emergent timing on the basis of the characterization of short-range correlation in PER series, and between self-paced and synchronization timing on the basis of long-range correlation properties.

4.1.1 Distinction between event-based and emergent timing. Several complementary approaches to timing control have supported the distinction between two forms of timing, namely, event-based and emergent timing, involved in the performance of discontinuous and continuous rhythmic movements, respectively (Delignières et al., 2004, 2008; Robertson et al., 1999; Ivry, Spencer, Zelaznik & Diedrichsen, 2002; Spencer & Ivry, 2005; Spencer, Zelaznik, Diedrichsen & Ivry, 2003; Zelaznik, Spencer & Doffin, 2000; Zelaznik, Spencer & Ivry, 2002; see also Huys, Jirsa, Studenka, Rheaume, & Zelaznik, 2008).

Event-based timing is assumed to involve a hierarchical structure with a prescriptive entity that is responsible for the explicit representation of temporal goals, independent of the motor execution itself. Cognitive ‘events’ are supposed to represent the informational basis for the timing of movements, so that the event-structure contained in discrete movements shows a similar pattern. This functioning is in accordance with the formulation of the Wing and Kristofferson (1973) model for self-paced tapping:

$$PER_{tap,\, n} = C_n + M_n - M_{n-1} \quad (1),$$

where discrete cognitive events delimit the successive time intervals (C_n) given by the timekeeper, and trigger the execution of the taps at the motor level. The execution of each tap is affected by a motor delay (M_n)

that is considered white noise. Thus, resulting periods of movement are affected by differenced white noise.

Emergent timing, in contrast, is assumed to need no explicit representation of temporal information. Timing emerges from the continuous regulation of the non-temporal parameters that determine the cycle duration of continuous movements such as oscillations or circle drawing. That is, there is no prescription from the cognitive level to the motor level (except a global setting of baseline stiffness), and movement implementation is an integral part of timing. Given the continuous regulation of movements during the cycle, a single error term ξ_n affects the cycle durations D_n determined by movement parameters, instead of affecting the two boundary events, so that the produced periods are given by (Delignières et al., 2004)

$$PER_{osc,\,n} = D_n + \xi_n \qquad (2)$$

Accordingly, the distinctive features of event-based and emergent timing are obvious in the comparison of the short-range correlation properties of PER series, notably in the lag 1 autocorrelation and the high-frequency region of power spectra. For tapping, *i.e.*, event-based timing, the differenced white noise that affects the successive periods (Equation 1) yields a negative lag 1 autocorrelation, and a positive slope in the high-frequencies of power spectra. For oscillations, *i.e.*, emergent timing, in contrast, the presence of a single white noise term instead of differenced white noise affecting the successive periods (Equation 2) should yield a zero or lowered lag 1 autocorrelation, and a zero or flattened high-frequency spectral slope, depending on the correlations carried by the timing mechanism (D) itself.

Our present results reproduced these distinctive properties. In unimanual tapping, PER series presented a negative lag 1 autocorrelation and a positive high-frequency slope. Both were shown to be statistically different from zero. These distinctive properties were similar in self-paced and synchronized tapping. In unimanual oscillations, in contrast, the lag 1 autocorrelation of PER series was positive or not significantly different from zero, and the high-frequency slope was not significantly different from zero or negative, in self-paced and synchronization conditions, respectively. Thus, the characterization of serial correlation in PER series evidenced the involvement of event-based timing in unimanual tapping, and emergent timing in unimanual oscillations, whatever the pacing conditions.

4.1.2 Distinction between self-paced and externally-paced movements. In self-paced movements, the variability in periods has for a long time been assumed to reflect uncorrelated white noise inherent in the involved timing processes. Analyzing long-range correlation, Gilden et al. (1995) first evidenced the presence of $1/f^{\beta}$ noise (*i.e.*, persistent long-range correlation) in PER series in unimanual self-paced tapping, revealed by a negative slope close to –1 in the low-frequency region in addition to the positive slope in the high-frequency region of power spectra. This result has been confirmed by a number of studies (Chen, Repp, & Patel, 2002; Delignières et al., 2004; Gilden, 2001; Madison, 2004, Yamada, 1995), and statistically attested by ARFIMA/ARMA modeling (Delignières et al., 2008; Lemoine et al., 2006). Note that this pattern of persistent correlations was sometimes attributed to the presence of drift in self-paced series (*e.g.*, Pressing & Jolley-Rogers, 1997). Obviously drift can contribute to the presence of persistent dependence in the series. However, Lemoine et al. (2006) showed that $1/f$ fluctuations were still present even when drift was removed from the series. Since the positive slope in the high frequencies was due according to the Wing and Kristofferson model to differenced white noise at the level of motor implementation, the $1/f^{\beta}$ variability was assumed to be in the timekeeping process (Delignières et al., 2004; Gilden et al., 1995; Gilden, 2001). Accordingly, the timekeeper has been modeled by the *shifting strategy model* (Wagenmakers, Farrell, & Ratcliff, 2004), an extension of classical threshold-activation models that provides the timekeeper periods C_n (Equation 1) with $1/f^{\beta}$ correlations (Delignières et al., 2008).

Unimanual self-paced oscillations have classically been modeled by a hybrid limit-cycle oscillator (Kay, Saltzman, Kelso, & Schöner, 1987), assuming that the variability in successive oscillation periods was uncorrelated. Recently, Delignières et al. (2004, 2008) showed that similar to self-paced tapping, PER series in self-paced oscillations present $1/f^{\beta}$ fluctuations: Power spectra of PER series presented the typical negative slope close to –1 in the low-frequency region in addition to the flattened slope in the high-frequencies that characterizes emergent timing, and ARFIMA/ARMA modeling attested the result statistically. Thus, mechanisms at the origin of emergent timing have been considered a source of $1/f^{\beta}$ noise, like event-based processes. Delignières et al. (2008) proposed to model emergent timing using the *hopping model* (West & Scafetta, 2003) for introducing a variable stiffness parameter with $1/f^{\beta}$ correlation over the successive cycles of the limit-cycle model, so that the resulting periods D_n (Equation 2) generated by the limit-cycle oscillator contained $1/f^{\beta}$ noise. A white noise term was added to account for the flat slope observed at high frequencies in the power spectrum.

Our present study reproduced previous results obtained in self-paced conditions: Spectral analysis of PER series collected in self-paced tapping and self-paced oscillations showed a negative slope characteristic of $1/f^{\beta}$ noise, and ARFIMA/ARMA modeling confirmed this result statistically.

In external pacing conditions, without the involvement of an effective synchronization process between movement and the pacing signal, the above characterized variability inherent to rhythmic movements would yield non-stationary asynchronies, since ASYN series are related to PER series by (Chen, Ding & Kelso, 1997, Pressing & Jolley-Rogers, 1997)

$$ASYN_n = ASYN_1 + \sum_{i=1}^{n-1}(PER_i - \tau) \qquad (3)$$

where τ is the constant period of the metronome. So, if PER series contained $1/f^{\beta}$ noise as in self-paced conditions, ASYN series would exhibit persistent fractional Brownian motion, *i.e.*, non-stationary series with positive correlations between the successive increments that correspond to the integration of $1/f^{\beta}$ noise.

Chen et al. (1997) first showed that periods in unimanual synchronization tapping contained anti-persistent noise instead of $1/f^{\beta}$ noise. Our present results are consistent with these results, showing that PER series in synchronization tapping contained anti-persistent noise, characterized by a positive slope in the low-frequency region of power spectra.

In contrast to synchronization tapping, synchronization oscillations have so far not been addressed from the point of view of serial correlation (as an exception, see a preliminary study by Torre et al., 2006); studies have essentially focused on the stability properties of the oscillator's limit-cycle dynamics and have notably addressed the *anchoring* phenomenon, *i.e.*, the local deformation of the phase plane trajectory in the vicinity of a landmark that is synchronized with the pacing signal (*e.g.*, Byblow, Carson, & Goodman, 1994; Carson, 1995; Roerdink, Ophoff, Peper, & Beek, 2008). Our present results revealed exactly the same effect of synchronization on serial correlation in oscillation as in tapping, PER series in synchronization oscillations containing anti-persistent noise.

Because ASYN series are the mathematical integration of PER series (Equation 3), it could be predicted that from the moment that the latter were anti-persistent noises, the former should present persistent, $1/f^{\beta}$-like correlations (Eke et al., 2000). This result has indeed been reported previously (Chen et al., 1997, 2001; Chen et al., 2002; Ding, Chen, & Kelso, 2002; Pressing & Jolley-Rogers, 1997), and the spectral analysis of ASYN series in synchronization tapping and oscillations gave consistent results in this study.

As mentioned above, despite these similar effects of synchronization on PER series in tapping and oscillations, the distinguishing features of event-based and emergent timing in the high frequency region were preserved in synchronization. In sum, the characterization of serial correlation in PER series provides statistical signatures allowing to distinguish between event-based and emergent timing, involved respectively in discontinuous and continuous movement tasks, and between absolute and synchronization timing processes, involved, respectively, in self-paced and externally paced conditions.

4.2 Persistence of the distinctive signatures of timing modes at the component level in bimanual coordination

Our results showed that PER series in bimanual coordination tasks presented consistent temporal patterns with specific correlation structures according to task modalities. These statistical properties were qualitatively the same as the distinctive signatures of event-based *versus* emergent, and self-paced *versus* synchronization timing evidenced in the corresponding unimanual tasks (see Figures 1 and 2).

In addition to the finding that the statistical properties were qualitatively the same in unimanual and bimanual series, we found no significant quantitative difference between the lag 1 autocorrelations, the high-frequency spectral slopes, and the low-frequency spectral slopes of unimanual and bimanual PER series, except for the low-frequency slopes in synchronization tapping (see Tables 1 and 2). These results showed that distinct event-based or emergent, absolute or synchronization timing processes were involved when participants performed bimanual coordination in tapping or oscillations, and in self-paced or externally paced conditions, respectively.

As mentioned in the introduction, obtaining a stable relative timing pattern given a particular structure of variability in the within-hand temporal patterns implies that this structure is highly similar or coherent across the two hands. The results of the spectral coherence analysis performed on PER series showed very high coherence coefficients ranging from 0.80 to 0.96 for the four conditions. As a comparison, we computed the spectral coherence in 20 randomly chosen pairs among simulated $1/f^{\beta}$ noise series[2] (with similar low-frequency spectral slopes about -0.9). The mean coherence coefficient computed over the low-frequency region of the spectra was only 0.29. Since the distinctive within-hand temporal patterns were similar in unimanual and bimanual tasks, the coordination or coupling process that yielded the strong coherence and the stable relative timing between the hands neither caused these correlations nor substantially modified them. In this view, we

[2] The simulations were performed using the algorithm proposed by Davies and Harte (1987), which generates 'exact' fractional Gaussian noise series with a known Hurst exponent (H). We set H = 0.9 for the present simulation.

assume that the different forms of timing control yielding specific within-hand temporal patterns constitute a basis for coordination control. Saying that we do not mean that the timing of components entirely determines, or ‘pilots’ coordination, but that assessing coordination without a close connection to timing processes might conceal the specific ways in which timing processes contribute to build the dynamics of rhythmic coordination.

4.3 Relationships between the temporal patterns at the component level and the coordinative level

A recent study of bimanual oscillations clearly demonstrated the presence of $1/f^{\beta}$ noise in RP series, in in-phase as well as in anti-phase coordination (Torre et al., 2007b). One prevailing account of the finding of $1/f^{\beta}$ noise across diverse domains of research including human cognition and movement science considers it as a typical property of the dynamics emerging from (nonlinear) interactions between components in a self-organized system. $1/f^{\beta}$ noise has been assumed to sign metastable patterns, *i.e.*, the relative coordination that arises from the compromise between the preservation of the independence of components, and their tendency to merge into a collective behavior (*e.g.* Beltz & Kello, 2006; Kello, Beltz, Holden, & Van Orden, 2007). In this sense, the finding of $1/f^{\beta}$ noise in RP series matches very well with the dynamical systems theory of bimanual coordination, and it might be tempting to conclude in general terms that $1/f^{\beta}$ noise is a signature of coordination dynamics.

However, these results of the ANOVA performed on the low-frequency spectral slopes of RP series, and ARFIMA/ARMA modeling, consistently characterized RP series as $1/f^{\beta}$ noise in oscillations, but not in tapping. At a first glance, this seems actually consistent with the idea that coordination in tapping and oscillations involves distinct event-based and emergent forms of control. Beyond this consideration, however, it is necessary to analyze the relationship between the relative timing pattern at the collective level and the temporal patterns at the component level, to assess the respective contributions of timing and coordination.

Indeed, RP series contained $1/f^{\beta}$ noise in bimanual oscillations but not in tapping even though our results showed that PER series contained $1/f^{\beta}$ noise both in self-paced tapping and in self-paced oscillations. In a similar way, the ANOVA showed no difference in the correlation structure of RP series between self-paced and externally paced coordination even though our results showed that PER series were qualitatively completely different as they contained $1/f^{\beta}$ noise in the self-paced conditions and anti-persistent noise in the synchronization conditions. With regard to these discrepancies, one may ask what relationship, or what kind of coupling between components would account for the correlation structures in RP series, given the specific temporal patterns at the component level across task modalities. Could an identical coupling process actually be at work in each particular case mentioned above?

For instance, let us consider the case of self-paced *versus* externally paced coordination and proceed by simulation. We simulated 40 series of $1/f^{\beta}$ noise (characterized by similar low-frequency spectral slopes of about –0.9), representing PER series in bimanual self-paced conditions, and 40 series of anti-persistent noise (characterized by similar low-frequency spectral slopes of about 1.1), representing PER series in bimanual synchronization conditions.[3] Then we computed the RP series for 20 randomly chosen pairs among the simulated series for the two conditions. Results showed that the relative phase between two uncoupled series of $1/f^{\beta}$ noise was non-stationary (${}^{low}PSD_{we}$ characterized the series as fractional Brownian motions, with β exponents higher than 1.0, see Eke et al., 2000), whereas the relative phase between two uncoupled series of anti-persistent noise was stationary (the series were characterized as fractional Gaussian noises, with β exponents of less than 1.0). Obviously, such discrepancies tend to question the idea that the same coupling applies without quantitative or even qualitative changes, whatever the conditions of performance of coordination and the corresponding structure in the temporal patterns of components.

4.4 Can current models of bimanual coordination account for empirical serial correlations?

Current bimanual coordination models have not been designed to account for serial correlation, as their aim is generally to account for stability properties at the collective or the component levels. Torre et al. (2007b) showed that, in their original formulations, neither the *multiple timer model* (Helmuth & Ivry, 1996; Ivry & Richardson, 2002; Ivry, Richardson, & Helmuth, 2002) for bimanual tapping nor the HKB model (Haken et al., 1985) for bimanual oscillations accounts for the experimentally observed serial correlations in relative phase.

The immediately following step would be to test whether the architectures of these models, notably the formalization of their respective coupling functions, could account for the observed correlations in RP series once the variability at the component level is provided with the appropriate task-specific correlation properties. Therefore, we incorporated the shifting strategy model at the timekeeper level of the multiple

[3] Simulations were performed using the algorithm proposed by Davies and Harte (1987). For the first set of series, we set H = 0.9. For the second, we used the same H value and differentiated the obtained series in order to simulate anti-persistent noise series similar to those experimentally observed in synchronization period series.

timer model, and the hopping model at the oscillator level of the HKB model. Both the shifting strategy model and the hopping model were shown to account for the $1/f^{\beta}$ correlations observed in unimanual self-paced tapping and oscillations (for details, see Delignières et al., 2008).

The multiple timer model for self-paced bimanual tapping is an extension of the Wing and Kristofferson model for unimanual tapping. The model assumes two (or more) co-acting timing processes, each associated with an effector. A *gating* process is responsible for the temporal coupling of these timers, by integrating the timing processes into a sequence of common cognitive 'events' that trigger the two limbs simultaneously, in the case of in-phase coordination. At the effector level, the execution of taps is affected by motor delays that have commonly been considered to contain uncorrelated white noise. Thus, the presence of correlation in RP series is necessarily related to the central timekeeper level. Obviously, because of the simultaneous triggering of the two effectors, the multiple timer model can not account for any form of correlation in RP series between the two hands, yielding white noise whatever the structure of variability generated by the two timing processes. Thus, experimentally observed correlations in relative phase suggest a 'weaker' form of coupling between timers, rather than their integration into a single common signal as assumed by the multiple timer model.

Recently, Torre and Wagenmakers (in press) proposed to model bimanual self-paced tapping by incorporating the shifting strategy model at the level of the two timers. The functioning of the shifting strategy model is similar to classical activation-threshold mechanisms, where an activation process grows linearly in time, until reaching a threshold level. The reaching of the threshold determines a particular event in time that triggers the tap and resets the activation process. The shifting strategy model simply assumes a threshold with plateau-like variations over time and an activation process whose speed varies between successive iterations, following an auto-regressive process. The authors proposed a continuous coupling of the thresholds of the two timers, and an alignment of the onsets of the two activation processes, and showed that this coupling solution accounts for the serial correlations of RP series in bimanual self-paced tapping (Torre & Wagenmakers, in press).

Further studies should test whether this model could be extended to bimanual synchronization tapping. Unimanual synchronization tapping has commonly been modeled by an auto-regressive error correction process (Vorberg & Wing, 1996). Recently, Torre and Wagenmakers (in press) showed that when considering the timekeeper as a source of $1/f^{\beta}$ noise, this error correction model accounts for the anti-persistent noise in PER series in synchronization tapping. Combining the Torre-Wagenmakers model with an auto-regressive error correction process might well account for the experimental correlation properties in bimanual synchronization tapping.

For bimanual self-paced oscillations, we tested in the present study the HKB model of coupled oscillators (Haken et al., 1985; Schöner & Kelso, 1988) which accounts for coordination by a nonlinear coupling between two hybrid limit-cycle oscillators, based on the two oscillators' state variables (position and velocity):

$$\begin{aligned} \ddot{x}_1 + \delta\dot{x}_1 + \lambda\dot{x}_1^{\,3} + \gamma x_1^{\,2}\dot{x}_1 + \omega^2 x_1 &= (\dot{x}_1 - \dot{x}_2)[a + b(x_1 - x_2)^2] \\ \ddot{x}_2 + \delta\dot{x}_2 + \lambda\dot{x}_2^{\,3} + \gamma x_2^{\,2}\dot{x}_2 + \omega^2 x_2 &= (\dot{x}_2 - \dot{x}_1)[a + b(x_2 - x_1)^2] \end{aligned} \qquad (4),$$

where x_i is the position of oscillator i, and the dot notation represents derivation with respect to time. The left side of the equations represents the limit cycle dynamics of each oscillator determined by a linear stiffness parameter (ω) and damping parameters (δ, λ, and γ), and the right side represents the coupling function determined by parameters a and b.

