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Contemporary theories of 1/*f* noise in motor control

Ana Diniz [a], Maarten L. Wijnants [b], Kjerstin Torre [c], João Barreiros [a], Nuno Crato [a], Anna M.T. Bosman [b], Fred Hasselman [b], Ralf F.A. Cox [b], Guy C. Van Orden [d], Didier Delignières [e,*]

[a] *Technical University of Lisbon, Portugal*
[b] *Behavioural Science Institute, Radboud University Nijmegen, The Netherlands*
[c] *McMaster University, Hamilton, Canada*
[d] *University of Cincinnati, OH, United States*
[e] *EA 2991 Motor Efficiency and Deficiency, University Montpellier I, Montpellier, France*

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ABSTRACT

1/*f* noise has been discovered in a number of time series collected in psychological and behavioral experiments. This ubiquitous phenomenon has been ignored for a long time and classical models were not designed for accounting for these long-range correlations. The aim of this paper is to present and discuss contrasted theoretical perspectives on 1/*f* noise, in order to provide a comprehensive overview of current debates in this domain. In a first part, we propose a formal definition of the phenomenon of 1/*f* noise, and we present some commonly used methods for measuring long-range correlations in time series. In a second part, we develop a theoretical position that considers 1/*f* noise as the hallmark of system complexity. From this point of view, 1/*f* noise emerges from the coordination of the many elements that compose the system. In a third part, we present a theoretical counterpoint suggesting that 1/*f* noise could emerge from localized sources within the system. In conclusion, we try to draw some lines of reasoning for going beyond the opposition between these two approaches.

1. Introduction

1/*f* fluctuations present an intriguing phenomenon that received a growing interest in biology, psychology, and movement sciences during the last decade. This kind of fluctuation is typically observed

* Corresponding author. Tel.: +33 467415754; fax: +33 467415708.
E-mail address: didier.delignieres@univ-montp1.fr (D. Delignières).

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during repeated performances of a given system, facing the same task in stable conditions for a prolonged period.

For a long time, variability was not *per se* considered a research interest for scientists. Attention focused on mean values, and their evolution with specific experimental conditions. Variability over successive trials was mainly conceived as the expression of methodological and experimental errors, or of the presence of unmeaning noise in the system under study. Variability was generally discarded by means of averaging, over participants or trials, or by filtering in the case of time series. Sometimes, however, variability was considered a variable of interest, and was assessed in terms of magnitude through the calculation of variance, standard deviation, or coefficient of variation. These measures of variability implicitly suppose that fluctuations are white noise, i.e., uncorrelated over time.

Three decades ago, a growing interest for the analysis of dependencies in time series appeared, especially in econometrics (Box & Jenkins, 1976). This approach focused on short-term dependence, meaning that the current value is only dependent of the previous value, or of a few previous values. These hypotheses were particularly developed through the so-called ARMA models, containing auto-regressive or moving-average processes, in isolation or in combination (Box & Jenkins, 1976).

In most cases, however, correlations in the successive performances of the system are not restricted to the short-term, but are visible over various time scales. In other words, a typical dependence in the series, for example a positive trend between successive values, appears nested with similar trends expressing at larger scales. Statistically similar fluctuations are potentially observed at the level of the second, the hour, the day, the week, the year, and the century. As such, the current value possesses the "memory" of the entire preceding values of the series. This phenomenon was termed, alternatively, as long-term memory, long-range dependence, fractal process, or $1/f$ noise.

$1/f$ noise has been discovered in a number of systems and in a number of situations. Such fluctuations have been found in heartbeat series (Peng et al., 1993), and in stride series during walking (Hausdorff et al., 1996). In the domain of experimental psychology, the seminal paper of Gilden, Thornton, and Mallon (1995) evidencing the presence of $1/f$ fluctuations in tapping tasks had a great impact on the development of research in this area. $1/f$ noise was evidenced in subsequent experiment in various situations, including mental rotation, lexical decision, or visual search (Gilden, 2001), simple reaction time (Van Orden, Holden, & Turvey, 2003), forearm oscillation (Delignières, Torre, and Lemoine, 2008), synchronization to a metronome (Chen, Ding, & Kelso, 1997; Torre & Delignières, 2008a), bimanual coordination (Torre, Delignières, & Lemoine, 2007), and serial force production (Wing, Daffertshofer, & Pressing, 2004). Delignières, Fortes, and Ninot (2004) evidenced $1/f$ fluctuations in the daily evolution of self-esteem.

The interest of scientists toward $1/f$ noise was reinforced by the discovery of the relationships between fractal fluctuations and health (Goldberger et al., 2002). $1/f$ fluctuations are generally evidenced in young and healthy systems, performing in a stable, unperturbed environment, and facing easy or overlearned tasks (Kello, Beltz, Holden, & Van Orden, 2007). In contrast, $1/f$ fluctuations seem to disappear with aging or disease (Hausdorff et al., 1997).

The aim of this paper is to present a general overview of current theories about $1/f$ noise, based on the contents of a symposium organized during the EWOMS 2009 congress in Lisbon. The first part of this paper develops a formal definition of $1/f$ noise, and presents the mathematical foundations of the methods used for evidencing the presence of such fluctuations and for measuring long-range correlations in the series. The second part introduces one of the prominent hypotheses in this domain of study, linking $1/f$ noise to the complexity of the system, and to the processes of coordination between their constituent self-systems. The third part presents an alternative point of view, suggesting that $1/f$ noise could take its origin in some specific sub-systems within the global system.

