

Biases in the simulation and the analysis of fractal processes

Clément Roume¹, Hubert Blain^{1,2} and Didier Delignières¹

1. Euromov, Univ. Montpellier

2. CHU, Montpellier

Fractal processes have recently received a growing interest, especially in the domain of rehabilitation. More precisely, the evolution of fractality with aging and disease, suggesting a loss of complexity [1] has inspired a number of studies that tried, for example, to entrain patients with fractal rhythms [2].

This kind of studies requires relevant methods for generating fractal signals, and for assessing the fractality of the series produced by the participants. In the present work we engaged a cross-validation of three generation methods and three analysis methods. Our rationale is that biases that are revealed by the three analysis methods should be attributed to the generation method, and conversely biases that appear whatever the generation method should originate in the analysis method.

We generated exact fractal series with (1) the Davies-Harte algorithm (DA, [3]), (2) the Spectral Synthesis Method (SSM, [4]), and (3) the ARFIMA synthesis method [5]. 120 series of 1024 data points were generated for fractional Gaussian noise (fGn) processes ranging from $\alpha = 0.1$ to $\alpha = 0.9$, by steps of 0.1, and for fractional Brownian motion (fBm) processes ranging from $\alpha = 1.1$ to $\alpha = 1.9$. Additionally, in order to analyze more closely the behavior of methods around to the $1/f$ boundary, we generated 120 series of 1024 data points of fGn from $\alpha = 0.91$ to 1.0, and fBm from 1.01 to 1.09, by steps of 0.01. The series were analyzed by (1) the evenly spaced Detrended Fluctuation Analysis (DFA, [6]), (2) the Power Spectral Density method (PSD, [7]), and (3) ARFIMA modeling [8].

Results show that some methods of generation present systematic biases: DA presented a strong bias toward white noise in fBm series close to the $1/f$ boundary (i.e. from $\alpha = 1.01$ to $\alpha = 1.1$). SSM produced series with a larger variability around the expected exponent, as compared with other methods. Concerning the methods of analysis, DFA tended to systematically underestimate fBm series. In contrast, PSD yielded overestimates for fBm series. With DFA, the variability of estimates tended to increase as α increased, and reached unacceptable levels for fBm series. The highest levels of variability were produced by PSD. Finally, ARFIMA methods generated the best series, and provided the most accurate and less variable estimates.

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