We combined the HKB model and the hopping model, providing the stiffness parameter ω of each oscillator with $1/f^{\beta}$ correlation over the successive cycles. The parameters of the hopping model were chosen in order to provide effectors period series with the experimentally observed correlations (for details, see Delignières et al., 2008). Parameters of the limit-cycle oscillator were set as $\delta = 0.5$, $\lambda = 0.02$, $\gamma = 1.0$, and $\omega = 4\pi$. Finally, for obtaining a variability of RP series similar to that experimentally observed, we set the coupling parameters $a = 4$ and $b = 4$. We simulated 100 series of 512 data points.[4]

This 'hopping-HKB' model provided a satisfactory account of our experimental results. Simulated PER series also contained $1/f^{\beta}$ noise, with a mean low-frequency spectral slope of -0.73 (-1.04 for experimental series), and ARFIMA/ARMA modeling detected long-range correlation in 89% of simulated series. The mean standard deviation of simulated PER series was 20 ms (22 ms for experimental series). Simulated RP series also contained $1/f^{\beta}$ noise, with a mean low-frequency spectral slope of -0.72 (–0.72 for experimental series), and long-range correlation detected in 99% of series. The mean standard deviation of simulated RP was 10° (9° for experimental series).

[4] Simulations were performed using a four-stage Runge-Kutta algorithm, following the scheme described by Burrage, Lenane, and Lythe (2007, pp. 11-12), for second-order stochastic differential equations with additive noise. We used a fixed step size of 0.001 sec.

For bimanual synchronization oscillations, we tested the *parametric driving model* (Fink et al., 2000; Jirsa et al., 2000), an extension of the HKB model including a parametric coupling to the metronome:

$$\ddot{x}_1 + \delta\dot{x}_1 + \lambda\dot{x}_1^{\,3} + \gamma x_1^{\,2}\dot{x}_1 + \omega^2 x_1 = (\dot{x}_1 - \dot{x}_2)[a + b(x_1 - x_2)^2] + e_1\cos\Omega t + e_2 x_1\cos\Omega t$$

$$\ddot{x}_2 + \delta\dot{x}_2 + \lambda\dot{x}_2^{\,3} + \gamma x_2^{\,2}\dot{x}_2 + \omega^2 x_2 = (\dot{x}_2 - \dot{x}_1)[a + b(x_2 - x_1)^2] + e_1\cos\Omega t + e_2 x_2\cos\Omega t \quad (5),$$

where Ω represents the frequency of external pacing. The first cosine term represents linear driving and the second a parametric driving, depending on effector position.

As for the HKB model of self-paced coordination, we introduced the hopping model at the oscillator level in the parametric driving model. Parameters of the hopping model, the limit-cycle oscillators, and the coupling function were identical to those used in the previous simulation. The driving parameters e_1 and e_2 were set to 10, and we simulated 100 series.

The 'hopping-parametric driving' model provided satisfactory account for our experimental results. In contrast to self-paced coordination, simulated PER series were anti-persistent noise, with a mean low-frequency spectral slope of 1.10 (1.00 for experimental series). The mean standard deviation of simulated PER series was 17 ms (18 ms for experimental series). Simulated RP series were $1/f^{\beta}$ noise, with a mean low-frequency spectral slope of -0.77 (-0.71 for experimental series), and ARFIMA/ARMA modeling detected long-range correlation 99% of series. The mean standard deviation of simulated RP series was 9° (9° for experimental series).

These results suggest that the coupling principles that underlie the HKB model and the parametric driving model can generate the correlation properties of RP series obtained in self-paced and in synchronization coordination, respectively. However, to cope with the variability generated at the oscillator level and to stabilize relative phase, we had to apply coupling parameters that were much stronger than those commonly reported in the literature (Assisi et al., 2005; Fink et al, 2000; Leise & Cohen, 2007). Further study should clarify whether these models conserve their basic stability properties (differential stability of in-phase and anti-phase, phase transition) under such parameter settings.

4.5 Bimanual coordination: A multifaceted paradigm

In sum, we have shown that the distinctive serial correlation properties of event-based *versus* emergent and self-paced *versus* synchronization timing found in unimanual tasks persist in the corresponding bimanual coordination tasks. This finding led us to distinguish the forms of timing control yielding the specific within-hand timing patterns from the control of coordination yielding a stable relative timing between the hands, considering the former as the background on which the latter builds.

Some authors have recently emphasized the need for detailed accounts of component dynamics in addition to collective dynamics for the understanding of bimanual coordination (Peper et al., 2004; Ridderikhoff, Peper, & Beek, 2005). Especially, Riley et al. (2001) suggested that correlated forms of noise at the component level might contribute to variability observed at the collective level. Our present results underline the need of analyzing the component and the collective levels simultaneously, for decomposing the respective contributions of timing control and coordinative processes in coordination. Integrating the two levels, our preliminary amended models seemed able to generate serial correlations similar to those observed experimentally.

Following from these considerations, the 'bimanual coordination paradigm' appears multifaceted, and it might be inappropriate for the studies to generalize theories and models of bimanual coordination over various instantiations of the bimanual coordination paradigm. Regarding the pacing conditions, research on the stability properties of coordination has often used a metronome to prescribe either a globally steady or a progressively increasing tempo on the rhythmic movements. In these cases, the use of the metronome has generally been considered *non-specific* (Fink et al., 2000), in the sense that it was assumed to have no effect on the dynamics of the effectors involved in coordination. One may wonder what confirms the actual non-specificity of metronome use in many studies, since Fink et al. (2000) and Jirsa et al. (2000) showed an effect on the within-cycle dynamics, and our present results showed moreover a qualitative change in the serial correlation properties of periods.

Regarding the discontinuous/continuous character of movements, one may wonder to what extent it is actually legitimate to generalize an event-based timekeeper model for bimanual tapping like the multiple timer model, to bimanual oscillations, considering it as an alternative to the coupled oscillator model (see, for example, Ivry & Richardson, 2002). Typically, the multiple timer model and the coupled oscillator model are in line with different theoretical perspectives on coordination, the information processing perspective and the dynamics systems perspective, respectively. The former has essentially conceptualized coordination dynamics in terms of congruent event-structure representing timing goals that order the movement of limbs over time, and the temporal relationship between them. The latter has considered coordination dynamics as the behavior that emerges from multiple interactions in a self-organized system, without specific regard to any timing function.

But at the same time, the former has essentially been based on bimanual tapping tasks, while the latter has essentially studied bimanual oscillation tasks. These theories have often been considered as contrasting - perhaps irreconcilable – accounts of the same phenomenon, bimanual coordination. Following the present concerns, one might assume that the methodological diversity that characterizes research on bimanual coordination, and the distinct ways for regulating temporal patterns that this diversity implies, have contributed to a certain extent to the parallel development of these theories. These results suggest that each approach possesses its own relevance, regarding a specific class of tasks, and that both should co-exist in a global account for timing and coordination control.

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7 MY THESIS IN A NUTSHELL

The recurring theme of the present thesis may be the outstanding need for trans-theoretical approaches of given objects of research. In our present approach, the central issue of the role of timing control in bimanual coordination gets tangled up in two further issues: on the one hand, the coexistence of two historically divergent approaches of bimanual coordination and timing and, on the other hand, the question of the relevance and the way to deal with long-range correlation in frameworks which have ignored it so far. With particular regard to these concerns, trans-theoretical approaches may enlighten a given issue by redefining the limits of relevance of different - on the best complementary but most of time rather hermetic - theoretical perspectives. In this concluding section, I wish to foreground three ideas and related perspectives of my thesis:

1) ***Serial long-range correlation should be considered in bimanual coordination, a research field where variability properties have constituted a central theme. The most useful and 'impacting' way to deal with long-range correlation, at least in the present state of art, follows a domain-specific, mechanistic approach.***

The consideration of serial long-range correlations drives to rethink the notion of stability and offers an original and pertinent entry point into behavioral neuroscience / experimental psychology research issues. However, one should be aware of the different, *nomothetic* and *mechanistic* perspectives on $1/f^{\beta}$ noise in literature, and their respective influences on research. This does not mean that one perspective is superior to the other one. Nomothetic and mechanistic perspectives address the issue of long-range correlation at different levels of analysis, which makes them complementary rather than competing. While nomothetic accounts of $1/f^{\beta}$ noise might appear more powerful at the first glance, mechanistic accounts allow to establish tight relationships with current theoretical frameworks by giving up the idea of a universal explanation of $1/f^{\beta}$ noise. Actually, the dialogue between the two perspectives on $1/f^{\beta}$ noise is quite apparent: for instance, one may argue that the development of low-dimensional models for $1/f^{\beta}$ noise should be motivated by some universal principle like metastability or self-organized criticality. Inversely, if different mechanistic approaches of $1/f^{\beta}$ noise in different domains of research come to develop quite simple and qualitatively similar models for $1/f^{\beta}$ noise, then this convergence may indicate the existence of a same generic principle that encompasses these mechanisms.

We have argued that long-range correlations are a property that theory and models should capture. Although current theories of coordination and timing have based to a large extent on empirical variability properties providing very valuable and powerful frameworks for thinking about the way behavior organizes, models of neither theoretical framework were developed with regard to the long-range correlation phenomenon. In this view, the value of the mechanistic approach is to allow to merge serial correlation and domain-specific theory by complying with the strengths of the established theoretical frameworks, in order to examine whether they could account – or could be amended to account – for the empirical correlations. Importantly, we did not wish to deal with empirical long-range correlation properties by adding new parameters to improve the data fit of models. As an example, the temporal coupling formalized in the *Multiple Timer Model* does not allow to account for any form of correlation in the relative phase in bimanual tapping; this is not a quantitative but a qualitative matter due to the form of absolute coupling assumed by this model. Then, we believe that the interest in assessing serial long-range correlation is to use the evidenced properties as criteria for probing the capacity of theories and models to qualitatively account for them.

2) ***Bimanual coordination performance engages a componential timing function and a coordinative function, the latter building on the former.***

Given empirical serial correlation properties, we have argued that single-hand timing processes constitute a basis on which coordination builds. That is, we oppose the idea that the observed within-hand timing is only the consequence of the timing of the coordinated limb activity. Of course, this is not meant to exclude that coupling also constrains the componential dynamics. Now, does our present thesis come down to previous hypotheses of the "timing basis" (Semjen, 2002), the "timing goals" (Semjen & Summers, 2002), generally speaking the temporal control hypothesis in bimanual coordination? We believe that it does not.

Indeed, an essential point in the body of studies in line with the temporal control hypothesis is that the control of time intervals is performed by a timekeeping structure separated from the implementation level. Then, it has been assumed that the same entities or processes which are responsible for the control of within-hand timing in unimanual tasks control the between-hand timing of coordinated bimanual movements. That is, timing processes are the same but control shifts from absolute time intervals to relative time intervals, so that the empirical properties of variability (*e.g.*, Weberian increase of variability with time interval duration and negative lag 1 auto-correlation) imputed to the two-level organization of timing control are revealed by examination of relative timing instead of absolute timing. Important in these accounts of coordination is that the within-hand timing becomes basically the result of the control of between-hand timing, which contrasts with our present hypothesis.

To reframe the issue of timing in coordination in our present approach, the evidence for an emergent form of timing besides event-based timing control has probably been crucial. The notion of emergent timing avoids an oversimplified distinction between 'temporal' coupling and 'trajectory' coupling, since in the emergent timing conceptualization, the properties of the effector system and its movements *make* timing. It further implies that the assumption of temporal control in coordination does not automatically require one to agree with the operation of a prescriptive level that is responsible for timing: in fact, it puts into perspective the hypothesis of sequentially, or hierarchically organized timing, with a central timekeeper which controls time independently of the implementation level. Timing indeed appears to be adaptive and controlled in distinct ways as well in unimanual movement, where within-hand timing constitutes the tasks goal, as in bimanual movements, where the componential timing is an integral (although not exclusive) part of coordination.

Previous efforts to overcome actual limitations of bimanual coordination theories proposed to associate different levels of explanation. On the one hand for instance, Semjen (2002) proposed to account for coordination in continuous movement tasks (as opposed to discrete movement tasks) by considering a 'trajectory-level coordination' in addition to 'timing-level coordination'. On the other hand, Beek et al. (2002), proposed to overcome limitations of the HKB coupled oscillator model by unfolding it into a four-oscillator system which combines a neural level of coupled oscillators and a limb-level of coupled oscillators.

Our present work leads us to think coordination neither in terms of a 'constraint-based' decomposition (temporal congruence / spatial congruence of limbs' movements) nor in terms of a 'structural-based' decomposition (neural level / limb level). Instead we propose a functional decomposition considering the respective contributions of a (single-limb) timing function and a coordinative function simultaneously involved in bimanual coordination; both being likely to be acted by processes at either of the neural or peripheral levels, and to act on either of the temporal or spatial aspects of movements[7].

[7] See also Diedrichsen, Criscimagna-Hemminger, & Shadmehr (2007) for an example of dissociation between timing and coordination as distinct functions involved in a unimanual task associating a arm reaching component and a thumb press component.

3) ***The limits of relevance of the two theoretical perspectives on bimanual coordination, i.e. the dynamical systems perspective and the representational perspective, should be reconsidered with respect to task definition, notably.***

Dynamical systems and information processing perspectives have traditionally developed in parallel and provided challenging accounts of bimanual coordination, the first conceptualizing coordination in terms of emergent behavior in self-organized systems and the second in terms of representation of timing goals ordering the movements of effectors. The way the two approaches have dealt with variability properties seems telling in this regard, since the properties on which theory has build on the one hand have mostly been disregarded by the alternative framework. However, either dynamical systems and representational models of coordination should live up to the evidence of all these empirical features (Krampe et al., 2002) which moreover include serial (long-range) correlation.

Previous efforts to conciliate these two alternative theories mainly aimed to show that given results which were usually accounted for by non-linear oscillators systems can also be explained by an information processing, or representational approach and, inversely, that some results which were traditionally considered as supporting the representational approach can also be explained by the dynamical systems framework (Rosenbaum, 2002). Were such studies indeed conciliation efforts or rather attempts to reinforce one theory by accounting for features which were so far acknowledged as proving the strength of the challenging theory...? Anyhow, these studies point the idea that the historically hermetic development of the two frameworks seems quite inappropriate and regrettable.

Our present approach leads us to consider an epistemological dimension of this theoretical divergence. Because of mathematical convenience the dynamical systems approach has essentially focused on cyclical, oscillatory movement; similarly, theories of coordination which follow the Wing and Kristofferson framework have mainly focused on brisk movement tasks entailing an apparent event-structure. As we showed that the distinctive correlation properties entailed by event-based *versus* emergent timing as a function of the discontinuous / continuous character of unimanual movement persist in the corresponding bimanual coordination tasks, our present work tends to support the idea that the theoretical gap has to a large extent been deepened by methodological discrepancy. Then, seeing that the different instantiations of bimanual coordination are not neutral with respect to the organization of processes underlying behavior, theoretical perspectives may loose part of their respective generalizability. That is, the limits of relevance of the dynamical systems and representational accounts of bimanual coordination and timing should be redefined in accordance.

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APPENDIX 1

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Fractal models for event-based and dynamical timers

Didier Delignières, Kjerstin Torre & Loïc Lemoine

EA 2991, University Montpellier I, France

Abstract: Some recent papers proposed to distinguish between event-based and emergent timing. Event-based timing is conceived as prescribed by events produced by a central clock, and seems to be used in discrete tasks (*e.g.*, finger tapping). Emergent or dynamical timing refers to the exploitation of the dynamical properties of effectors, and is typically used in continuous tasks (*e.g.*, circle drawing). The analysis of period series suggested that both timing control processes possess fractal properties, characterized by self-similarity and long-range dependence. The aim of this article is to present two models that produce period series presenting the statistical properties previously evidenced in discrete and continuous rhythmic tasks. The first one is an adaptation of the classical activation/threshold models, including a plateau-like evolution of the threshold over time. The second one is a hybrid limit cycle model, including a time-dependent linear stiffness parameter. Both models reproduced satisfactorily the spectral signatures of event-based and dynamical timing processes, respectively. The models also produced auto-correlation functions similar to those experimentally observed. Using ARFIMA modeling we show that these simulated series possess fractal properties. We suggest in conclusion some possible extensions of this modeling approach, to account for the effects of metronomic pacing, or to analyze bimanual coordination.

Key words: Event-based timers, Dynamical timers, Fractal processes, Model, Variability

Address for correspondence:
Pr. Didier Delignières (PhD),
EA 2991 Motor Efficiency and Deficiency
Faculty of Sport Sciences,
University Montpellier I,
700 avenue du Pic saint Loup, 34090 Montpellier – France
didier.delignieres@univ-montp1.fr

Timing was for a long time considered a general-purpose ability (Keele & Ivry, 1987; Treisman, Faulkner, & Naish, 1992; Ivry & Hazeltine, 1995). This idea was supported by a number of correlation studies showing that timing variability was significantly correlated across a range of different rhythmic tasks (Keele, Pokorny, Corcos, & Ivry, 1985; Keele, Ivry, & Pokorny, 198; Franz, Zelaznik, & Smith, 1992). These studies suggested the presence of a common timing process, a fundamental timing ability that could be shared by a variety of motor tasks. Nevertheless, several complementary studies addressing the structure of variability in time series, neuro-anatomical contributions, formal distinctions, and spectral properties of time series, provided consistent insights that oppose the assumption of a unique timing process.

Robertson et al. (1999) analyzed timing variability in finger tapping and in continuous circle drawing. They showed that individual differences in the variability of timing in the tapping task were not significantly correlated with the individual differences in the variability of timing in drawing (see also Zelaznik, Spencer, & Doffin, 2000). The authors suggested that these two tasks could involve distinct timing control processes. Finger tapping was considered as representative of a class of tasks where timing follows from the operation of an 'internal clock', engaging an explicit representation of the temporal goal. This mode of timing control was then referred to as explicit. The key feature of this class of tasks could be related to their discreteness: for example finger tapping appears as the concatenation of discrete movements, in response to cognitive events. In contrast, other tasks require smooth and continuous movements. In this case, timing has been assumed to be controlled on the basis of a different process, involving the dynamical properties of the effector, with non-temporal parameters such as muscle stiffness. The authors proposed to refer to this latter type of timing as implicit. In order to establish the importance of the distinction between continuous/discontinuous movements, Zelaznik, Spencer, and Ivry (2002) developed a intermittent circle drawing task, where participant were instructed to pause between each circling cycle. Despite the similarity between intermittent and continuous circle drawing, the variability of periods in the intermittent drawing task correlated with finger tapping while continuous drawing correlated neither with tapping nor with intermittent drawing.