Note that the authors of this paper commit to different theoretical positions, and develop rather distinct, sometimes competing approaches of $1/f$ noise. We did not try to propose any kind of consensus between our points of view. We decided to present as clearly as possible the rationale of each theoretical position, in order to allow the reader to understand the meaning of the debate.

2. Formal mathematical definition of 1/*f* noise

An important issue in several scientific areas is the change of phenomena over time. The classical methods of analysis are based on descriptive statistics, such as the mean and the variance, and ignore the dimension of time. In contrast, the time series methods, in the so-called time and frequency domains, focus on the dynamical behaviors across time and allow for modeling and inference. In the biological field, numerous studies have revealed results typical of a particular type of structure called long-term memory or long-range dependence. To assess the fundamental properties of the observed signals, it is natural to consider that they are realizations of stochastic processes. For the signals under study, it is usually supposed that they have a kind of stability that can reasonably be modeled by stationary processes.

In the time domain analysis, a central concept is the autocorrelation function of the process which gives the correlation between variables of the process at two different times. Formally, a stochastic process is said to be stationary (in the wide sense) if its mean is constant across time and its autocorrelation function depends only on the time lag between the variables. In this case, the stochastic memory of the process can be defined as the speed of the decay of the autocorrelation function. Formally, a stationary process is said to have long memory if its autocorrelation function $\rho(.)$ satisfies the power law

$$\rho(k) \sim ck^{-(1-2d)}, \quad k \to \infty, \tag{1}$$

where c and d are two constants such that $c \neq 0$, $d \neq 0$, and $d < 0.5$, and k is the lag. This means that the function $\rho(.)$ decays to zero very slowly with a hyperbolic decay. Moreover, the process is said to have persistent long memory if $0 < d < 0.5$, so that $\Sigma_{k=-\infty}^{\infty} \rho(k) = \infty$, reflecting the fact that the remote past has a strong influence into the present.

In the frequency domain analysis, a key concept is the spectral density function of the process which gives the amount of variance accounted for by each frequency in the process and corresponds mathematically to the Fourier transform of the autocorrelation function. The spectral density function allows for identifying dominant frequencies in the process that may be associated to hidden periodicities. Formally, a long-memory process can be defined as a process whose spectral density function $S(.)$ satisfies the power law

$$S(f) \sim cf^{-2d}, \quad f \to 0, \tag{2}$$

where c and d are two constants such that $c \neq 0$, $d \neq 0$, and $d < 0.5$, and f is the frequency. This means that the function $S(.)$ has a pole at zero if $0 < d < 0.5$, that is $S(0) = \infty$, signifying that the low frequencies predominate and therefore long-term oscillations are expected. These processes whose function $S(.)$ has the form $S(f) \sim f^{-\alpha}$, where $\alpha = 2d$ and so α is a constant such that $\alpha < 1$, are usually known as $1/f^{\alpha}$ noise. Note that the definitions in Eqs. (1) and (2) are mathematically equivalent.

In continuous time, a long-memory process is self-similar. Formally, a stochastic process $\{Y_t\}$ is said to be self-similar with parameter H if, for any constant $c > 0$, it satisfies the relation

$$Y_{ct} =_d c^H Y_t, \quad t \in R, \tag{3}$$

where d denotes equality in distribution. This means that the process $\{Y_t\}$ has identical statistical properties independent of the scale of observation. For a long-memory process, the parameter H relates to the parameter d through the expression

$$H = d + 0.5. \tag{4}$$

In conclusion, a long-memory process has special properties, in the time and in the frequency domains, which are very distinct from those of other traditional stationary processes. Apart from the definitions of long memory presented here, there are other possible definitions. Some interesting details can be found in Baillie (1996) and Guégan (2005).

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3. Some methods for the detection and estimation of exponents

Many methods, in the time and in the frequency domains, have been proposed to estimate the long-memory and the self-similarity parameters d and H (e.g., Eke, Herman, Kocsis, & Kozak, 2002). Among these methods, there are some heuristic techniques, non- or semi-parametric, mainly useful as first diagnostic tools for checking the existence of long memory, and more refined techniques, parametric and model-dependent, useful for estimating the long-memory and the scale parameters. For reference, four methods are presented below, namely the rescaled range methodology (R/S), the detrended fluctuation analysis (DFA), the Geweke and Porter–Hudak regression (GPH), and the maximum likelihood estimation (MLE). These methods have been widely used in the literature and provide suitable and complementary tools for the study of long-memory time series.

The R/S method was initially developed by Hurst (1951) in a study of the levels of the Nile River and it was explained by Mandelbrot (1965) with the introduction of fractional models. This method is one of the better known methods and it is based on the rescaled adjusted range. For a time series $\{Y_1 \ldots Y_n\}$ and a positive integer $n_s \leqslant n$, the R/S statistic is defined as

$$Q(n_s) = R(n_s)/S(n_s), \tag{5}$$

with

$$R(n_s) = \max_{1 \le r \le n_s} \sum_{t=1}^{r} (Y_t - \bar{Y}) - \min_{1 \le r \le n_s} \sum_{t=1}^{r} (Y_t - \bar{Y}) \tag{6}$$

and

$$S(n_s) = \left[\sum_{t=1}^{n_s} (Y_t - \bar{Y})^2 / n_s \right]^{1/2}, \tag{7}$$

where $\bar{Y}$ is the sample mean. This signifies that the statistic $Q(.)$ records the integrated series range adjusted for the mean and normalized by the standard deviation in blocks of length n_s.

For a persistent long-memory process, the statistic $Q(.)$ satisfies the power law

$$E[Q(n_s)] \sim c\ n_s^b, \quad n_s \to \infty, \tag{8}$$

or, equivalently,

$$\log E[Q(n_s)] \sim \log c + b \log n_s, \quad n_s \to \infty, \tag{9}$$

where c and b are two constants such that $c > 0$ and $0.5 < b < 1$ (for short-memory, $b = 0.5$). The parameter b is called the Hurst exponent or the self-similarity parameter H.

To estimate H through the R/S method, proceed as follows: (i) divide the time series of length n into contiguous blocks of length n_s with starting points $p_j = (j - 1)\ n_s + 1$, $j = 1 \ldots [n/n_s]$; (ii) compute, for each block, the value of $Q(n_s)$ and determine the mean $\bar{Q}\ (n_s)$. Then, repeat this procedure over all possible block lengths n_s (in practice, $10 \leqslant n_s \leqslant [n/2]$) and plot log $\bar{Q}\ (n_s)$ against log n_s. Finally, fit a linear-regression model to the points, obtain the slope $\hat{b}$ with a least-squares method, and set $\hat{H} = \hat{b}$ (Delignières, Torre, & Lemoine, 2005; Taqqu, Teverovsky, & Willinger, 1995).

The DFA method was established by Peng et al. (1993) in a study of the behavior of the heartbeat. This method is based on the detrended values fluctuation. For a time series $\{Y_1 \ldots Y_n\}$ and a positive integer $n_s \leqslant n$, the DFA statistic is given by

$$F(n_s) = \left[\sum_{r=1}^{n} [W_r - \hat{W}_r(n_s)]^2 / n \right]^{1/2} \tag{10}$$

with

$$W_r = \sum_{t=1}^{r} (Y_t - \bar{Y}) \text{ and } \hat{W}_r(n_s) = \hat{a}_0(n_s) + \hat{b}_0(n_s) r, \quad r = 1 \ldots n, \tag{11}$$

where $\bar{Y}$ is the sample mean, $\hat{a}_0\ (n_s)$ and $\hat{b}_0\ (n_s)$ are the integrated series estimators of the coefficients of linear-regression models. This signifies that the statistic $F(.)$ records the integrated series variability adjusted for local trends in blocks of length n_s.

For a persistent long-memory process, the statistic $F(.)$ satisfies the power law

$$E[F(n_s)] \sim cn_s^b, \quad n_s \to \infty, \tag{12}$$

or, equivalently,

$$\log E[F(n_s)] \sim \log c + b \log n_s, \quad n_s \to \infty, \tag{13}$$

where c and b are two constants such that $c > 0$ and $0.5 < b < 1$ (for short-memory, $b = 0.5$). The parameter b is the self-similarity parameter H.

To estimate H through the DFA method, proceed as follows: (i) divide the time series of length n into contiguous blocks of length n_s with starting points $p_j = (j - 1)\ n_s + 1$, $j = 1\ldots[n/n_s]$; (ii) compute, for the length n_s, the value of $F(n_s)$. Then, repeat this procedure over all possible block lengths n_s (in practice, $10 \leqslant n_s \leqslant [n/2]$) and plot log $F(n_s)$ against log n_s. Finally, fit a linear-regression model to the points, obtain the slope $\hat{b}$ with a least-squares method, and set $\hat{H} = \hat{b}$ (Delignières et al., 2005; Taqqu et al., 1995).

The GPH regression was introduced by Geweke and Porter-Hudak (1983). This method involves a regression of the logarithm of the periodogram on the logarithm of a function of the frequency. Recall that, for a time series $\{Y_1\ldots Y_n\}$, the periodogram $I(.)$ is defined as

$$I(f_j) = \left|\sum_{t=1}^{n} Y_t e^{itf_j}\right|^2 /(2\pi n),\ f_j = (2\pi j)/n, \quad j = 1\ldots[n/2], \tag{14}$$

where $I(f_j)$ represents the intensity of the frequency f_j. It is well know that the periodogram is an estimator of the spectral density function (Brockwell & Davis, 1991).

For a persistent long-memory process, the spectral density function $S(.)$ satisfies the relation

$$S(f) = S^*(f)|1 - e^{-if}|^{2b}, \quad |f| \le \pi \quad (S(f) \sim cf^{2b},\ f \to 0), \tag{15}$$

where $S^*(.)$ is an even function that is finite and nonzero at zero, and b is a constant such that $b = -d$ and $0 < d < 0.5$, that is, b is the negative of the long-memory parameter d (for short-memory, $b = 0$ and $d = 0$); the periodogram $I(.)$ satisfies the relation

$$\log I(f_j) = a + b \log|1 - e^{-if_j}|^2 + e_j, \quad f_j \approx 0,\ j = 1\ldots m, \tag{16}$$

where a and b are two constants such that $b = -d$ and $0 < d < 0.5$, e_j are independent and identically distributed random variables, and $m = [n^{0.5}]$.

To estimate d through the GPH method, compute the values of $I(f_j)$ and plot log $I(f_j)$ against $\log|1 - e^{-if_j}|^2$. Then, fit a linear-regression model to the points, obtain the slope $\hat{b}$ with a least-squares method, and set $\hat{d} = -\hat{b}$ (Crato & Ray, 2000; Taqqu et al., 1995). The power spectral density method (PSD) is very similar to this method but it is based on the asymptotic distribution of the spectral density function instead of the exact distribution shown in Eq. (15) (Delignières et al., 2005).