Spencer and Ivry (2005) noted that 'explicit' timing should not be considered as conscious. To avoid confusion, they proposed to refer to this process as *event-based* timing. Similarly, they proposed to qualify 'implicit' timing as *emergent*, considering that in this case timing results from the operation of peripheral, non-cognitive parameters. Spencer, Zelaznik, Diedrichsen, and Ivry (2003) provided a neurological support for this distinction, showing that patients with cerebellar damage were selectively impaired in discontinuous rhythmic tasks, but were unimpaired in a continuous task (see also Spencer, Ivry & Zelaznik, 2003; Spencer & Ivry, 2005).

On a more formal basis, Schöner (2002) proposed a similar distinction between event-based and *dynamical* timers. The former is based on the hypothesis of 'internal clocks' generating periodic timing events. In contrast, dynamical timers exploit the limit-cycle dynamics of effectors, which are considered as self-sustained oscillators. Within the limit cycle, particular events (such as movement reversals) can be used to delimit time intervals. The duration of the time intervals produced by this model depends on a single control parameter, the linear stiffness.

Delignières, Lemoine and Torre (2004) evidenced additional properties of event-based vs. dynamical timing processes. They analyzed the spectral properties of period series produced in continuation tapping and in rhythmic forearm oscillations, the first being supposed to exploit event-based timing, and the second dynamical, or emergent timing. They showed that the two kinds of timing processes were characterized by specific signatures in the log-log power spectrum (Fig. 1). For tapping series, the log-log spectrum was characterized by a positive slope in the high frequency region, whereas for oscillation series, the high-frequency slope remained slightly negative. In both cases, the low-frequency region of the spectrum presented a negative linear slope, close to minus one, which is generally considered as revealing the presence of $1/f$ noise in the series.

This typical log-log spectrum for tapping series has been already evidenced in a number of previous experiments (see, for example, Chen, Repp, & Patel, 2002; Gilden, 2001; Gilden, Thornton, & Mallon, 1995; Yamada, 1995). Gilden et al. (1995) interpreted this result on the basis on the model proposed by Wing and Kristofferson (1973), which supposes that the production of each interval is based on two independent processes: an internal clock, which provides a series of temporal intervals C_i, and a motor component, responsible for the execution of the tap i at the expiration of the interval C_i. This motor component does not operate instantaneously, and all taps have an assigned motor delay M_i. The observed period I_i then depends on both components

$$I_i = C_i + M_i - M_{i-1} \quad (1)$$

In the original model, C_i and M_i were assumed to be uncorrelated white noise processes. According to Gilden et al. (1995), the positive slope observed in the high-frequency region of the log-log spectrum

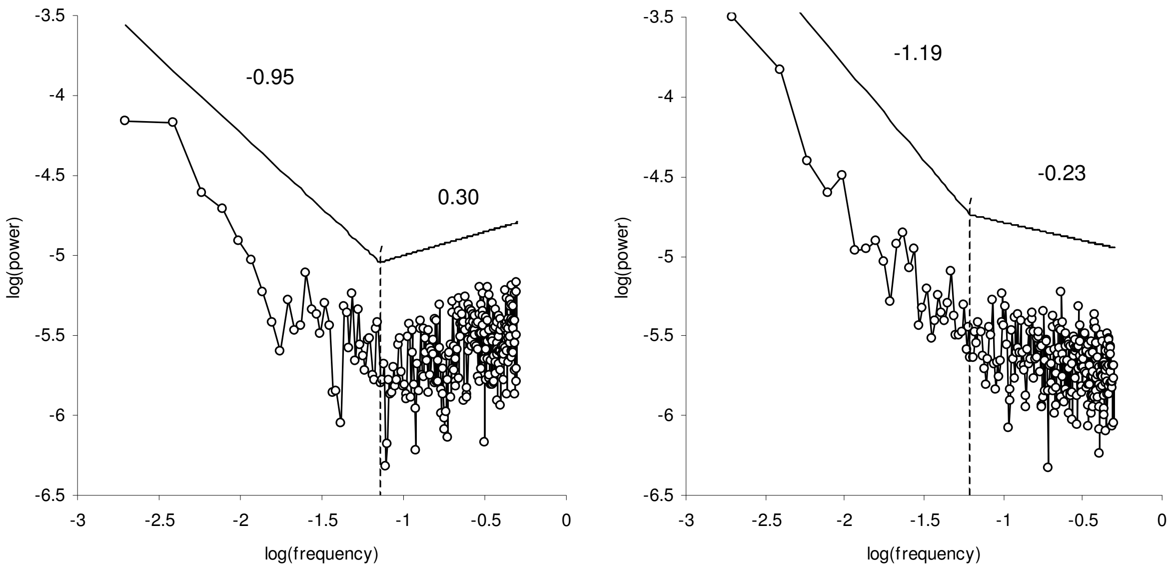

Figure 1: Averaged power spectra (in log-log coordinates), for the tapping task (left) and the oscillation task (right). From Delignières et al. (2004).

could arise from the presence of a differenced white noise process in the series ($M_i - M_{i-1}$). This result reinforces an essential aspect of the Wing-Kristofferson model, and especially the idea that tapping performance is governed by an event-based timing process: each tap is triggered by a discrete cognitive event, and consequently each inter-tap interval is determined by two boundary events and their associated motor delays. Regarding the low-frequency power spectrum, Gilden et al. (1995)'s interpretation suggested that the internal clock should be considered as a source of 1/*f* noise, and not as a white noise process. In the log-log spectrum of oscillation data, the obtaining of a flattened, but negative slope in the high-frequency region was interpreted by Delignières et al. (2004) as the typical signature of a dynamical timing process. As previously indicated, the duration of the time intervals produced by such processes depends only on the linear stiffness parameter. In order to account for the inherent variability of biological systems, one can suppose that a Gaussian white noise should be added to the model, leading to random, uncorrelated fluctuations in the series of produced time intervals. The simplest formulation of such a dynamical model should read as following:

$$I_i = D_i + \xi_i \qquad (2)$$

where D_i represents the duration of cycle i and ξ_i a Gaussian white noise term. In this case the error term affects directly the time interval, and not the successive events that delimit the interval, as in the Wing-Kristofferson model. A simple flattening of the slope in the high frequency region reveals the presence of this unique white noise term. Because Delignières et al. (2004) evidenced that time interval series in oscillation show 1/*f* noise in the low frequency region of power spectra, D_i can be assumed to be responsible for these 1/*f* fluctuations.

The discovery of 1/*f* noise in psychological series was questioned by several authors (Pressing & Jolley-Rogers, 1997; Wagenmakers, Farrell, & Ratcliff, 2004). 1/*f* noise is characterized by a highly structured variability, possessing properties of self-similarity and long-range dependence. Self-similarity signifies that the series do not present a specific time scale (this property is suggested by the linear slope observed in the log-log power spectrum), and long-range dependence that the current value is not dependent on a limited set of previous values, but possesses the memory of the whole previous history of the series. Evidencing these properties in experimental data remains difficult, and Pressing and Jolley-Rogers (1997) suggested that the 1/*f* shape of the spectra could be an artifact due to series non-stationarity. Recently, nevertheless, Lemoine, Torre and Delignières (2006), applying ARFIMA/ARMA modeling (Torre, Delignières, & Lemoine, 2007), showed that continuation tapping data contained genuine long-range dependence.

Our aim in the present paper was to develop and to test specific models for event-based timing and dynamical timing, that would be able to generate time interval series reproducing the statistical properties observed in experimental tapping and oscillation data. A number of generic models have been proposed for generating series presenting long-range correlations. Hausdorff and Peng (1996) showed that under certain conditions, the association of white noise processes acting at different time scales could produce 1/*f*-like fluctuations. Another multi-scaled model was proposed by Pressing (1999) and Wing, Daffertshofer and Pressing (2004), suggesting that the aggregation of moving average processes could also generate fractal series. This model will be examined in detail in the last section of this paper. Granger (1980) showed that the aggregation of AR(1) processes by random sampling of the autoregressive parameters from a beta distribution can lead to 1/*f* noise. Kaulakys (1999, 2000) proposed another model based on AR(1) processes, that consists of pulses whose recurrence times obey an autoregressive process with very small damping.

Our goal in this paper was not to arbitrarily select a formal model. Rather we derived from the models that have previously been proposed for accounting for event-based and dynamical timing processes. This approach seemed essential to provide our models with a certain biological or psychological plausibility. These models should not only be able to reproduce the classically reported experimental features in tapping or oscillation tasks, but also to generate time interval series possessing the 1/*f* properties evidenced in the aforementioned studies. For event-based timing, we tested the 'shifting-strategy' model recently suggested by Wagenmakers, Farrell and Ratcliff (2004). This model is a rather simple extension of the activation-threshold model (Ivry, 1996). For dynamical timing, we worked on an adaptation of the 'hopping' model proposed by West and Scafetta (2003) for modeling walking step variability. We combined this model with a classical limit-cycle model for limb oscillation (Kay, Saltzman, Kelso & Schöner; 1987).

A fractal event-based timer model

The activation-threshold model (Ivry, 1996) represents the simplest way for conceiving timing control. In this model an activation level is supposed to increase linearly in time. The attainment of a particular activation threshold determines a first event, and resets the activation

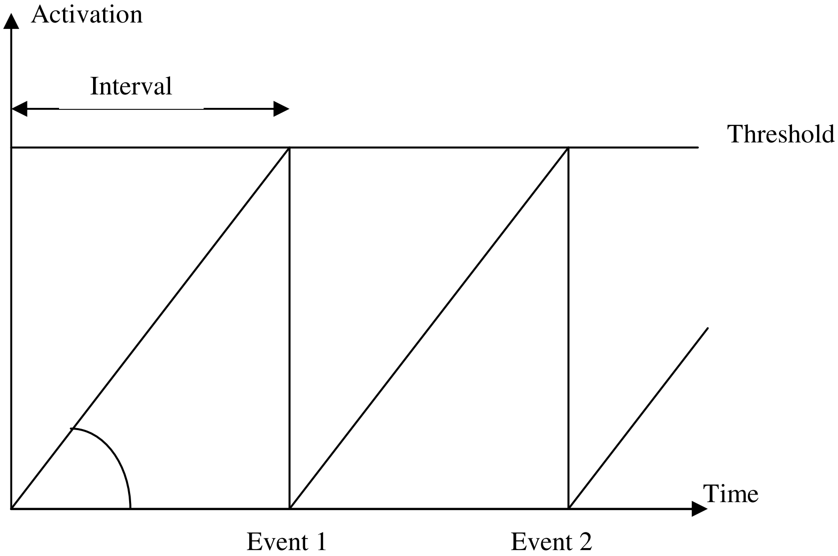

Figure 2: A simple activation-threshold model. An activation level is supposed to increase linearly in time. The attainment of a particular activation threshold determines a cognitive event, and the resetting of the activation level.

level. The iteration of this simple process produces a succession of periodic events, regularly spaced in time (Fig. 2).

Wagenmakers et al. (2004) introduced a 'shifting-strategy model', which seemed able to generate $1/f$ fluctuation. This model assumes that the threshold level could evolve in time, as a consequence of the successive adoption of different strategies for controlling the interval duration (Fig. 3). Each strategy is characterized by a particular threshold that can be modeled by sampling uniformly from an interval centered on a baseline level. These successive strategies are employed during a limited and variable period of time (*i.e.*, number of produced events). This shifting in strategy can also be modeled by sampling from a uniform distribution of usage times, bounded by a minimal and a maximal usage duration. Each iteration of the activation process is then realized until the reaching of a threshold T_i:

$$T_i = T_0 + T'_i \qquad (3)$$

where T_0 represent the baseline threshold, and T'_i the deviation from this baseline, sampled from a uniform distribution of range R. These strategy shifts produce local plateaus in performance.

It is further assumed that the speed v with which activation grows over time is variable from one interval to the other, following an auto-regressive process of order one:

$$v_i = v_0 + \varphi(v_{i-1} - v_0) + p\varepsilon_i \qquad (4)$$

where v_0 represents the baseline speed, φ is the auto-regressive parameter, and ε_i a centered white noise with unit variance. The time interval C_i produced by iteration i is then simply given by

$$C_i = T_i / v_i \qquad (5)$$

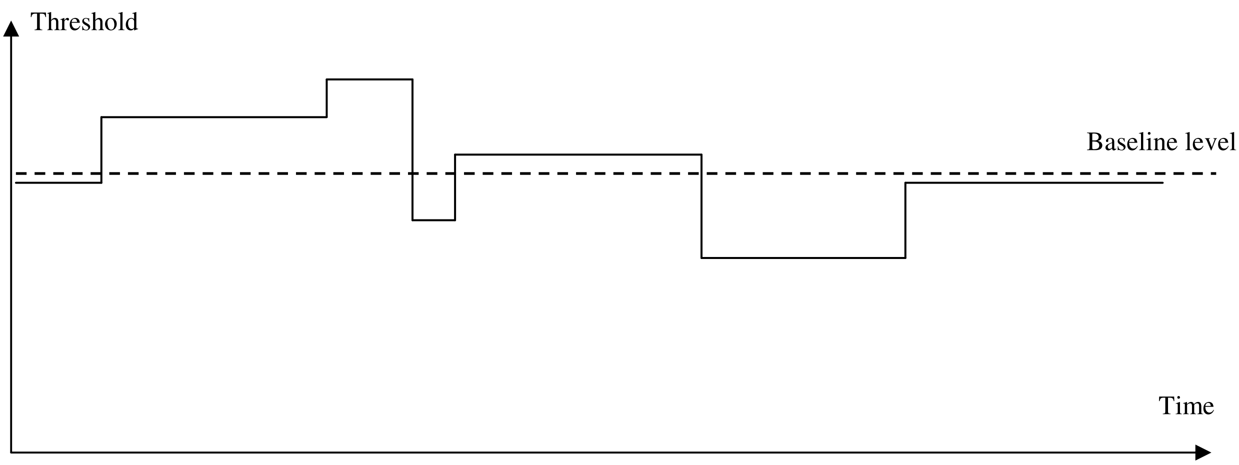

Figure 3: The shifting strategy model: the threshold evolves in time, as a consequence of the successive adoptionof different strategies for controlling duration.

Finally, according to the Wing-Kristofferson model (Eq. 1), we added a differenced white noise term:

$$I_i = C_i + \lambda(M_i - M_{i-1}) \quad (6)$$

where M_i is a centered white noise process of unit variance.

A fractal dynamical timer model

The starting point of the second model is the 'hybrid' model proposed by Kay et al. (1987), for accounting for the dynamics of a single oscillating effector. This model obeys the following equation:

$$\ddot{x} = \alpha \dot{x} - \beta \dot{x} x^2 - \gamma \dot{x}^3 - \omega^2 x \quad (7)$$

where x represents position. The dot notation indicates differentiation with respect to time. In this second-order differential equation $\omega^2 x$ represents stiffness, $\alpha \dot{x}$ linear damping, $\beta \dot{x} x^2$ is a nonlinear van der Pol damping term, and $\gamma \dot{x}^3$ a nonlinear Rayleigh damping term. A noise term of strength Q is added to the model in order to simulate the perturbations that affect all dynamical systems. In the present notation, all coefficients are supposed to be positive. Under these conditions, this model yields a limit cycle attractor of frequency ω with amplitude:

$$A = 2\sqrt{|\alpha| / (3\gamma\omega^2 + \beta)} \quad (8)$$

This model was proven to provide a satisfying account for the empirically observed relationships between frequency, amplitude, and peak velocity in oscillation (Kay et al., 1987). Note that this approach averages the system over a period of oscillation, considering perturbations as reasonably small around the stable limit cycle.

The correlation structure of the series of periods produced by this model was analytically studied by Schöner (1994) and Daffertshofer (1998). Schöner (1994) suggested that the relaxation properties of such an oscillator could induce a negative short-term dependence in the series of successive periods: If the relaxation time of the limit cycle is close to the cycle duration, then relaxation should generate a negative dependence between successive periods. Indeed, Daffertshofer (1998) showed that a van der Pol oscillator, when perturbed by a white noise source, could exhibit slight oscillations around the basic frequency during the relaxation onto the limit cycle, leading to the expected negative lag-one autocorrelation. Nevertheless, this serial dependence was obtained with rather unrealistic stiffness parameters. Note that in both cases the aim of the authors was to check whether nonlinear oscillators could represent a plausible alternative to internal clock models, in producing the typical negative lag-one autocorrelation observed in tapping experiments. In contrast, our aim was not to produce this kind of auto-correlation function, but instead a typical long-range auto-correlation function presenting a positive value at lag one, and a power-law decay over time. The aforementioned studies clearly showed that the limit cycle dynamics *per se* could not produce this kind of dependence.

Considering that in a dynamical timing model the period is mainly determined by the linear stiffness parameter, a solution could be to provide ω with fractal properties over time. Such a solution was explored by Ashkenazy, Hausdorff, Ivanov and Stanley (2002) and West and Scafeta (2003), in the domain of locomotion. They developed the so-called 'hopping' model that seemed able to generate a series of stiffness values possessing long-range correlation properties. The key element of the model is a linear Markov process δ_j, generated by a first-order auto-regressive equation:

$$\delta_j = \phi\delta_{j-1} + \eta\varepsilon_j \quad (9)$$

where $0< \phi <1$ is a constant, and ε_j a white noise process with zero mean and unit variance. This process could be conceived as a chain of possible states of the effector, neighboring states being determined by similar factors and mutually correlated. The chain then contains "correlated zones" of typical size r:

$$r = -1/\log\phi \quad (10)$$

The successive states of the system are supposed to be activated by a random walk along the chain, whose jump sizes follow a Gaussian distribution of width ρ (Fig. 4). This random walk generates a series δ_i, representing the state adopted by the effector for each successive cycle i. In this process, correlations within the δ_i series increase as the size of correlation within the chain (r) increases, and decrease as the width ρ of the distribution of jumps increases.