The maximum likelihood methods, in the time and in the frequency domains, are based on parametric models and allow for the estimation of the long-memory parameter as well as scale parameters. Suppose that $\{Y_t\}$ is a Gaussian stationary process with mean $\mu = 0$ and autocovariance function $\gamma(.)$ whose model comes from a parametric family with parameter vector β. Suppose, in addition, that $Y_n = (Y_1\ldots Y_n)'$ is a realization of the process with covariance matrix $\Gamma_n(\beta) = [\gamma(i - j)]_{i,j=1\ldots n}$. Then the likelihood function is equal to

$$L_n(\beta) = (2\pi)^{-n/2}[\det\Gamma_n(\beta)]^{-1/2} \exp[-1/2Y_n'\Gamma_n^{-1}(\beta)Y_n] \tag{17}$$

and the log-likelihood function is equal to

$$\log L_n(\beta) = -n/2 \log 2\pi - 1/2 \log\det\Gamma_n(\beta) - 1/2Y_n'\Gamma_n^{-1}(\beta)Y_n. \tag{18}$$

The maximum likelihood estimator of β is obtained by maximizing $L_n(\beta)$ or $\log L_n(\beta)$ with respect to β. However, the maximization of $L_n(\beta)$ or $\log L_n(\beta)$ requires the calculation of the determinant and the inverse of the matrix $\Gamma_n(\beta)$ which can pose computational problems, in particular for long time series with long memory. These problems can be minimized with some analytical algorithms, such as the Durbin–Levinson algorithm (Durbin, 1960).

An alternative to maximizing the exact likelihood function in the time domain is to maximize an approximation to that function in the frequency domain. Suppose that $S(.;\beta)$ is the spectral density function of the process, $I(.)$ is the periodogram of the realization, and $f_j = (2\pi j)/n$. With some approximations proposed by Whittle (1953) and some Riemann sums, the negative of the log-likelihood function is approximated up to a constant by

$$L_n(\beta) = \sum_{j=1}^{[n/2]} \log S(f_j;\beta)2/n + \sum_{j=1}^{[n/2]} I(f_j)/S(f_j;\beta)2/n. \tag{19}$$

The approximate maximum likelihood estimator of β is obtained by minimizing $L_n(\beta)$ with respect to β. When it is not easy to specify a parametric model and a spectral density function for the observations, it is possible to use a similar semi-parametric method for estimating the parameters of interest as long as the shape of the spectral density is known (Robinson, 1995). Thus, this is a very flexible method which can be used even in additive models widely found in motor control theories (Diniz, Barreiros, & Crato, 2009, 2010).

In sum, there are various methods to estimate the long-memory and the self-similarity parameters. The heuristic techniques are based on specific properties of the time series, whereas the maximum likelihood type techniques are based on parametric models. The methods reviewed here are some of the most widely used in the estimation of the parameters. Some additional techniques can be seen in Taqqu et al. (1995) and Delignières et al. (2005).

4. Looking at 1/*f* noise as a signature of system complexity

The origin of 1/*f* noise in the behavior of biological systems remains an issue of debate across scientific disciplines. The differences in approach that feed the debate, as in the current article, are partially due to the fact that a number of different mechanisms are able to effectively produce 1/*f* noise in system dynamics (i.e., both simple and complex systems). Complex systems are systems that consist of a set of interrelated and interdependent parts with an almost infinite amount of degrees-of-freedom that cohere into a coordinated functional system. The parts dynamically interact in non-linear ways, a conceptual metaphor referred to as interaction dominance (e.g., Van Orden et al., 2003). Defining features include self-organization (the spontaneous organization that coordinates system behavior in the absence of a central controller) and emergence (the appearance of features that are not implicit in the parts of the system). On the other hand, simple systems are systems that contain a number of distinct components whose internal dynamics, when integrated, account for the observed performance. This way of thinking is the more conventional conceptual metaphor to think about movement control and may be referred to as component dominance because "the intrinsic activities of the components are held to be much more dominant in determining the observed performance than the interactions among components" (Turvey, 2007, p. 690).

Some theorists attempt to compromise between these approaches. For example, Delignières and colleagues (see Section 5) commit to the idea of local interactions as a mechanism underlying 1/*f* noise, without committing to the idea of multiplicative interactions among components. Delignières et al. rather conceive such local interactions as within-component coordination dynamics. In this section, Van Orden and colleagues argue that the widely observed 1/*f* noise in human behavior is the fingerprint of a complex system in the true physical sense; that is, a system comprising fully interdependent feedback processes among components (e.g., Turvey, 2007). Accepting this premise, 1/*f* noise necessarily results from the intrinsic dynamics that govern human behavior (Kello et al., 2007; Van Orden et al., 2003; Wijnants, Bosman, Hasselman, Cox, & Van Orden, 2009). Accepting the origin of 1/*f* noise in complexity, it may be postulated that, far from being mere noise, 1/*f* noise is actually the signature of strongly emergent coordination (e.g., Buzsaki, 2006; Kello et al., 2007;

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Van Orden et al., 2003; Wijnants, Bosman, Hasselman, et al., 2009). This general hypothesis is illustrated in the five predictions listed next, to evaluate 1/*f* noise as a metric for coordination in human behavior.

4.1. 1/f noise is ubiquitous in human performance

Any behavioral phenomenon will reveal long-range dependence if measured over a sufficient duration in time (usually 2^{10} data points suffice, Delignières, Lemoine, & Torre, 2004). If 1/*f* noise originates from a complex system, any component process stems from the mutual interactions that govern the system. The implication is that any process should yield 1/*f* noise in its dynamics. To date, dozens of studies have been published on long-range dependence in cognitive and motor performance, all demonstrating widespread, perhaps ubiquitous 1/*f* noise (e.g., Kello et al., 2007; Van Orden, Kloos, & Wallot, 2010, are reviews).

4.2. 1/f noise is obscured when sources of external variation are increased

Intrinsic fluctuations which govern the cognitive system are obscured when external variation in an experiment increases. For example, external manipulations of task demands may constitute sources of white noise. Such random perturbations to behavior caused by external factors disrupt the intrinsic dynamics, thereby obscuring their signature. Thus, unsystematic changes across trial measurements show themselves as "whitened" signals of 1/*f* noise, as they reduce the presence of 1/*f* noise in the now de-correlated behavioral signal (e.g., Clayton & Frey, 1997; Correll, 2008). Conversely, when unsystematic sources of external perturbation are minimized, white noise is reduced and 1/*f* noise is more clearly present (e.g., Kello, Beltz et al.; Kiefer, Riley, Shockley, Villard, & Van Orden, 2009; Ward & Richard, 2001). This prediction stems from the broad association between 1/*f* noise and intentionality (Van Orden et al., 2003): Adding external constraints reduces the demands for voluntary control and the presence of 1/*f* noise in a behavioral signal, resulting in a whitened signal, regardless of the specificity of a certain task.

Also more systematic trial-by-trial perturbations reduce the presence of 1/*f* noise. For example, providing feedback in a time estimation task constrains responding sufficiently to reduce demands for voluntary control and a whitened signal is the result (N. Kuznetsov, & S. Wallot, personal communication, September 15, 2009). Similarly, entrainment reduces the need for voluntary control and also whitens behavioral signals. In continuation tapping participants tap in synch to a training beat and then tap from memory after the metronome is turned off. Leaving the metronome on throughout cedes control of tapping to the environment via entrainment, which reduces the demands of voluntary control and consequently reduces the presence of 1/*f* noise in the asynchronies to the metronome. A whiter signal is observed in entrained signals compared to continuation tapping and compared to a control condition of syncopated tapping between the beats (Chen, Ding, & Kelso, 2001; see also Hausdorff et al., 1996).

Following this logic, one may expect clearer signals of 1/*f* noise in tasks that emphasize voluntary control, such as measurement trials that do not include a response cue. For example, in tasks like precision aiming or spatial and temporal estimation, the "stimuli" (targets) remain in front of the participant throughout the task and one sees clear signals of 1/*f* noise (Gilden et al., 1995; Wijnants, Bosman, Hasselman, et al., 2009) compared to simple response or word-naming tasks (Van Orden et al., 2003) which do include a response cue at each trial. Hence, the presence of 1/*f* noise changes in predictable ways across tasks as a function of external constraints, a finding which seems to require a domain-general explanation.

4.3. More stable and coordinated behaviors reveal a clearer 1/f noise signature

The premise that the behavior of a complex system is determined by the interactions among components leads to the prediction that these interactions are more apparent in the intrinsic dynamics of the system, if the system operates in more coordinated and efficient ways. This prediction is in line with results obtained in other disciplines: 1/*f* noise in living systems is generally accepted as an emergent pattern of coordination (West & Brown, 2005).

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For instance, physiological systems reveal healthy and coordinated functioning in the presence of 1/*f* noise. When a human heart deviates from 1/*f* noise in an inter-beat interval sequence, in either the direction of randomness (white noise) or over-regularity (brown noise), this deviation from the adaptive healthy baseline indicates pathological, life-threatening states like atrial fibrillation and congestive heart failure, respectively (Goldberger, 1996; Goldberger et al., 2002; Havlin et al., 1999). The clear relation between complexity and coordination is not unique to heart beat dynamics. Other examples include epilepsy, fetal distress syndrome, major-depressive disorder, attention deficit and hyperactivity disorder, falling, and slow transit constipation, among others (Gilden & Hancock, 2007; Goldberger, 1996; Hausdorff, 2007; Linkenkaer-Hansen et al., 2005; Yan, Yan, Zhang, & Wang, 2008), all of which are associated with a deviation from the healthy fractal pattern of 1/*f* noise.

The issue of coordination in physiological processes parallels the coordination of motor behavior. For instance, 1/*f* noise is less prominent in stride intervals of human gait with disease. Both Huntington's and Parkinson's Disease patients reveal reliably more randomness and less 1/*f* noise in the time series collected from their gait cycles compared to healthy controls (Hausdorff, 2007). Furthermore, 1/*f* noise is strongly correlated with the severity of the illness.

If more stable and coordinated behaviors are associated with the clearer presence of 1/*f* noise, one would expect that 1/*f* noise emerges less clearly in more challenging tasks. There is some evidence suggesting that more effortful task conditions indeed reduce the presence of 1/*f* noise (Clayton & Frey, 1997; Correll, 2008). Likewise, one may expect that 1/*f* noise emerges more clearly as task performance improves with learning. For example, extensive practice of a motor task yields a reduced signal of white noise and an enhanced signal of 1/*f* noise (Wijnants, Bosman, Hasselman, et al., 2009). Wijnants, Bosman, Cox, Hasselman, and Van Orden (2009) successfully replicated this finding in a different domain, word-naming. The study was based on the robust observation that word repetition facilitates word-naming performance. When word stimuli are named repeatedly (over three identical blocks), again 1/*f* noise emerged more clearly over blocks of practice.