Finally, the frequency of the limit cycle is determined, for each successive cycle i, by

$$\omega_i = \omega_0 + \mu\delta_i + \theta\xi_i \quad (11)$$

where ω_0 represents the baseline frequency, μ is a constant, and ξ_i is a white noise with zero mean and unit variance. The addition of this white noise process to the series of stiffness values was motivated by the observation of a flattening of the log-log power spectrum in high frequencies (Delignières et al., 2004), suggesting the presence of

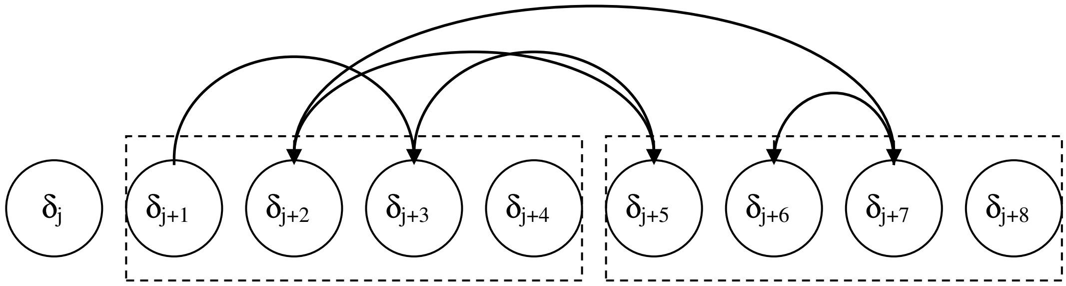

Figure 4: Illustration of the hopping model. The random walk selects successively the variables δ_{j+1}, δ_{j+3}, δ_{j+5}, δ_{j+2}, δ_{j+7}, and δ_{j+6}. The dashed boxes indicated the size (here r = 4) of the correlated zones.

a high-frequency random fluctuation (see Eq. 4). The parameter θ allows controlling the relative strength of this noise term. This series of linear stiffness parameters is then injected into the hybrid model (Eq. 7).

One could note that in both cases our strategy was to enrich the classical model with a module that was supposed to insert long-range dependence in the timing-generating parameter (*i.e.* the internal clock in the Wing-Kristofferson model, and the stiffness parameter in the limit-cycle model). However this enrichment should not be considered as simply additive, and hence trivial. The capacity of these modules to generate genuine long-range dependence has to be statistically proven, and additionally, their combination with the initial models, generating their own dependencies, could lead to unexpected results in the finally produced series.

The aims of the present work were then (1) to collect series of periods produced in uni-manual finger tapping and uni-manual forearm oscillation, (2) to simulate with the two models, using appropriate parameter settings, series possessing similar Gaussian statistics (mean and variance) than those obtained in the corresponding experimental series, and (3) to check whether experimental and simulated series shared, in both conditions, similar statistical features related to self-similarity and long-range dependence. We chose finger tapping and forearm oscillation tasks because they have previously been proven to induce the exploitation of event-based and dynamical timing, respectively (Delignières et al., 2004). The main aim of the experimental work was to provide a clear statistical characterization of empirical series, in terms of Gaussian statistics and fractal signatures, in order to guide the setting of the parameters of the models and to provide a reference for the assessment of the temporal structure of simulated series.

Method

Participants

12 participants (mean age 29.0 +/- 7.2) were involved in the experiment. None of them had particular expertise or extensive practice in music. They signed an informed consent form, and were not paid for their participation.

Procedure

The participants performed two continuation tasks, in finger tapping and in forearm oscillation. In the tapping task participants tapped with the index finger of their dominant hand. In the forearm oscillation task the time intervals were produced through the oscillations of a joystick, which could only be moved in the frontal plane, by forearm pronation/suppination movements. The joystick was manipulated with the dominant hand, and the key-events delimiting intervals were the reversal points of maximal pronation.

Before each trial, participants had to carefully observe a 30-second video sequence that displayed finger tapping or forearm oscillations performed at a frequency of 2 Hz. These video sequences gave only visual information, about the task to perform and the tempo to follow. We suppressed any metronomic information in order to avoid the artifactual induction of an event-based timing control, especially in the oscillation task. Participants were instructed to not perform the task during this first stage. Then, without any external pacing signal, they had to tap or oscillate regularly, following the requested tempo. In the two conditions, the task was pursued up to the recording of 700 successive time intervals, leading to trial duration of approximately 6 minutes. The order of presentation of the two tasks was systematically counterbalanced. A resting period of 5 minutes was given between the tasks.

Experimental device

The experiment was individually performed in a quiet room. In the tapping task participants had to tap on a rectangular (4cm X 4cm) pressure sensor. For the oscillation task, we used a 15-cm wooden joystick. Participants were asked to perform regular oscillations, with amplitude of about 45 degrees on each side of the vertical position. The angular movements were recorded with a potentiometer located at the axis of the joystick. In both experimental conditions, data were recorded with a sampling frequency of 300Hz.

Data reduction

In the tapping condition, the time of each tap was identified as the reaching of a threshold in the pressure time series. The threshold was determined in order to distinguish the initiation of the taps from the background noise present in the collected voltage data. Time intervals were then computed as the differences between the times of successive taps. For the oscillation condition, a bi-directional low-pass Butterworth filter (cut-off frequency 15 Hz) was applied to the collected voltage data. An appropriate algorithm, locating the local extrema corresponding to maximal pronation was then used for delimiting time intervals. In both conditions the first hundred points of the series was ignored, in order to avoid the initial drift that frequently occurs in the continuation paradigm (Ogden & Collier, 1999). Analyses were then conducted on the 512 next points of the series.

Simulations

Simulations were performed with the shifting strategy model and the hopping model. For the hopping model, we used a four-stage Runge-Kutta algorithm, following the scheme described by Burrage, Lenane and Lythe (2007, pp. 11-12), for second-order stochastic differential equations with additive noise. We used a fixed step size of 0.001 sec.

In both cases parameters were chosen in order to reproduce the statistical characteristics of the experimental series, in terms of means and variances. One hundred series of 512 points were generated with each model.

Analyses

We used four different analyses, aiming at revealing specific signatures of self-similarity or long-range dependence. Note that the use of complementary methods is highly recommended in fractal analysis, for an unambiguous characterization of series (Rangarajan & Ding, 2000). We first applied spectral analysis, in order to check whether our experiment replicated the spectral signatures of event-based and dynamical timing, previously evidenced by Delignières et al. (2004). We used the method proposed by Fougère (1985) and modified by Eke et al. (2000), which includes some preprocessing operations before the application of the Fast Fourier Transform (Appendix A; for details, see Delignières et al., 2006). The power spectrum was then represented in bi-logarithmic coordinates, and the low- and high-frequency slopes were estimated separately. The power estimates obtained for frequencies below 1/8 of maximal frequency were considered for determining the low frequency slope, and those obtained for frequencies above 1/2 of maximal frequency were considered for the high frequency slope. $1/f$ noise yields a negative linear trend in the low frequency region, with a slope close to –1.0. As reported by Delignières et al. (2004) we expected to obtain a positive slope in high frequencies in tapping, and a negative slope in forearm oscillation.

In complement to spectral analysis we applied the *Detrended Fluctuation Analysis* (DFA, Peng et al., 1993), because this method was proven to give better estimates of the fractal exponent than spectral analysis, especially when the analyzed series were contaminated by noise (Delignières et al., 2006). This method is based on the analysis of the relation between the mean magnitude of fluctuations in the series and the length of the intervals over which these fluctuations are observed (see Appendix A). DFA provides an index α, which allows determining whether the analyzed series can be considered as stationary ($\alpha < 1$) or non-stationary ($\alpha > 1$). $1/f$ noise is characterized by α values close to 1.0. The application of DFA constitutes a useful precaution, because ARFIMA modeling can only be applied on stationary signals.

Thirdly, we applied the ARMA/ARFIMA modeling procedure proposed by Farrell et al. (2005) and Torre et al. (2007), which allows providing a statistical test for the presence of long-range dependence in the series. This method consists in fitting 18 models to the series: nine are ARMA (p,q) models, p and q varying systematically from 0 to 2, and the other nine are the corresponding ARFIMA (p,d,q) models. This method selects the best model on the basis of a goodness-of-fit statistic that is based on a trade-off between accuracy and parsimony: the best model is the one that gives a good account of the data with a minimum number of free parameters. We used in the present study the Bayes Information Criterion (BIC, see Appendix B for details). For allowing comparisons between models, BIC raw values were transformed into normalized weights (Wagenmakers & Farrell, 2004; see Appendix B). Note that the normalized weights computed among a given set of models sum to one. ARFIMA modeling also allows testing the value of the d parameter estimated from ARFIMA (p,d,q), considering the corresponding ARMA (p,q) model as the null hypothesis.

Two complementary criteria were taken in account for detecting the presence of long-range dependencies in the series: (1) the best model (*i.e.* the model with the highest weight) that should be an ARFIMA (p, d, q), d being significantly different from 0, and (2) the sum of weights of ARFIMA models. Torre et al. (2007) proposed to accept the hypothesis of long-range correlation if at least of 90% of series are best fitted by an ARFIMA model, and if the mean sum of ARFIMA weights exceeds 0.90.

Models' fitting was conducted using the ARFIMA package (Doornik & Ooms, 1999; Ooms & Doornik, 1998) for the matrix computing language Ox (Doornik, 2001). We used, with some minor adaptations, the Ox code provided by Simon Farrell, available at http://eis.bristol.ac.uk/~pssaf/ (for details, see Farrell et al., 2005).

Finally, we computed the auto-correlation function of the obtained series. Auto-correlation functions are supposed to reveal the presence of long-range dependence in the series, with a typical power-law decay of auto-correlation over time. Moreover, these auto-correlation functions can also be useful for distinguishing between event-based and dynamical timers: the former are known to present a typical negative lag-one auto-correlation (Wing & Kristofferson, 1973), while the latter are supposed to contain persistent dependence at all lags.

Further testing of the models.

Finally we analyzed the fractal properties of our two models, by examining systematic combinations of different values for their key parameters (R and φ for the shifting-strategy model, and ρ and r for the hopping model). The respective analyzed ranges were chosen on the basis of the results of the first simulations. One hundred series of 1024 points were generated for each combination. Analyses focused on DFA and ARFIMA modeling, in order to detect and characterize long-range dependence in the series.

Results

Experimental tapping series and shifting- strategy model

We present in Fig. 5 (upper panel) an example of experimental series collected in the tapping condition. In order to obtain simulated series possessing similar mean and variance than the experimental series, we set the models parameters to the following values: $T_0 = 1000$, $v_0 = 2$, $R = 40$, $\beta = 0.05$, $\varphi = 0.4$, and $\lambda = 17$. One of these simulated series is presented in Fig. 5 (lower panel). The mean standard deviation was 28.83 ms for experimental series ($N = 12$), and 28.24 ms for simulated series ($N = 100$).

Fig. 6 (left panel) represents the mean power spectrum, obtained by a point-by-point averaging of the 12 individual spectra, and plotted in log-log coordinates. The right panel represents the mean power spectrum obtained from 12 simulated series, randomly selected from the set of series produced by the shifting strategy model. The spectra present similar shapes, with a negative linear slope in the low frequencies, and a positive slope in the high frequencies. The mean slope in low frequencies was –0.66 ($SD = 0.53$) for experimental series ($N = 12$), and –0.71 ($SD = 0.24$) for simulated series ($N = 100$). The mean slope in high frequencies was 0.42 ($SD = 0.25$) for experimental series, and 0.52 ($SD = 0.21$) for simulated series.

Detrended Fluctuation Analysis yielded mean exponents of 0.70 (SD = 0.14) for experimental series, and 0.68 (SD = 0.07) for simulated series.

ARFIMA modeling selected an ARFIMA model as best model for 11 experimental series over 12, and the mean sum of weights of ARFIMA models was 0.95. Long-range dependence was detected in all simulated series, and the mean sum of weights of ARFIMA models was 0.96. In both experimental and simulated series the most often selected model was a very simple (0,d,1) model, containing a fractional integration and a first-order moving-average process.

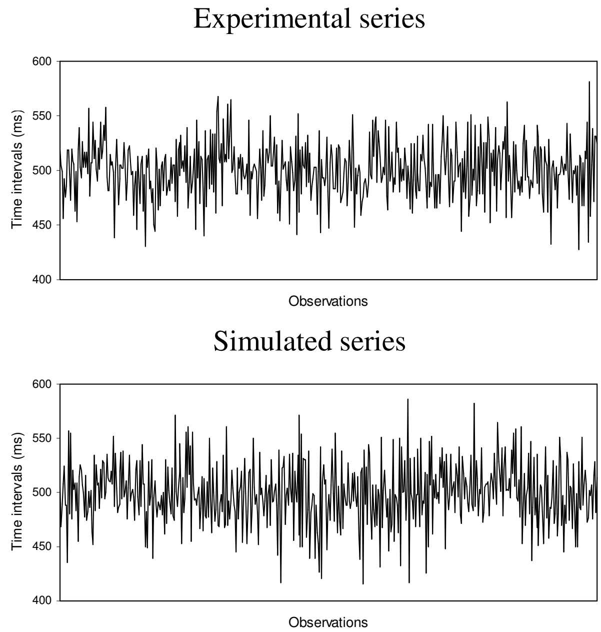

Figure 5: Upper panel: An example of individual experimental series of time intervals produced in the tapping condition. Lower panel: an example of simulated series obtained with the shifting strategy model. Parameters values: $T_0 = 1000$, $v_0 = 2$, $R = 40$, $\beta = 0.05$, $\varphi = 0.4$, and $\lambda = 17$.

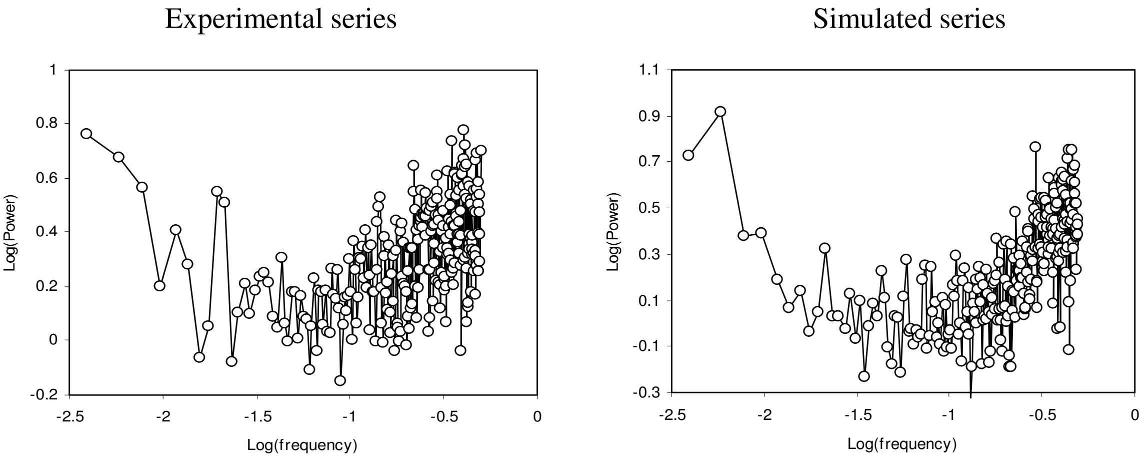

Figure 6: Averaged power spectra, in log-log coordinates. Left: experimental tapping series (N = 12), right: simulated series (shifting strategy model, N = 12).

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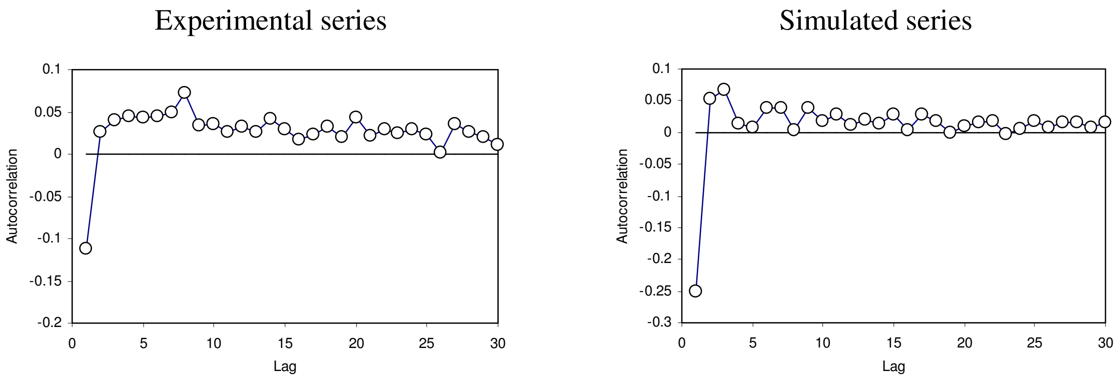

Figure 7: Averaged auto-correlation functions. Left: experimental tapping series (N = 12), right: simulated series (shifting strategy model, N = 12).

Fig. 7 (left panel) represents the mean auto-correlation function, up to lag 30, obtained by a point-by-point averaging of the 12 experimental auto-correlation functions. As can be seen, this auto-correlation function presents a negative value at lag one and then quite low, but positive values for lags superior to one. The right panel represents the mean auto-correlation function obtained from the 12 randomly selected simulated series. This function presents a roughly similar shape than the function obtained for experimental series

Experimental oscillation series and hopping model

We present in Fig. 8 (upper panel) an example of experimental series collected in the oscillation task. In order to obtain simulated series possessing similar mean and variance than the experimental series, we set the model parameters to the following values: $\alpha = 0.5$, $\beta = 1.0$, $\gamma = 0.02$, $\omega_0 = 4\pi$, $Q = 0.1$, $r = 25$, $\eta = 0.1$, $\rho = 25$, $\mu = 1.0$, and $\theta = 0.015$. One of these simulated series is presented in Fig. 8 (lower panel). The mean standard deviation was 18.86 ms for experimental series (N = 12), and 20.05 ms for simulated series (N = 100).

Fig. 9 (left panel) represents the mean power spectrum, obtained by a point-by-point averaging of the 12 experimental spectra, and plotted in log-log coordinates. The right panel represents the mean power spectrum obtained from 12 simulated series, randomly selected from the set of series produced by the hopping model. The two spectra present similar shapes, with a negative linear slope in the low frequencies, and a flattening trend in the high frequencies.

Further testing of the shifting-strategy model

Parameters T_0, v_0, β, and λ were set to previous values. We then simulated series of time intervals for values of R ranging from 0 to 100, by steps of 20, and for three values of φ (0.6, 0.4, and 0.2). 100 series of 1024 points were simulated for each combination of R and φ. We applied DFA and ARFIMA modeling on the simulated series. For each combination of R and φ, we computed the mean DFA exponent, and the percentage of series detected as ARFIMA. The results of these analyses are reported in Table 1. As could be expected, the DFA exponent increased when R increased. The influence of φ appeared less marked, with, nevertheless, a slight increase of exponents when φ increased. ARFIMA modeling suggested the effective presence of long-range dependence for most combinations, with percentages exceeding the 90% threshold proposed by Torre et al. (2007). Long-range dependencies were absent, nevertheless, in the series generated with R = 0 (no strategy shifting) and low values of φ, and also for the series resulting from the combination of the highest R values and the lowest φ.