From an interaction-dominant perspective, it is argued specifically that the discussed association of 1/*f* noise and coordination processes is too general to be captured by task-specific explanations. These findings rather seem to suggest a broad connection between 1/*f* noise and the self-organization across processes of mind and body (Van Orden et al., 2010).

4.4. 1/f noise should be accompanied by additional evidence for emergence and self-organization

If the relation between 1/*f* noise and coordination can be understood as the coupling or interdependence of components, as opposed to their independence, 1/*f* noise should covary with other dynamical measures. Examples include reduced entropy, a decrease in system dimensionality and a more efficient recycling of kinetic energy in sequences of rhythmical movement (e.g., Wijnants, Bosman, Hasselman, et al., 2009; Wijnants, Cox, Hasselman, Bosman, & Van Orden, 2009. We may expect additional surprises in this vein if we truly confront interaction-dominant dynamics and complexity (e.g., Ihlen & Vereijken, 2010). Altogether, such system properties may support or reject the postulation that the relative presence of 1/*f* noise constitutes a sensitive metric for emergent coordination. Unfortunately, in most studies that incorporate 1/*f* noise, other dynamical measures are not evaluated. However, it is exactly the convergence between such measures, which reveals the full-blown complexity of the cognitive system, thereby posing specific challenges to the development of contemporary theories and models of motor control.

Note that fractal methods do not replace traditional measures based on means and standard deviations, because they provide orthogonal and complementary pieces of information about the behavior of the system. Several studies suggest, however, that 1/*f* may constitute a more sensitive metric compared to measures of central tendency to discriminate between groups and experimental manipulations (Anderson, Lowen, & Renshaw, 2006; Hausdorff, 2007; Kiefer et al., 2009).

4.5. Indefinite numbers of 1/f signals exist in any behavior

According to the premise, 1/*f* noise is a generic property of system interactions that give rise to all behaviors. According to emergent coordination, 1/*f* noise is not restricted to some domain-specific

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process or measure of cognition. Any and all behavioral signals should yield 1/*f* noise under conditions of intrinsic fluctuation, even if there are multiple distinct signals. Thus, one should be able to find multiple, parallel streams of 1/*f* noise under conditions of intrinsic fluctuation. Kello, Anderson, Holden, and Van Orden (2008) instructed participants to repeat an utterance (here, the word bucket) many times in order to elicit intrinsic fluctuations from one utterance to the next. The authors took over 100 acoustic measures of each word utterance and analyzed the fluctuations in those measures from one *bucket* to the next. Every single measure was found to fluctuate as 1/*f* noise, including dozens of parallel yet uncorrelated 1/*f* fluctuations. The findings of 1/*f* noise throughout the intrinsic fluctuations of speech, and in two different key response measures (Kello et al., 2007), are parsimoniously explained by emergent coordination: 1/*f* noise is prevalent wherever intrinsic fluctuations are measured. Does each signal require its own mechanism?

5. An alternative hypothesis: 'localized' sources of 1/*f* noise

The preceding part of this paper develops a strong theoretical point of view, considering 1/*f* fluctuations as the natural expression of coordination within complex systems (Kello et al., 2007, 2008; Van Orden, Holden, & Turvey, 2005; Van Orden et al., 2003). According to this point of view, 1/*f* noise is related to very general properties of complex systems, such as self-organized criticality and metastability. These properties are supposed to express in all complex systems, and as such provide a satisfying explanation for the ubiquity of 1/*f* noise. The authors contest the necessity of domain-specific hypotheses for fractal fluctuations: as claimed by Kello et al. (2007), "1/*f* scaling [noise] is too pervasive to be idiosyncratic" (p. 551). As a consequence, the authors oppose the idea of a structural localization of 1/*f* sources within the system: "Pink [1/*f*] noise cannot be encapsulated; it is not the product of a particular component of the mind or body. It appears to illustrate something general about human behavior" (Van Orden et al., 2003, p. 345). This so-called *nomothetic perspective* to 1/*f* noise (Torre & Wagenmakers, 2009) seeks for general explanations, regardless of the specificity of any hypothesized sub-system.

Torre, Delignières, and collaborators adopted a different point of view (Delignières et al., 2004, 2008; Delignières & Torre, 2009; Torre & Delignières, 2008a, 2008b; Torre & Wagenmakers, 2009). The main specificity of their approach is to combine the analysis of fractal, long-range correlations, with that of short-term dependence in the series. Their work initiated in the study of timing tasks, and especially finger tapping. The finger tapping task has a long history in experimental psychology. In the most basic experimental condition, participants are instructed, after a short period during which a metronome provides a given tempo, to continue tapping following the same rhythm despite the removing of the metronome. The most famous model for this so-called 'continuation' condition was proposed by Wing and Kristofferson (WK model, 1973). This model is composed of two components: a cognitive timekeeper that generates series of time intervals C_i, and a motor implementation process in charge of the execution of the tap at the end of each interval. The motor component is supposed to present a delay M_i. According to this model, the produced inter-tap interval is given by the period provided by the timekeeper, plus the difference between the time delays that characterize the two taps that limit the interval:

$$I_i = C_i + M_i - M_{i-1}. \tag{20}$$

In the initial formulation of the model, C_i and M_i were both considered as uncorrelated white noises. This model especially allowed to account for the typical negative lag-one autocorrelation in inter-tap interval series (due to the presence of the same M_i term, but of opposite signs, in successive inter-tap intervals).