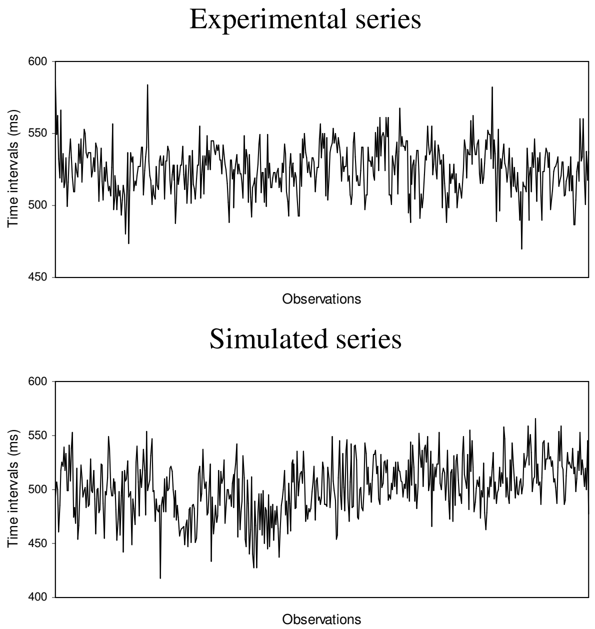

Figure 8: Upper panel: An example of individual experimental series of time intervals produced in the oscillation condition. Lower panel: an example of simulated series obtained with the hopping model. Parameters values are: $\alpha = 0.5$, $\beta = 1.0$, $\gamma = 0.02$, $\omega_0 = 4\pi$, $Q = 0.1$, $r = 25$, $\eta = 0.1$, $\rho = 25$, $\mu = 1.0$, and $\theta = 0.015$.

The mean slope in low frequencies was -1.11 (*SD* = 0.43) for experimental series (N = 12), and -0.85 (*SD* = 0.52) for simulated series (N = 100). The mean slope in high frequencies was -0.13 (*SD* = 0.32) for experimental series, and -0.31 (*SD* = 0.20) for simulated series.

Detrended Fluctuation Analysis yielded mean exponents of 0.86 (*SD* = 0.18) for experimental series, and 0.82 (*SD* = 0.17) for simulated series.

ARFIMA modeling selected an ARFIMA model as best model for all experimental series. The mean sum of weights of ARFIMA models was 0.92. For the simulated series, long-range dependence was detected in 97% of series. The mean sum of weights of ARFIMA models was 0.93. In both experimental and simulated series the most often selected model was a very simple (0,*d*,0) model, containing only a fractional integration process.

Fig. 10 (left panel) represents the mean auto-correlation function, up to lag 30, obtained by a point-by-point averaging of the 12 experimental auto-correlation functions. This function presents a positive auto-correlation at lag one (about 0.20), and then a slow decay with increasing lags. The right panel represents the mean auto-correlation function obtained from the 12 randomly selected simulated series. This function presents a similar shape than that observed for experimental series.

Further testing of the hopping model

Parameters α, β, γ, ω_0, q, η, μ and θ were set to previous values. We then simulated series of time intervals for values of ρ ranging from 15 to 45, by steps of 10, and for values of r ranging from 5 to 45, by steps of 10. One hundred series of 1024 points were simulated for each combination of ρ and r. As previously, we applied DFA and ARFIMA modeling on the simulated series. The results of these analyses are reported in Table 2. As expected, DFA exponent increased as r increased and as ρ decreased. However, long-range dependence tended to disappear for the combinations of low r and high ρ, and conversely for high r and low ρ. In other words, long-range dependence seems related to a kind of equilibrium between the two parameters.

Table 1: Mean DFA exponent and percentage of ARFIMA models, for different combinations of R and φ in the shifting-strategy model. One hundred series of 1024 points were simulated for each combination. The other parameters in the model were set as following: $T_0 = 1000$, $v_0 = 2$, $\beta = 0.05$, and $\lambda = 17$.

			R					
			0	20	40	60	80	100
DFA exponent	φ	0.6	0.52	0.62	0.71	0.85	1.00	0.99
		0.4	0.47	0.59	0.65	0.85	0.88	0.90
		0.2	0.43	0.49	0.62	0.80	0.93	0.94
% ARFIMA	φ	0.6	97%	98%	98%	99%	100%	92%
		0.4	66%	92%	100%	92%	97%	78%
		0.2	60%	93%	100%	89%	87%	63%

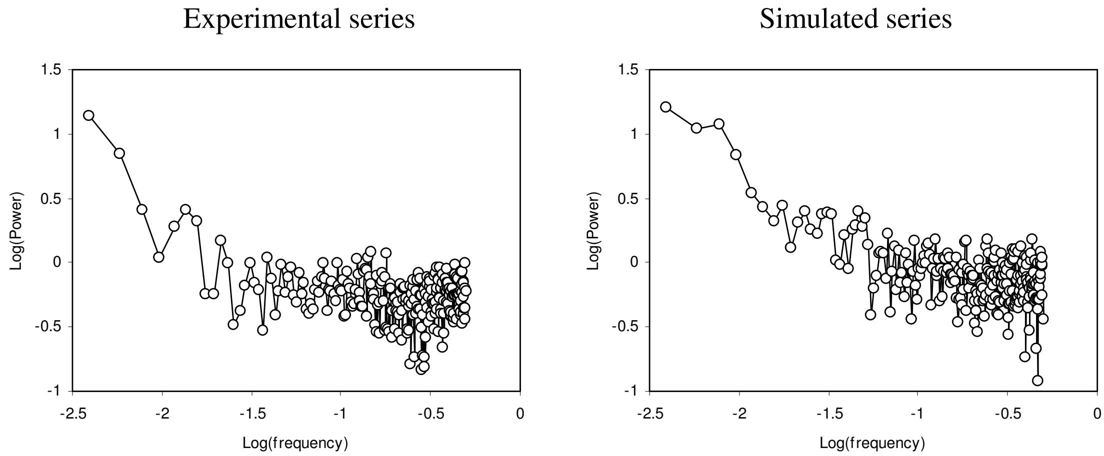

Figure 9: Averaged power spectra, in log-log coordinates. Left: experimental oscillation series ($N = 12$), right: simulated series (hopping model, $N = 12$).

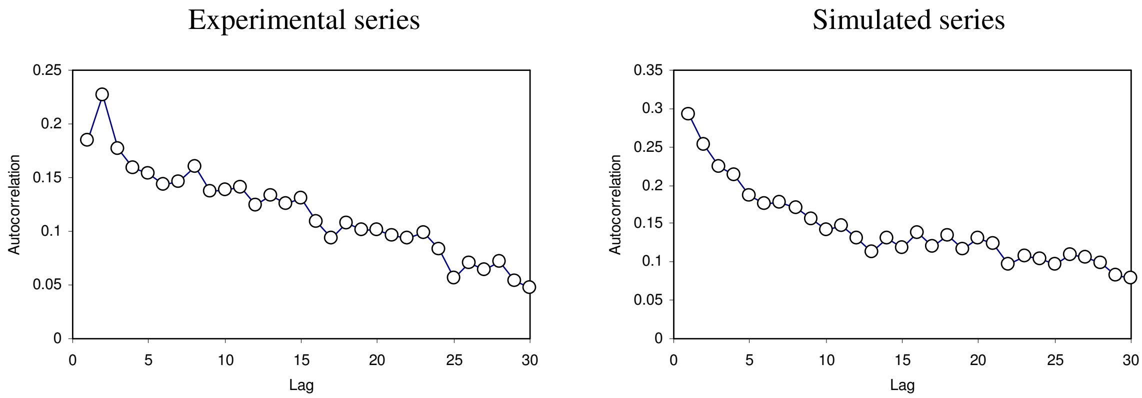

Figure 10: Averaged auto-correlation functions. Left: experimental oscillation series ($N = 12$), right: simulated series (hopping model, $N = 12$).

Table 2: Mean DFA exponent and percentage of ARFIMA models, for different combinations of r and ρ in the hopping model. One hundred series of 1024 points were simulated for each combination. The other parameters in the model were set as following: $\alpha = 0.5$, $\beta = 1.0$, $\gamma = 0.02$, $\omega_0 = 4\pi$, $Q = 0.1$, $\eta = 0.1$, $\mu = 1.0$, and $\theta = 0.015$.

			r				
			5	15	25	35	45
DFA exponent	ρ	15	0.64	0.79	0.86	0.89	0.93
		25	0.59	0.74	0.83	0.90	0.88
		35	0.58	0.74	0.77	0.83	0.86
		45	0.55	0.65	0.76	0.79	0.85
% ARFIMA	ρ	15	99%	88%	87%	78%	71%
		25	94%	96%	97%	82%	89%
		35	85%	98%	97%	88%	88%
		45	82%	99%	96%	98%	95%

Discussion

The present experiment confirmed, qualitatively and quantitatively, the results obtained by Delignières et al. (2004) in the analysis of extended series of time intervals produced in tapping and in oscillation tasks: the spectral analysis revealed a completely different behavior in high frequencies, suggesting the use of different kinds of timing control in the two situations. In tapping, the positive slope in high frequencies suggested the presence of a differenced white noise term in the series, as proposed in the Wing and Kristofferson (1973)'s event-based model. Conversely, a simple flattening of the spectrum in the high frequencies was observed in the oscillation task, and this result was consistent with the dynamical timing hypothesis (Schöner, 2002). Moreover, we confirmed that the time interval series obtained in both conditions contained genuine $1/f$ fluctuations. The application of ARFIMA modeling, which was not used by Delignières et al. (2004), gave a statistical evidence for the presence of long-range dependence in the series produced in both conditions.

The two proposed models were able to produce series presenting statistical features similar to those obtained in experimental conditions. The log-log spectra presented similar shapes, with comparable slopes in the low- and high frequency regions, and DFA gave similar exponents for experimental and simulated series. The application of ARFIMA modeling confirmed the presence of long-range dependence in the series produced by the two models. Finally, we showed that the auto-correlation functions of experimental and simulated series were identical, with similar shapes and comparable correlation values for the first lags.

Obtaining fractal correlations in simulated series should not be considered a trivial result. We tested a number of alternative models, which failed to produce series possessing such fractal properties. For example, an activation/threshold model including a trial-to-trial auto-regressive evolution of the threshold did not produce the expected properties. As well, a cycle-to-cycle autoregressive evolution of stiffness did not provide the hybrid model with long-range dependence.

We also tested some models recently proposed in the literature for simulating $1/f$ series. For example, Wing, Daffertshofer and Pressing (2004) proposed a very simple model, involving summation of moving average processes on multiple time scales. One of the simplest formulations of such model is as following:

$$y_t = \varepsilon_t + w\langle\varepsilon_t\rangle_n + w^2\langle\varepsilon_t\rangle_{n^2} \qquad (12)$$

where ε_t is an uncorrelated white noise process, w and n are two constants, and $\langle\varepsilon_t\rangle_n$ represents the moving average of a window of length n:

$$\langle\varepsilon_t\rangle_n = (\varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \ldots + \varepsilon_{t-n+1})/n \qquad (13)$$

We simulated series with this model for w = 5, 10, and 15, and n = 5, 10 and 15. One hundred series were simulated for each (w, n) combination. DFA provided quite acceptable graphical results, with a linear relationship between interval length and fluctuations, in log-log coordinates. As expected by Wing et al. (2004), the mean DFA exponent increased with increasing values of w. Windows length (n) had no influence for the lower w values, but tended to increase the DFA exponent for the highest values of w. The application of spectral analysis yielded similar results, and the pattern of results was consistent with those presented by Wing et al. (2004).

Nevertheless, the application of ARMA/ARFIMA modeling showed that the samples of simulated series never reached the criterion of 90% of preferred ARFIMA models. The best result (60%) was obtained for the combination w = 10 and n = 5, a percentage clearly insufficient for supporting the presence of long-range dependence in the series produced by this model (Table 3).

Nevertheless, the present modeling effort shows that genuine $1/f$ fluctuations can be obtained by quite simple formal models. A recent controversy opposed two conceptions about the origin of long-range dependence in complex systems. Wagenmakers et al. (2005) claimed that such fluctuations could arise from the presence of a sufficiently large number of independent components. Van Orden, Holden and Turvey (2005) considered $1/f$ noise as the signature of self-organized criticality. The present results offer a possible alternative, suggesting that such complex

Table 3: Percentage of ARFIMA models for different combinations of n and w in the weighted moving averaged model (Wing et al., 2004).

		n		
		5	10	15
w	5	52%	18%	5%
	10	60%	28%	16%
	15	40%	47%	35%

fluctuations could arise from quite simple macroscopic mechanisms. Note that in both models the production of fractal fluctuations arose from the combination of an auto-regressive process and a random walk.

Importantly, the proposed models are just simple extensions of two classical models that have previously been proven to satisfactorily reproduce a number of statistical features observed in the respective empirical paradigms of finger tapping and uni-manual oscillations. As such these models possess a certain psychological and biological plausibility. Additionally, the enrichment of these models was performed on basic assumptions that could allow a systematic experimental testing. For example, the shifting-strategy model suggests that the intensity of long-range dependence in the series is closely related to the range of variation of the threshold, with the successive adoption of different strategies. One could then suggest that an experimental instruction inducing the persistent adoption of a selected strategy should lead to a decrease in long-range dependence. In the related paradigm of tapping in syncopation with a metronome, Chen, Ding and Kelso (2001) showed that imposing a strategy to participants (for example, to tap "in the air" in synchrony with the metronome, just in-between the two effective syncopation taps), led to a significant decrease of the spectral exponent of series of errors to the metronome. This result seems consistent with the assumptions that can be derived from the model. As well, one could suppose that an extended practice of tapping would induce the stabilization of strategies, and result in a decrease of long-range dependence.

Concerning oscillations, one could suppose that the size of correlation (r) within the Markov chain is related to the physiological state of the effector. West and Scafetta (2003) suggested that a physiological stress could induce an increase of the correlation size within the chain, thus inducing an increase of long-range dependence in time interval series. This evolution of long-range dependence was observed in locomotion, when participants were instructed to walk or to run below, or conversely above, their preferred frequency (Hausdorff et al., 1996; Jordan, Challis & Newell, 2006).

These hypotheses have to be tested experimentally, in order to validate the basic assumptions of our models. It is important to note that at this point, the only certitudes we have are (1) the presence of long-range dependence in the time interval series produced by central clocks, and (2) the presence of long-range dependence in the series of stiffness during uni-manual oscillation. We proposed simple formal models that allowed simulating these dependencies, but other candidate models could produce similar results. Deciding between models should be based on their ability to simulate experimentally induced effects, such as those evoked above. Note also that a final validation of our models should also consider their capacity to account for the classical features of event-based and dynamical timing, as, for example, the typical evolution of cognitive and motor variances with time interval duration in tapping tasks (Wing & Kristofferson, 1973), or the stability properties of the limit-cycle trajectories during limb oscillation (Kelso, 1995).

The interest of these models is not restricted to the uni-manual (tapping or oscillation) continuation paradigm which was used in the present work. A number of extensions of the present models could be considered, as for example for the synchronization paradigm, where participants have to tap in synchrony with a metronome, during the whole trial. A simple extension of the Wing-Kristofferson model, with the addition of an auto-regressive correction process, was proven to give a satisfying account for the structure of short series of errors to the metronome collected in a synchronization task (Pressing & Jolley-Rogers, 1997 ; Vorberg & Wing, 1996). However, some recent studies have shown that the series of errors to the metronome had fractal properties (Chen, Ding, & Kelso, 1997, 2001). Clearly, the auto-regressive model proposed by Vorberg and Wing presents some limitations for taking into account long-range dependence in the series of errors to the metronome. Torre, Lemoine and Delignières (2006), in a preliminary study, showed that the combination of the shifting-strategy model with an auto-regressive correction process could adequately model the fractal properties of extended series produced in synchronization tapping.

Similarly, synchronization with a metronome can be performed in oscillation. This paradigm was recently studied by Assisi, Jirsa, and Kelso (2005), who proposed to couple the hybrid model with a periodic function representing the external pacing given by the metronome. This model was proven to give a satisfactory account for the phase-plane trajectory of the oscillator. Nevertheless, Torre et al. (2006) showed that similarly to the results obtained in tapping, the series of errors to the metronome presented long-range dependence. Assisi et al. (2005)'s model should be tested in this regards, but considering the apparent impossibility to generate long-range dependence on the basis of the classical hybrid model, one could suppose that the model proposed by Assisi et al. (2005) should be unable to satisfy this criterion. Another solution could be to introduce in the hopping model an auto-regressive correction mechanism similar to that used by Vorberg and Wing (1996) for synchronization tapping. Torre et al. (2006) showed that this solution offered a satisfactory alternative for modeling errors and time intervals series collected during oscillations in synchrony with a metronome.

Another extension of the present study could involve the analysis of the bimanual versions of the two experimental tasks. Bimanual tapping has been extensively studied, and some experiments evidenced a so-called “bimanual advantage”: within-hand variability appears lower in bimanual than in uni-manual tapping. Helmuth and Ivry (1996) proposed a model for accounting for this rather counter-intuitive result. According to this model, each effector possesses its own timing mechanism. During bimanual tapping, the outputs of these two timers are integrated prior to the movement and the output of this process is imposed to both effectors. This integration is supposed to result in the observed bimanual advantage.

Bimanual oscillations have also been widely studied, especially from a dynamical systems perspective (Kelso, 1995). Some models have been proposed for accounting for the dynamics of between-hand coordination (deGuzman & Kelso, 1991; Haken, Kelso, & Bunz, 1985; Schöner, Haken, & Kelso, 1986). These models formalized a system of coupled oscillators, and were able to reproduce the main features of bimanual coordination (differential intrinsic stabilities of in-phase and anti-phase patterns, transition from anti-phase to in-phase pattern under an increase of driving frequency, etc.).

For these bimanual versions also, the window of observation we propose can offer new insights on the principles that underlie effectors coordination. For example, in the case of bimanual oscillations, Torre, Delignières and Lemoine (in press) evidenced the fractal properties of relative phase series, and showed that the aforementioned models were unable to produce this kind of fluctuations in relative phase. One could suppose that a coupled version of the hopping model could be able to account for the fractal nature of relative phase. Several coupling options could be conceived, and the discovery of an efficient coupled model should provide new hypotheses about coordination between oscillating limbs. A similar investigation could be proposed for bimanual tapping, by exploring the properties of a coupled version of the shifting-strategy model.

Conclusion

The original models that constituted the starting point of the present work were proposed for taking into account the short-term dependence in tapping series, for the Wing-Kristofferson model, and the phase portrait properties for Kay et al. (1987)’s hybrid oscillator model. The present approach focuses on long-range dependence, through the analysis of extended experimental series. As such, it opens a new window of observation, revealing new statistical properties that were previously neglected. We proposed simple extensions of the original models that seem able to take into account these properties, revealed by fractal analyses and auto-correlation functions. This approach offers a theoretical and methodological basis for the reassessment of current theories, in the domains of uni-manual motion and bimanual coordination.

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Appendix A

Spectral analysis

We used the $^{low}PSD_{we}$ method initially proposed by Fougère (1985) and modified by Eke et al. (2000), which includes some preprocessing operations before the application of the Fast Fourier Transform (FFT): First the mean of the series was subtracted from each value, and then a parabolic window was applied: each value in the series was multiplied by the following function:

$$W(j) = 1 - \left(\frac{2j}{N+1} - 1\right)^2 \qquad \text{for } j = 1, 2, \ldots, N. \qquad \text{(A1)}$$

This transformation induces a tapering of the series and is supposed to reduce the leakage in the periodogram. Spectral leakage is the term used to describe the loss of power of a given frequency to other frequency bins in the FFT. There are edge effects arising from the discontinuity at the bounds that cause spectral leakage. It implies that windowing in the time domain corresponds to smoothing in the frequency domain. This smoothing reduces sidelobes associated with the window. Finally, a linear detrending was applied to the resulting series. The FFT algorithm was then applied on the obtained series.

A fractal series is characterized by the following power law:

$$S(f) \propto 1/f^{\beta} \qquad \text{(A2)}$$

where β is the spectral exponent, f the frequency and $S(f)$ the correspondent squared amplitude. β is estimated by calculating the negative slope ($-\beta$) of the linear regression of log (S(f)) against log f. β equals 0 for white noise, 2 for ordinary Brownian motion, and 1 for 1/f noise. As proposed by Eke et al. (2000) we excluded in the fitting of β the high-frequency power estimates (f > 1/8 of maximal frequency). This method was proven to provide more reliable estimates of the spectral exponent. Finally, in order to avoid any effect of the logarithmic distribution of the points, we averaged the log-log spectrum into non-overlapping intervals of 0.1 log(Hz) before computing the regression slopes.

Detrended Fluctuation Analysis

DFA is a fractal method that is supposed to be not affected by non-stationarity. The algorithm of DFA consists first in integrating the series $y(t)$, calculating for every t the cumulated sum of the deviations of the mean:

$$Y(i) = \sum_{t=1}^{i} \left[y(t) - \bar{y} \right] \qquad \text{for i = 1, 2, 3….}N \qquad \text{(A3)}$$

where N corresponds to series length. This integrated series is then divided in non-overlapping intervals of length n. In each interval, a least squares line is fit to the data (representing the trend in the interval). The $Y(t)$ series is then locally detrended by subtracting to all values the theoretical value $Y_{th}(t)$ given by the regression. For all interval length n, the characteristic magnitude of fluctuation $F(n)$ is calculated by:

$$F(n) = \sqrt{\frac{1}{N} \sum_{k=1}^{N} \left[Y(k) - Y_{th}(k) \right]^2} \qquad \text{(A4)}$$

This computation is repeated over all possible interval lengths n (in practice, the shortest length is around 10, and the largest N/2, giving two adjacent intervals). Typically, $F(n)$ increases with interval length n. For fractal series, a power law is expected, as

$$F(n) \propto n^{\alpha} \qquad \text{(A5)}$$

where α is the scaling exponent. α is estimated by the slope of the graph representing $F(n)$ as a function of n, in log-log coordinates. α equals 0 for white noise, 1.5 for ordinary Brownian motion, and 1 for 1/f noise.

Appendix B

Model selection in ARMA/ARFIMA modeling

The method proposed by Farrell et al. (2005) consists in fitting 18 models to the studied series. Nine of these models are ARMA (p,q) models, p and q varying systematically from 0 to 2. The other nine models are the corresponding ARFIMA (p,d,q) models, differing from the previous ARMA models by the inclusion of the fractional parameter d, representing persistent serial correlations.

Fitting a particular time series involves maximizing the likelihood of a given model with respect to the autocovariance function of the series. Nevertheless, the examination of the maximum likelihood scores provided by the fitting procedure is not sufficient, as the capacity of models to account for the data is partly related to their number of free parameters. The selection of models has to be based on a trade-off between accuracy and parsimony: the best model is the one that gives a good account of the data with a minimum number of free parameters.

We used in the present paper the Bayes Information Criterion (BIC), defined as:

$$BIC = -2\log L + k\log N \qquad \text{(B1)}$$

where L represents the maximum likelihood for the model under study, k the number of free parameters in the model, and N the number of observations in the series. As can be seen, the first term rewards accuracy, and the second penalizes the lack of parsimony. The lower the BIC, the better the model.

The raw values of this criterion remain difficult to interpret and to compare between models. Wagenmakers and Farrell (2004) proposed a convenient transformation of the raw values in weights. Consider that the goal is to select the best model among m candidates. The first step is to compute the difference, for each model, between the criterion for this model and for the best model. That is, for the i^{th} model:

$$\Delta_i(BIC) = BIC_i - \min BIC \qquad \text{(B2)}$$

This difference in BIC can then be converted in an estimate of relative likelihood through the following transform :

$$L_i(BIC) \propto \exp\left\{ -\frac{1}{2} \Delta_i(BIC) \right\}$$

(B3)

Finally, these relative likelihoods are transformed into weights by normalization (*i.e.* by division by the sum of the relative likelihood of all models):

$$w_i(BIC) = \frac{L_i(BIC)}{\sum_{j=1}^{m} Lj(BIC)} \qquad \text{(B4)}$$

$w_i(BIC)$ can be conceived as the probability for the i^{th} model to be the best model given the data and the set of candidate models (Wagenmakers & Farrell, 2004). Note that the weights computed among a given set of models sum to one.

On the basis of these weights, two criteria could be proposed for detecting the presence of long-range dependence in the series: (1) the best model (*i.e.* the model with the highest weight) should be an ARFIMA (*p*, *d*, *q*), *d* being significantly different from 0, and (2) the sum of the weights of the ARFIMA models should be higher than the sum of the weights of the ARMA models.

Torre et al. (2007) showed that BIC gave better results than the Akaike Information Criterion (AIC) initially proposed by Wagenmakers, Farrell and Ratcliff (2004).

APPENDIX 2

Oscillating in synchrony with a metronome: serial dependence, limit cycle dynamics, and modeling

Kjerstin Torre & Didier Delignières

EA 2991, Motor Efficiency and Deficiency, University Montpellier I

Abstract: We analyzed serial dependencies in periods and asynchronies collected during oscillations performed in synchrony with a metronome. Results showed that asynchronies present 1/*f* fluctuations, and periods series anti-persistent dependence. The analysis of limit cycle dynamics revealed a specific asymmetry induced by synchronization. We propose a hybrid limit cycle model including a cycle-dependent stiffness parameter provided with fractal properties, and a parametric driving function based on velocity. This model accounts for most experimentally evidenced statistical features, including serial dependence and limit cycle dynamics. We analyze the implications of these results within the event-based vs emergent timing distinction. Finally we discuss the impact of evidencing fractal fluctuations in limb oscillations, notably regarding bimanual coordination.

Key Words: Oscillation, asynchrony, 1/*f* noise, limit cycle model, parametric driving

Send correspondence to:
Kjerstin Torre
Faculty of Sport Sciences – University Montpellier 1
EA 2991, "Motor Efficiency and Deficiency"
700 Avenue du Pic Saint Loup
34090 Montpellier – France
Email: ktorre@univ-montp1.fr

Introduction

So far, studies of single-limb oscillations have essentially focused on oscillators within-cycle dynamics, especially through the analysis of the phase-plane representation of motion (limit-cycle dynamics). Serial dependence (*i.e.* cycle-to-cycle dynamics) has been largely disregarded, except in a few studies focusing on self-paced oscillations (Daffertshoffer, 1998; Delignières et al., 2004, 2008; Schöner, 1994). Especially, Delignières et al. (2008) revealed the presence of fractal serial dependencies in series of oscillation periods, that could not be accounted for by classical limit cycle models. We focus in the present paper on oscillations performed in synchrony with a metronome. Our first aim was to combine the analyses of within-cycle and cycle-to-cycle dynamics, for providing a complete characterization of the impact of synchronization on limb's dynamics. In a second step, we try to assess the capability of some candidate models to account for observed results.

The dynamics of a single oscillating effector has classically been modeled by a hybrid differential equation, associating Rayleigh and van der Pol damping terms (Kay, Saltzman, Kelso, & Schöner, 1987):

$$\ddot{x} = \alpha \dot{x} - \beta \dot{x} x^2 - \gamma \dot{x}^3 - \omega^2 x \qquad (1)$$

This equation describes the intrinsic dynamics of a self-sustained oscillator, whose frequency is mainly determined by the linear stiffness term ω^2. This model provides a satisfying account for the empirically observed relationships between frequency, amplitude, and peak velocity during self-paced limb oscillations.

When limb oscillations have to be performed in synchrony with a metronome, the external pacing has been shown to modify the limit-cycle dynamics determined by the above equation (Assisi, Jirsa & Kelso, 2005; Byblow, Carson & Goodman, 1994; Fink, Foo, Jirsa & Kelso, 2000): a decrease of the spatial variability in the movement cycle is typically observed, with a thinning of the phase plane trajectory in the neighborhood of the occurrence of the metronome signal. This *anchoring effect* was also shown to affect coordination in paced bimanual oscillations, with a decrease of relative phase variability at the anchored point, and a global stabilization of coordination revealed by a decrease of the occurrences of transitions from anti-phase to in-phase modes (Fink et al., 2000).

Schöner and Kelso (1988) conceived the metronome as an environmental information attracting the dynamics of the effector toward the prescribed frequency. They proposed to add a linear cosine term to drive the limit cycle oscillator. Jirsa, Fink, Foo and Kelso (2000) showed that a more complex model including a parametric driving term was more appropriate to account for the effect of external pacing on the limit cycle dynamics. The simplest formulation of the parametric driving model obeys the following equation (Fink et al., 2000; Assisi et al., 2005):

$$\ddot{x} = \alpha \dot{x} - \beta \dot{x} x^2 - \gamma \dot{x}^3 - \omega^2 x + \epsilon_1 \cos \Omega t + \epsilon_2 x \cos \Omega t \tag{2}$$

where Ω is the frequency at which the metronome is presented and ϵ_1 and ϵ_2 are the strengths of the linear and parametric driving terms, respectively. Notably, this model accounted for the decrease of spatial variability in the phase plane at the anchored point (Assisi et al., 2005). However, the criteria used for assessing the validity of this model remained limited in scope, since they focused exclusively on the analysis of the stability properties of limit cycle dynamics.

While serial dependence has rarely been studied in the domain of oscillatory motion, it represents a key feature in research on human gait, as in the close domain of finger tapping, where the analysis of serial short-range and long-range correlations and their evolution according to experimental manipulations have been considered a crucial step for understanding the underlying processes (Chen, Ding, & Kelso, 1997, Delignières, Torre & Lemoine, 2004, 2008; Gilden, Thornton, & Mallon, 1995; Hausdorff et al, 1996; Slifkin & Newell, 1998; West & Scafetta, 2003). With close connection to the present issue, studies contrasting self-paced and synchronized conditions evidenced typical changes in the correlation structure of experimental series, caused by external pacing. For example Hausdorff et al. (1996) showed that stride interval series, in self-paced walking, presented $1/f$ fluctuations. However, when participants had to walk following an auditory metronome, this correlation structure vanished and successive step durations became uncorrelated. In tapping, Chen et al. (1997) evidenced $1/f$ fluctuations in inter-tap interval series in self-paced condition. In synchronization condition, inter-tap interval series became anti-persistent, and $1/f$ fluctuations were evidenced in the series of asynchronies to the metronome. These results suggest that external constraints such as metronome driving are likely to specifically alter serial dependencies, revealing the actual nature of their influence

Note that the above results obtained in finger tapping cannot be generalized to oscillatory motion, although tapping and oscillation tasks have sometimes been considered as different instantiations of timing in rhythmic movements. Indeed, several studies have supported the idea that these two tasks belong to different classes of rhythmic movement. Finger tapping is representative of discontinuous movement tasks where timing is controlled on an event-based mode. A central timekeeper is supposed to determine periodic cognitive events that trigger discrete motor responses. The mechanism of event-based timing has nicely been captured by the well-known Wing and Kristofferson (1973) model. In contrast, the timing of continuous movement tasks like oscillations or circle drawing seems achieved following an emergent mode, exploiting the dynamical properties of movement trajectory (Delignières et al., 2004, 2008; Robertson et al., 1999; Schöner, 2002; Spencer, Zelaznik, Diedrichsen, & Ivry, 2003; Zelaznik, Spencer, & Ivry, 2002). As a consequence of this distinction between event-based and emergent timing, the way in which synchronization is performed in an oscillation task could hardly be inferred from empirical evidences obtained with tapping tasks.

With regard to single-limb oscillations, the only published experimental results assessing serial dependence were obtained in self-paced conditions (Delignières et al., 2004, 2008), where the series of periods produced by rhythmic forearm oscillations were shown to be $1/f$ noise. Delignières et al. (2008) proposed to account for this serial behavior by providing the stiffness parameter of a hybrid self-sustained oscillator (see Eq. 1) with $1/f$ fluctuations over successive cycles. This model, and especially the fractal generator that injects long-range dependence in the dynamics of the oscillator, will be presented in details in the modeling section of this paper.

One could predict that the correlation structure of oscillation periods observed in self-paced condition should be inevitably altered from the moment that oscillations are performed in synchrony with an external pacing signal. It is important to note that whatever the performed task, in synchronization conditions asynchrony series is the mathematical integration of the corresponding period series (Chen et al., 1997). In terms of serial correlation structure the integration of $1/f$ noise corresponds to persistent fractional Brownian motion, a typically non-stationary series. Consequently, if periods in synchronization were still 1/*f* noise, asynchronies would be non-stationary, that is inconsistent with the performance of synchronized movements.

What are the possible changes in the correlation structure of periods? One can conceive of three possible hypotheses: According to the first one, the periodic rhythm imposed by the metronome could override the intrinsic dynamics of the oscillator. In that case, the series of periods, in synchronization, should present only uncorrelated fluctuations around a mean stable value. Such a result was evidenced in externally paced walking by Hausdorff et al. (1996). A second hypothesis suggests that the synchronization process could induce by itself a specific pattern of correlation. This hypothesis was sustained by Chen et al. (1997) in their study on synchronized tapping. In that case the intrinsic dynamics of the oscillator is overridden as well, and serial correlations are supposed to emerge exclusively from the synchronization process. Finally, a third hypothesis could be that the pattern of correlation observed in synchronization arises from the combination of the intrinsic dynamics of the oscillator, and the coupling mechanism. In that view, the process that causes 1/*f* fluctuation in self-paced oscillations is supposed to be still at work in synchronization, but expresses differently due to coupling.

The aim of the present article was to determine a model of synchronized single-limb oscillations that accounts simultaneously for the characteristic within-cycle dynamics and cycle-to-cycle dynamics. In a first step, we propose a complete characterization of synchronized oscillations, including limit-cycle dynamics and the structure of serial correlation. In a second step, we test the capability of the parametric driving model (Jirsa et al., 2000) to account for the evidenced features, notably the empirical serial correlations in oscillation periods and asynchronies. We confirm that this model allows accounting for limit-cycle dynamics of synchronized oscillations, but not for the empirical correlation properties. Accordingly, we assume that the serial correlation structures evidenced in asynchronies and periods are partly due to the intrinsic dynamics of the oscillator. In the third step, we thus propose to model synchronized oscillations by combination of the hopping model that has previously been shown to account for 1/*f* noise in self-paced oscillations (Delignières et al., 2008), and a modified version of the parametric driving model (Jirsa et al., 2000) that was designed for accounting for the limit cycle dynamics in synchronized oscillations.

Experiment

Procedure

Data used here were obtained as part of a larger study, crossing tapping *vs* oscillation, self-paced *vs* synchronized conditions, and unimanual *vs* bimanual performances (Torre & Delignières, in press). In the present paper we focus on data collected in unimanual self-paced and synchronized oscillations.

Participants and experimental design

12 participants (mean age 29.0 ± 7.2) were involved in the experiment. None of them had particular expertise or extensive practice in music. They declared no recent upper limb traumatism or neurological injury. They signed an informed consent form, and were not paid for their participation.

The experiment was individually performed in a quiet room. In the two conditions, forearm oscillations were performed with the dominant hand, by manipulating a 15-cm wooden joystick which could only be moved in the frontal plane. For self-paced oscillations, the prescribed tempo of 2 Hz was initially presented using a short video sequence of about 30 secs, during which the participants did not perform any movement. Then, they had to perform oscillations following the prescribed tempo as accurately and regularly as possible, without metronome. In the synchronization condition, participants had to perform forearm oscillations in synchrony with an auditory signal generated by a PC-driven metronome and delivered at a frequency of 2 Hz. Participants had to synchronize the reversal points of maximal pronation with the metronome signals. In both conditions they were asked to perform regular oscillations, with amplitude of about 45 degrees on each side of the central (vertical) position. Note that the 8 conditions of the original experiment (tapping *vs* oscillation, self-paced *vs* synchronized, and uni-manual *vs* bimanual), were presented in random order to each participant.

The angular movements were recorded with a potentiometer located at the axis of the joystick, with a sampling frequency of 300Hz. In both conditions the task was pursued up to the recording of 600 successive cycles.

Analyses

Data reduction. A bi-directional low-pass Butterworth filter (cut-off frequency 15 Hz) was applied to the collected voltage data. An appropriate algorithm was then used for peak detection. The variables of interest were series of periods and asynchronies. Periods were computed as the differences between the times of two successive peaks of maximal pronation. Asynchronies were defined as the difference between the times of the maximal pronation peak and of the corresponding auditory signal.