Gilden et al. (1995) applied spectral analysis to prolonged series of inter-tap intervals, and showed that the log–log power spectrum presented a negative linear trend in the low-frequency region, indicative of 1/*f* noise, and a positive trend in the high-frequency region. This result was confirmed by a number of subsequent studies (Delignières et al., 2004; Lemoine, Torre, & Delignières, 2006; Yamada, 1996; Yamada, 1995; Yamada & Yonera, 2001; Yoshinaga, Miyazima, & Mitake, 2000; see Fig. 1). According to Gilden et al. (1995), the positive slope in high frequencies is typical of differenced white

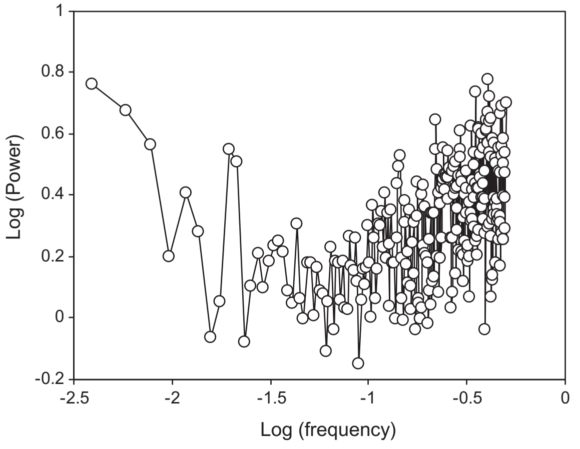

Fig. 1. Log–log power spectrum of series of inter-tap intervals in continuation finger tapping (Delignières et al., 2008). This average spectrum presents a negative slope in the low- frequency region, indicative of 1/*f* noise, and a positive slope in high frequencies typical of differenced white noise.

noise, and thus, can be attributed to the $M_i - M_{i-1}$ part of the WK model. The authors concluded that the timekeeper C_i should be a 1/*f* source, responsible of the fractal fluctuations in inter-tap interval series. They finally stated that the cognitive component should be considered a complex dynamical system, composed of multiple interacting components. Note that in contrast with the basic assumptions of Van Orden, Kello, and their collaborators, Gilden's approach suggested that the source of 1/*f* noise could be "localized" within the global system, in a sub-system that interacts with other components for producing the final outcome.

A second aspect of the tapping paradigm consists of asking participants to tap in synchrony with the sounds emitted by a metronome. This experimental condition allows collecting two variables: the series of inter-tap intervals, and the series of asynchronies to the metronome. Chen et al. (1997) showed that the series of inter-tap intervals, in synchronization tapping, were no more 1/*f* noise, but were negatively correlated. In contrast, they discovered 1/*f* fluctuations in the series of asynchronies to the metronome (see Fig. 2). Surprisingly, Chen et al. did not try to relate the finding of 1/*f* noise in asynchronies in their experiment with the presence of fractal fluctuations in inter-tap

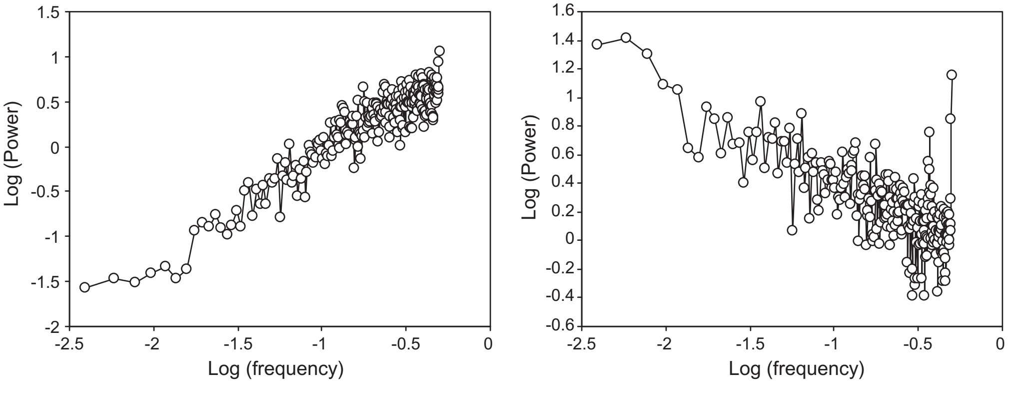

Fig. 2. Log–log power spectra of series of inter-tap intervals (left) and asynchronies (right), in synchronization finger tapping (Torre & Delignières, 2008a). The positive slope, for inter-tap interval series, reveals the presence of negative serial correlations between successive values. The negative slope for asynchronies series is indicative of the presence of 1/*f* noise.

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intervals in continuation, as evidenced by Gilden et al. (1995). They considered that synchronization could *per se* induce $1/f$ noise, as the natural outcome of the complex system formed by experimental constraints. They proposed that complex systems could be characterized by "essential variables" (e.g., asynchronies in synchronization tapping), and that $1/f$ fluctuations appeared at the level of these essential variables. This proposition is consistent with the nomothetic approach of Kello, Van Orden, and collaborators: each experimental condition is supposed to establish a new set of constraints, determining a complex system that expresses itself in time through $1/f$ fluctuations. From this point of view, however, the presence of $1/f$ noise in periods in continuation tapping, and in asynchronies in synchronization tapping are just independent phenomena.

In contrast, Torre and Delignières (2008a) proposed a unifying framework to account for the presence of $1/f$ fluctuations in both continuation and synchronization tapping. They started from the linear phase correction model proposed by Vorberg and Wing (1996). We first present the rationale of this model.