Serial dependence. For each series, we applied four complementary analyses aiming at a thorough characterization of serial dependence. We first examined the spectral properties of series using $^{low}PSD_{we}$ (Eke et al., 2000), an improved version of the classical spectral analysis. The spectral exponent β was estimated by the negative of the linear regression slope of the power spectrum in bi-logaritmic coordinates. As proposed by Eke et al. (2000) we excluded the high-frequency power estimates (f > 1/8 of maximal frequency) for the fitting of β. 1/*f* fluctuations (*i.e.* persistent long-range correlations) are characterized by β exponents ranging from 0.5 to 1.5, and negative exponents reveal anti-persistent (negative) correlation in the series.

Secondly, we applied *Detrended Fluctuation Analysis* (DFA, Peng, Havlin, Stanley, & Goldberger, 1995), working in the time domain. This method is based on the analysis of the relationship between the mean magnitude of fluctuations in the series and the length of the intervals over which these fluctuations are determined. For fractal series, a power relationship, characterized by a scaling exponent α, is expected between the mean magnitude of fluctuations and interval length. 1/*f* fluctuations are characterized by α exponents ranging from 0.75 to 1.25. Exponents comprised between 0 and 0.5 reveal anti-persistent correlation in the series.

Thirdly, we used ARFIMA/ARMA modeling (*Auto-regressive Fractionary Integrated Moving Average*, Wagenmakers, Farrel & Ratcliff, 2004; Torre, Delignières & Lemoine, 2007a) in order to evaluate the statistical evidence for the presence of long-range correlation in the series. This method consists in fitting 18 models to the studied series: nine are ARMA (*p*,*q*) models, *p* and *q* varying systematically from 0 to 2, and the other nine are the corresponding ARFIMA (*p*,*d*,*q*) models, where *d* is the fractional integration parameter. The best model is then selected on the basis of a goodness-of-fit statistic that assesses the trade-off between accuracy and parsimony: the best model is the one that gives a good account of the data with a minimum number of free parameters. We used the Bayes Information Criterion (BIC), which was proven to be the most reliable for detecting long-range dependence (Torre et al., 2007a).

Finally, we computed the auto-correlation function of series, from lag 1 to lag 30. Auto-correlation functions reveal the presence of long-range dependence in series by a typical power-law decay over time.

Limit-cycle dynamics. In order to obtain a clear characterization of limit cycle dynamics, we summarized each trial in a normalized average cycle, according to the procedure adopted by Mottet and Bootsma (1999), and Nourrit, Delignières, Deschamps, Caillou and Lauriot (2003). We retained for this analysis the 10,000 first points of each collected position series (corresponding to 33.33 sec, and approximately 66 oscillation cycles). Each cycle was normalized in time using 150 equidistant points, by means of linear interpolation, and rescaled within the interval [-1,+1]. Point-by-point averaging of these normalized cycles allowed computing a normalized average cycle of 150 points. The first derivative was then computed from this average cycle, and rescaled within the interval [-1, +1]. The limit cycle was portrayed by plotting average velocity against average position. This analysis was performed for both self-paced and synchronization conditions.

Results

Serial dependence

Periods in self-paced oscillations. The mean period was 493 ms, with a mean within-series standard deviation of 19 ms. An example of individual series is portrayed in Figure 1 (left column, upper graph). The log-log power spectrum presented a negative linear regression slope in low frequencies and flattened in high frequencies (Figure 1, left column, middle graph). The mean β exponent was 1.11 (± 0.43). DFA yielded a mean α of 0.86 (± 0.18). ARFIMA/ARMA modeling confirmed statistically that series were 1/*f* noise, detecting long-range correlation in all series. The auto-correlation function presented a positive auto-correlation at lag one (about 0.20), and then a slow decay with increasing lags (Figure 1, left column, bottom graph).

Asynchronies in synchronization. The mean asynchrony was -10.90 ms, with a mean within-series standard deviation of 42.11 ms. An example individual series is portrayed in Figure 1 (middle column, upper graph). $^{low}PSD_{we}$ yielded a linear regression over the entire range of frequencies in the log-log power spectrum (Figure 1, middle column, middle graph). The mean β exponent was 0.78 (± 0.34). DFA confirmed this result with the obtaining of a perfectly linear diffusion plot, with a mean α of 0.87 (±0.15). ARFIMA/ARMA modeling detected long-range correlation in 9 series over 12. Finally, the mean auto-correlation function presented a very slow decay over time (Figure 1, middle column, bottom graph), typical of long-range dependence in series.

Periods in synchronization. The mean period was 498.73 ms, with a mean within-series standard deviation of 17.42 ms. An example individual series is portrayed in Figure 1 (right column, upper graph). $^{low}PSD_{we}$ provided log-log power spectra characterized by a linear, positive slope in the low frequency region, and a flattened slope in high frequencies (Figure 1, right column, middle graph).

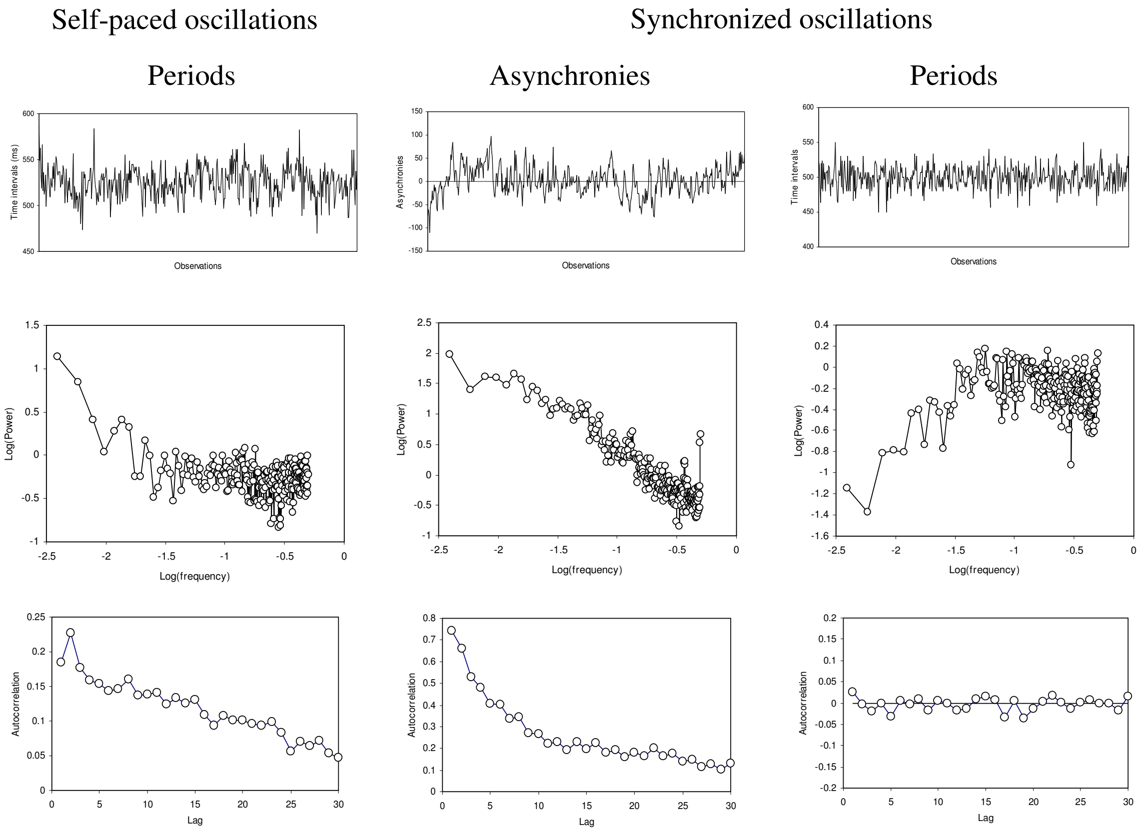

Figure 1: Graphical results obtained from experimental data in self-paced oscillations and synchronized oscillations. Upper graphs: representative example of individual series. Middle graphs: averaged log-log power spectra. Bottom graphs: averaged auto-correlation functions.

The mean β exponent was -1.03 (± 0.64), and DFA yielded a mean α of 0.31 (±0.22), showing consistently that series were anti-persistent noise. The mean auto-correlation function presented values close to zero, for the whole range of examined lags (Figure 1, left column, bottom graph).

Limit cycle dynamics

Figure 2 (upper graphs) displays the phase portraits of two single series obtained in self-paced (left) and synchronization (right) oscillations. The anchoring effect is obvious on the right side of the synchronization cycle that corresponds to the occurrence of the metronome signals. Figure 2 (bottom graphs) also presents the normalized average cycles obtained for the two conditions. Note that these cycles were averaged over the 12 participants, but all individual cycles presented similar shapes. At first glance, the average cycle looked symmetrical and circular in self-paced condition. In synchronization, in contrast, an asymmetry appeared between the out phase (semi-cycle departing from the reversal on the metronome) and the back phase (semi-cycle towards the metronome) of oscillations. The former (negative velocity values, from the right to the left) was quasi-circular with, nevertheless, a slight shift of the (negative) peak velocity in the first part of the semi-cycle. The latter (positive velocity values, from the left to the right) presented a stronger deformation, with a delayed peak velocity, occurring in the second part of the semi-cycle.

Discussion

The obtained pattern of results displays a number of features that were already observed in tapping experiments (Chen et al., 1997; Torre & Delignières, 2008). In self-paced oscillations as in self-paced tapping, the period series present persistent long-range correlations (Delignières et al., 2004, 2008). Moreover, as in tapping, the synchronization to the metronome induces a major alteration of the correlation structure of the oscillation periods which present anti-persistent dependence in that case. Finally, all methods of serial correlation analysis gave consistent results, characterizing asynchronies as 1/*f* noise. However, the present results differ from those obtained in tapping in terms of short-range correlations. The flattening of the log-log power spectrum in the high frequency region, considered as the typical signature of emergent timing processes (Delignières et al., 2004), was clearly observed for self-paced period series (in contrast with the positive slope that is typically observed in tapping), and a similar flattening was obtained in the high-frequency region of the power spectrum for periods in synchronization, a phenomenon that has never been observed in tapping.

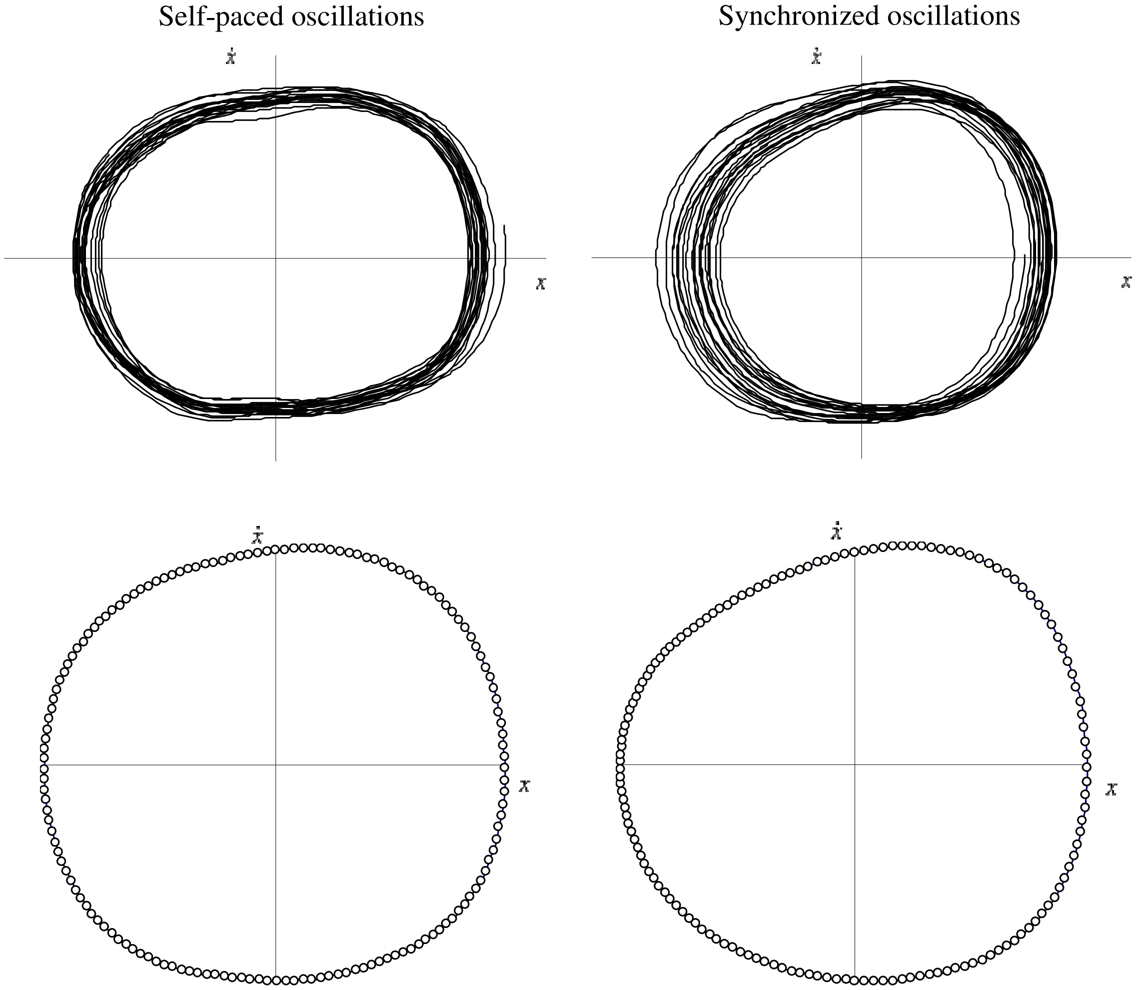

Figure 2: Limit-cycle dynamics of experimental self-paced and synchronized oscillations. Upper graphs: example individual limit cycles. Bottom graphs: normalized average limit cycles ($N = 12$).

In sum, the present results show that the patterns of long-range dependence obtained in self-paced and synchronization conditions, were similar in tapping and oscillation tasks, despite some discrepancies due to the different timing processes that are engaged in the two tasks. In tapping, the synchronization process has been conceived as discrete, acting through the correction of the following period on the basis of the current asynchrony (Vorberg & Wing, 1996). Torre and Delignières (2008) showed that this auto-regressive correction process, combined with a timekeeper possessing fractal properties, gave a satisfying account for the pattern of serial dependencies observed in synchronization tapping. With regard to the evidenced serial dependencies only, one could indeed propose to similarly account for the synchronization of oscillations. Nevertheless, such a discrete auto-regressive correction would modify the periods of the oscillator without affecting its limit cycle dynamics, thus being unable to account for the deformation of the limit cycle we observed in synchronization condition. Our results regarding the limit cycle dynamics, in contrast, support the hypothesis that synchronization is achieved on the basis of a continuous coupling, rather than a discrete cycle-to-cycle correction. Moreover, the typical asymmetry of the limit cycle, in synchronization, suggests that a linear coupling function should be insufficient, and that a parametric driving, as proposed by Jirsa et al. (2000) should be necessary for producing this complex dynamics. Nevertheless, the exact form of this parametric driving term remains to be determined with respect to our present empirical data.

In sum, the present experiment shows that in synchronization, asynchronies and periods display a rich pattern of serial correlations, that allows rejecting the first hypothesis of a complete overriding of the system's dynamics by the regularity of the metronome. Our results show that coupling plays a major role in the evolution of serial correlations, and give strong indications in favor of a continuous form of coupling. We cannot conclude, however, on the relative contributions of the intrinsic dynamics of the oscillator and the coupling function. We try in the following modeling section to address this question. The present examination of experimental data encompassed serial dependence and limit-cycle dynamics, allowing a rather complete characterization of the impact of synchronization on single-limb oscillations, and providing a demanding set of criteria for assessing candidate models.

Modeling

Our second hypothesis suggested that coupling, by itself, could generate the serial correlation pattern evidenced in synchronized oscillations. Considering that our results gave a strong support for a continuous form of coupling, we first examined the properties of the parametric driving model (see Eq. 2), proposed by Jirsa et al. (2000). We used the following parameters: $\alpha = 0.5$, $\beta = 1.0$, $\gamma = 0.02$, and $\omega = 4\pi$, $\epsilon_1 = 0.1$ and $\epsilon_2 = 3$, and included an additive white noise term with variance $Q = 0.1$. These parameter values were consistent with those used in recent simulation[1] experiments (Jirsa et al., 2000, Leise & Cohen, 2007).

These simulations allowed reproducing the typical anchoring phenomenon in the limit cycle dynamics. But while this model accounted for the limit-cycle dynamics of synchronized oscillations, it failed to account for the presence of 1/*f* noise in asynchronies. Applying $^{low}PSD_{we}$ to simulated asynchrony series yielded a mean β of -0.96 (± 0.21). Consistently, the DFA yielded a mean α of 0.24 (± 0.07), suggesting the presence of anti-persistent dependence. This pattern of results showed that the parametric driving model was *per se* unable to generate the persistent long-range correlation pattern observed in the experiment.

It can be concluded that the pattern of correlation observed in synchronization conditions emerges necessarily from the combination of the intrinsic dynamics of the oscillator (characterized in self-paced condition by 1/*f* fluctuations), and the driving function that induces anti-persistent dependence. Then, a solution could be to start with a model accounting for fractal fluctuations in self-paced oscillations, and then to enrich this first model with a synchronization process.

Delignières et al. (2008) proposed a model for accounting for the presence of long-range dependence in self-paced oscillations. The starting point is the hybrid model by Kay et al. (1987) that accounts for the dynamics of a single oscillating effector (Eq. 1). A noise term of strength Q is added to this model in order to simulate the perturbations that affect all dynamical systems. Schöner (1994) and Daffertshofer (1998) showed that such model could under some parameter specifications yield a negative lag-one auto-correlation, but not the long-range dependence observed in self-paced oscillations. Considering that in this limit cycle model the period is mainly determined by the linear stiffness parameter ω^2, Delignières et al. [3] proposed to provide this parameter with fractal properties over the successive cycles. Such a solution was previously explored by Ashkenazy, Hausdorff, Ivanov and Stanley (2002) and West and Scafeta (2003), in the domain of locomotion. These authors developed a so-called 'hopping' model that was able to generate a fractal series of stiffness values.

The key element of the hopping model is a linear Markov process δ_j, generated by a first-order auto-regressive equation:

$$\delta_j = \phi\delta_{j-1} + \eta\varepsilon_j \qquad (3)$$

where $0< \phi <1$ is a constant and ε_j is a white noise process with zero mean and unit variance. The chain then contains "correlated zones" of typical size r:

$$r = -1/\log\phi \qquad (4)$$

The successive states of the system are supposed to be activated by a random walk along the chain, whose jump sizes follow a Gaussian distribution of width ρ. This random walk generates a series δ_i, representing the state adopted by the effector for each successive cycle i. In this process, correlations within the δ_i series are assumed to increase with the size of correlation within the chain (r), and to decrease as the width ρ of the distribution of jumps increases.

Finally, the frequency of the limit cycle is determined, for each successive cycle i, by

$$\omega_i = \omega_0 + \mu\delta_i + \theta\xi_i \qquad (5)$$

where ω_0 represents the baseline frequency, μ is a constant, and ξ_i is a white noise with zero mean and unit variance. The addition of this white noise process of strength θ to the series of stiffness values was motivated by the observation of the flattening of the log-log power spectrum in the high frequencies (Delignières et al., 2004), suggesting the presence of high-frequency random fluctuations. This series of linear stiffness parameters is then injected into the hybrid model (Eq. 1). Delignières et al. (2008) showed that this model allowed simulating the 1/*f* structure of periods in self-paced oscillations. Figure 3 (left column) presents the results of this model using $\alpha = 0.5$, $\beta = 1.0$, $\gamma = 0.02$, and $Q = 0.1$ for the hybrid limit cycle model, and $\omega_0 = 4\pi$, $r = 25$, $\eta = 0.1$, $\rho = 25$, $\mu = 1.0$, and $\theta = 0.01$ for the hopping

[1] All simulations in this modeling section, were performed using a four-stage Runge-Kutta algorithm, following the scheme described by Burrage, Lenane and Lythe (2007, pp. 11-12), for second-order stochastic differential equations with additive noise, with a fixed step size of 0.001 sec. 100 series of 512 data points were generated for each proposed model.

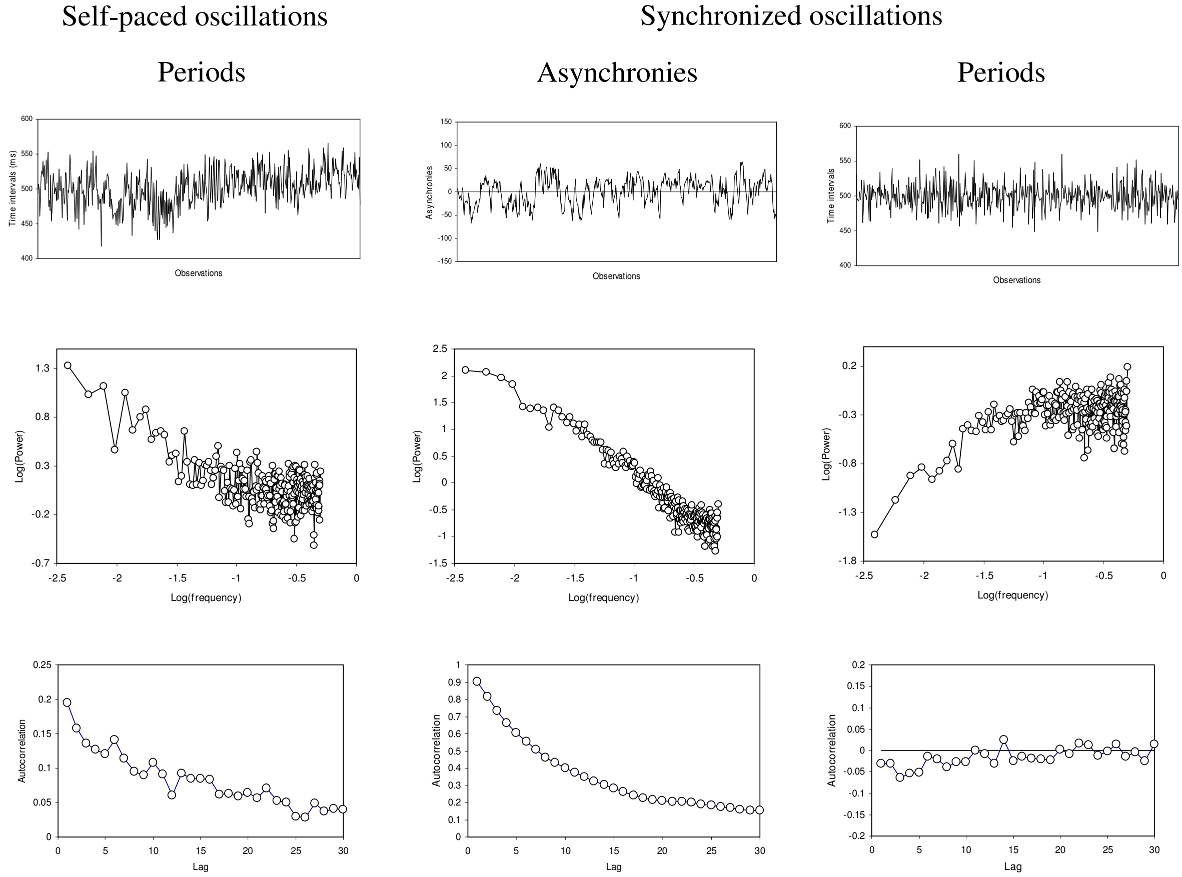

Figure 3: Graphical results obtained from simulations of self-paced oscillations and synchronized oscillations. Upper graphs: representative example of individual series. Middle graphs: averaged log-log power spectra. Bottom graphs: averaged auto-correlation functions. Power spectra and auto-correlation functions were computed from 12 randomly selected series.

model. The mean power spectrum and the auto-correlation function were similar to those obtained during the experiment for self-paced oscillations (see figure 1, left column). The mean β exponent was 0.85 (*SD* = 0.52), and DFA yielded a mean α exponent of 0.82 (*SD* = 0.17). ARFIMA modeling detected long-range dependence in 97% of the simulated series. We display in Figure 4 (left column) an example of phase-plane representation for one realization of the model, and an average normalized limit cycle, computed over 12 randomly selected realizations.

Once we accounted for 1/*f* fluctuation in the intrinsic dynamics of the oscillator, the next step consisted in accounting for the synchronization process. Given the ability of the parametric driving model to account for some essential aspect of the coupling to the metronome, we analyzed the properties of series generated by a combination of the hopping model and the parametric driving model. In a first step, we used the simplest formulation of the model, including a linear driving term and a position-based parametric driving term (see Eq. 2). This solution yielded a satisfying pattern of serial dependence, with persistent long-range correlations in asynchronies, and anti-persistent dependence in periods. The model moreover produced an asymmetric deformation of the limit cycle, similar to that observed in the experimental series, with the characteristic shifts of peak velocities in the two semi-cycles. Nevertheless, the anchoring phenomenon appeared at the opposite reversal point of the cycle (*i.e.*, on the left in our conventions), discarding this modeling solution.

The very specific deformation of the limit cycle that was observed in experimental series provided a precise criterion for selecting the best way for modeling synchronization. The simplest solution was to use a higher order parametric driving term (indexed on velocity rather than on position), and to use a sine driving function for this term, rather than a cosine function. This model can be written as follows:

$$\ddot{x} = \alpha\dot{x} - \beta\dot{x}x^2 - \gamma\dot{x}^3 - \omega_t^2 x + \epsilon_1\cos\Omega t + \epsilon_3\dot{x}\sin\Omega t + \sqrt{Q}\varepsilon_t \quad (6),$$

ω_t being determined, cycle-by-cycle, by the hopping model. We used the following parameters: $\alpha = 0.5$, $\beta = 1.0$, $\gamma = 0.02$, $\Omega = 4\pi$, $\epsilon_1 = 0.3$ and $\epsilon_3 = 0.05$, and $Q = 0.1$. The parameters of the hopping model were the same than those previously used for the simulation of self-paced oscillation periods.

Using this set of parameters provided satisfying results. Simulated series presented similar Gaussian properties than those observed during the experiment, with mean asynchrony of -3.50 ms (± 39.07), and a mean period of 499.98 ms (± 0.08). Representative examples of simulated series are presented in Figure 3 (upper graphs, asynchronies: middle column; periods: right column).

Regarding simulated asynchronies, the $^{low}PSD_{we}$ yielded a linear regression in the log-log power spectrum, over the entire range of frequencies (Figure 3, left column, middle graph). The mean β exponent was 0.93 (± 0.13). DFA gave consistent results with a perfectly linear diffusion plot, and a mean α of 0.93 (± 0.14). ARFIMA/ARMA modeling detected long-range correlation in 88% of series. Finally, the mean auto-correlation function presented a very slow decay over the successive lags (Figure 3, left column, bottom graph), similar to that obtained with experimental series.

For simulated period series, as for empirical series, the $^{low}PSD_{we}$ provided log-log power spectra characterized by a linear, positive slope in the low frequency region, and a flattened zone in high frequencies (Figure 3, right column, middle graph). The mean β exponent was - 1.24 (± 0.36), and DFA yielded a mean α of 0.19 (± 0.05). The mean auto-correlation function presented values close to zero, for the whole range of examined lags (Figure 3, left column, bottom graph).

Finally, Figure 4 (right column, upper graph) displays an example of limit cycle obtained from simulated series. The anchoring phenomenon appeared clearly when the oscillator's position reached its maximum. We computed an average normalized limit cycle, on the basis of 12 randomly chosen realizations (Figure 4, right column, bottom graph). This average limit cycle clearly presented an asymmetry similar to that observed in the experimental cycles, with a precocious velocity peak in the out phase, and, conversely, a delayed velocity peak in the back phase.

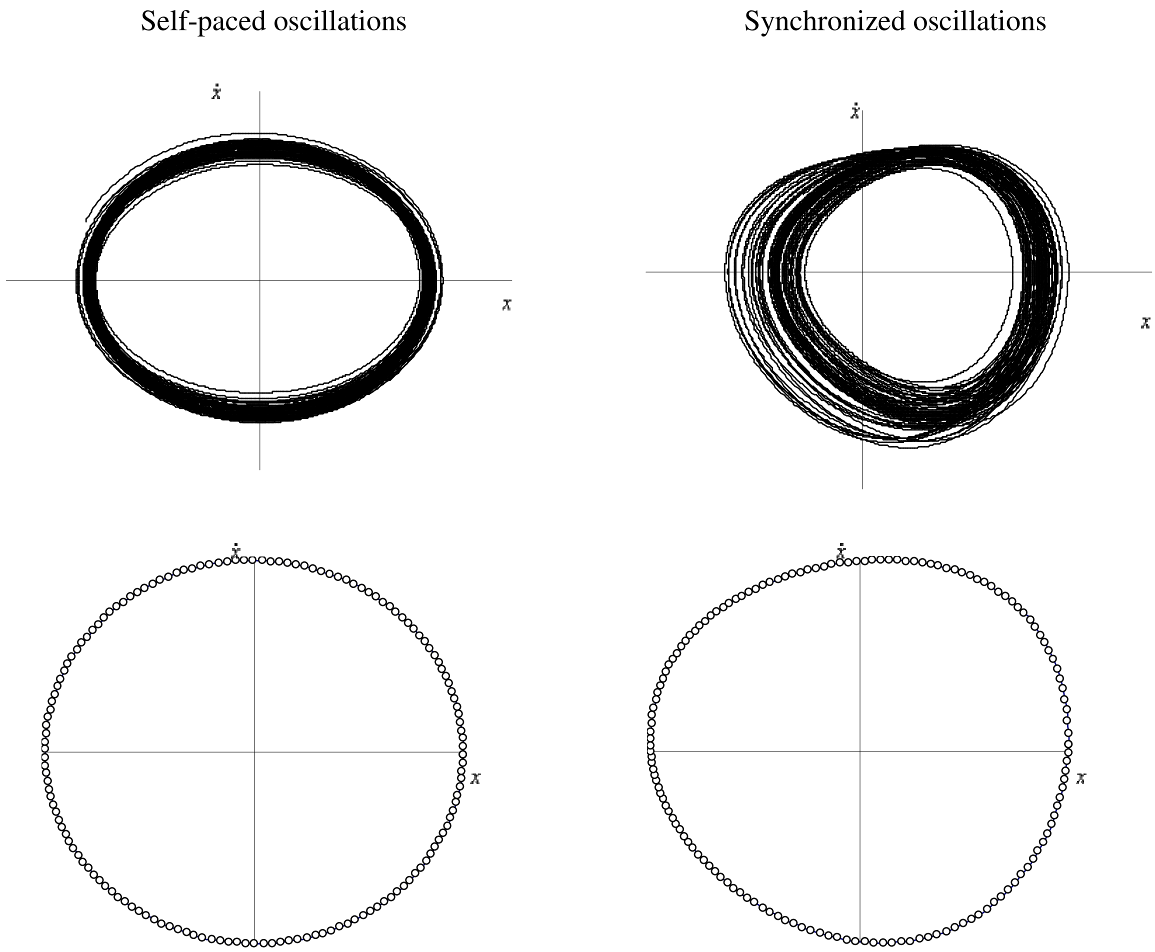

Figure 4: Limit-cycle dynamics of simulated self-paced (left) and synchronized (right) oscillations. Upper graph: example simulated limit cycles (50 cycles). Bottom graph: normalized average limit cycles, computed from 12 simulated series of 100 cycles.

General discussion

Experimental data allowed us to collect a number of dynamic signatures of self-paced and synchronized forearm oscillations. Results confirmed that the series of periods in self-paced oscillations presented genuine long-range dependence (Delignières et al., 2004). In the synchronization condition, the correlation structure of periods became anti-persistent, whereas 1/*f* fluctuations appeared at the level of asynchronies. These correlation structures were evidenced through the combination of spectral, temporal, and auto-correlation analyses (Delignières et al., 2006), and attested for by ARFIMA modeling (Torre et al., 2007a). These results paralleled those obtained in self-paced and synchronized tapping, and could therefore allow supporting the hypothesis of a similar mechanism underlying the two synchronization processes (*i.e.*, a discrete cycle-to-cycle correction of periods, based on the preceding asynchrony). The analysis of limit cycle dynamics, nevertheless, suggested that synchronization should rather involve a continuous coupling mechanism, as proposed by Jirsa et al. (2000).

Simulations allowed to test and refine these hypotheses. We first showed that the parametric driving model proposed by Jirsa et al. (2000) was not able to generate the experimentally observed correlation structures. This result was in line with the conclusions of Daffertshofer (1998) that showed that limit cycle models could only generate short-term dependence. A reasonable solution for this problem was to provide the stiffness parameter of the model with fractal properties, as suggested by West and Scafetta (2003). In this view, we used the 'hopping model', that was proven to generate genuine fractal correlations in the series of periods produced by a limit-cycle model (Delignières et al., 2008). In the domain of human locomotion, West and Scafetta (2003) suggested that this 'hopping model' could represent the activity a *central pattern generator*, an intraspinal network of neurons capable of producing a syncopated output. The central pattern generator hypothesis has been supported by a number of studies on animal locomotion (see, for example, Collins & Richmond, 1994; Collins & Stewart, 1993), but was never conceived as necessary in the domain of upper limb oscillation in humans. The present results show that forearm oscillations and limb oscillations during locomotion share similar statistical features that could reveal a general property of biological oscillators, characterized by a fractal evolution of stiffness over time. This fractal property could represent an essential ingredient for understanding the serial dynamics of limb oscillations. The formal architecture of the hopping model suggests that effectors present a set of possible neighboring states that are determined by similar factors and mutually correlated. A simple random walk among this set could produce the fractal correlation structure observed during repetitive oscillations (Delignières et al., 2008).

After testing different forms of continuous coupling functions, we identified a parametric driving based on oscillator's velocity as the simplest solution for accounting, simultaneously, for the empirical limit-cycle dynamics and the serial correlation structure. Combined with the hopping model, this coupling function allowed to generate series reproducing most of the experimentally established signatures.

The present results, combined with those recently obtained in synchronized tapping, yield a more complete picture of the distinction between event-based and emergent timing that has originally been formulated on the basis of self-paced movement tasks. As evoked in the introduction, these two kinds of processes refer to two distinct ways of timing control during rhythmic movements: the event-based processes engage a central and effector-independent representation of time, defining cognitive events that trigger the successive motor responses, and the emergent timing processes refer to a direct control of time through the intrinsic stability properties of the effector's trajectory. The analysis of synchronization tasks shows that despite the similarity in the serial dependence signatures obtained in synchronized tapping and synchronized oscillations, synchronization is realized differently in the two conditions, with a discrete correction process in the case of event-based timing (Torre & Delignières, 2008; Vorberg & Wing, 1996), and a continuous coupling in the case of emergent timing.

Our simulation study allowed us to deepen previous works on synchronization modeling. We confirm that parametric driving is necessary for accounting for the anchoring phenomenon in limit cycle dynamics (Jirsa et al., 2000). However, simulations led us to select a different parametric driving different that those previously used. Fink et al. (2000) and Assisi et al. (2005) choose to couple metronome input to limb position (Eq. 2), but they suggested that a parametric driving term coupling the stimulus to velocity rather than to position should produce qualitatively similar results (Fink et al., 2000). The present work suggests that the position-based coupling function did not allow to reproduce the specific deformation of the limit cycle observed in experimental synchronization series, while a velocity-based coupling between the oscillator and the stimulus yielded more satisfactory results. A deeper analytic approach of this solution remains necessary for testing its relevancy.

A straightforward extension of these results concerns bimanual coordination (Kelso, 1984; 1995). The most popular model was proposed by Haken, Kelso and Bunz (HKB model, 1985); it consists of a system of two coupled oscillating limbs whose dynamics are represented by limit cycle equations. This model was proven to satisfactorily account for essential features in bimanual coordination (*i.e.*, the differential stability of in-phase and anti-phase patterns, and the transition from anti-phase to in-phase pattern under an increase of oscillation frequency). Nevertheless, Torre et al. (2007b) evidenced that relative phase series in externally paced in-phase as well as in anti-phase coordination presented long-range correlations, and showed that the HKB model in its current form was unable to generate such correlation patterns. Considering that limb oscillations, under diverse experimental conditions, spontaneously produce fractal correlations over time, one could consider that an enrichment of the HKB model, providing effectors limit cycle equations with fractal properties, could give a more satisfying account for serial dependence in relative phase series. Finally, note that in most bimanual coordination studies a metronome has been used in a *non-specific* way for pacing movements (Fink et al., 2000). Considering the effect of synchronization on serial dependence in unimanual oscillations, one may wonder whether this effect on effectors's dynamics also holds in bimanual coordinations tasks. Such effects may indeed appear determinant, as accounting for different correlation structures at the component level in self-paced *versus* externally-paced coordination is likely to modify the stability properties of bimanual coordination models.

Acknowledgments

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