In synchronization tapping, each inter-tap interval (I_i) corresponds to the difference between its previous and next asynchronies (A_{i-1} and A_i), plus the period (τ) imposed by the metronome:

$$I_i = A_i - A_{i-1} + \tau. \tag{21}$$

The main assumption of the model is that the preceding asynchrony is taken into account by a linear phase correction: The interval produced by the timekeeper is corrected by a fraction of the preceding asynchrony:

$$C_i^* = C_i - \alpha A_{i-1}. \tag{22}$$

As proposed in the WK model, the produced interval results from the combination of this corrected cognitive interval and the two successive motor delays:

$$I_i = C_i^* + (M_i - M_{i-1}). \tag{23}$$

Combining Eqs. (21–23) leads to the following expression for current asynchrony:

$$A_i = (1 - \alpha)A_{i-1} + C_i + (M_i - M_{i-1}) - \tau. \tag{24}$$

The strength of this model is to offer a unifying framework for continuation and synchronization tasks: the Vorberg–Wing model is an extension of the basic WK model for continuation, and both models include the timekeeper entity initially postulated by Wing and Kristofferson (1973). As such, a timekeeper possessing fractal properties could explain the presence of fractal correlations in inter-tap interval series in continuation on the one hand, and in asynchrony series in synchronization on the other hand. Indeed, Torre and Delignières (2008a) proposed to enrich the Vorberg and Wing (1996)'s model by providing C_i with fractal properties, and showed that this "$1/f$-linear phase correction model" was able to adequately reproduce the complex correlation structures of period and asynchrony series. The most important, at this point, is to note that the WK model (Eq. (20)) and the Vorberg–Wing model (Eq. (24)) are able to account for the complex patterns of serial correlations in continuation and synchronization tapping, respectively, from the moment that a single element in both models is provided with fractal properties.

A similar approach was developed for another kind of rhythmic task: forearm oscillations. In a first step, Delignières et al. (2004) applied spectral analysis to series of periods collected during self-paced oscillations. Results showed that, like in tapping, period series presented $1/f$ fluctuations with a negative linear slope in the low-frequency region of the log–log spectrum. In the high-frequency region, in contrast, the authors observed a simple flattening of the log–log spectrum, the slope remaining slightly negative (see Fig. 3).

This comparison between tapping and oscillation was motivated by the distinction established some years ago between event-based and emergent timing processes (Robertson et al., 1999; Zelaznik, Spencer, & Doffin, 2000). Event-based timing is typically exploited in tasks involving serial discrete movement, especially finger tapping. In this case, timing is supposed to require an explicit representation of time. Emergent timing, in contrast, is supposed to be exploited in tasks involving smooth and continuous cyclical movements (as for example, circle drawing, or forearm oscillations). In that case,

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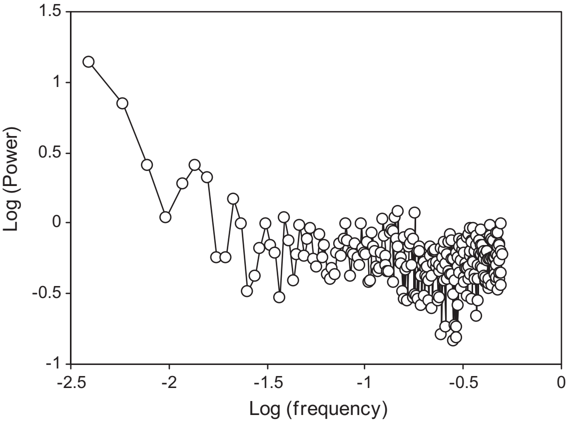

Fig. 3. Log–log power spectrum of series of periods during self-paced forearm oscillations (Delignières et al., 2008). This average spectrum presents a negative slope in the low-frequency region, indicative of 1/*f* noise, and a flattening in high frequencies typical of additive white noise.

timing seems governed by the dynamical properties of effectors, considered as self-sustained oscillators.

According to Delignières et al. (2004), the flattening of the slope of the log–log spectrum in high frequency shows that in oscillation motor variability affects directly interval durations via the movement trajectory (in contrast, in tapping it affects successive interval boundaries via discrete motor implementation delays). As such, the authors considered the high frequency slope of the log–log spectrum as a distinctive signature between event-based and emergent timing.

Delignières et al. (2008) proposed to model forearm oscillations with a hybrid self-sustained oscillator (Kay, Saltzman, Kelso, & Schöner, 1987):

$$\ddot{x} = \alpha\dot{x} - \beta\dot{x}x^2 - \gamma\dot{x}^3 - \omega^2 x + \sqrt{Q}\xi_t. \quad (25)$$

where x represents position and the dot notation differentiation with respect to time. α represents linear damping, β and γ the van der Pol and Rayleigh non-linear damping terms, respectively, ω^2 represent stiffness, and ξ_t a white noise term of strength Q. Delignières et al. (2008) showed that in its initial form, the series of periods produced by this hybrid model fluctuates randomly around a baseline period determined by the stiffness parameter. They suggested that the presence of 1/*f* noise in oscillations' periods could be due to cycle-to-cycle fluctuations of stiffness. Introducing 1/*f* noise in the ω^2 parameter, they showed that the oscillator produced series of period possessing dynamical signatures similar to those experimentally observed.

More recently, Torre, Balasubramaniam, and Delignières (2010) examined the effect of synchronization to a metronome on forearm oscillations. Basically, the impact of synchronization was similar to that observed in tapping: the series of periods became anti-persistent, and the series of asynchronies to the metronome presented 1/*f* fluctuations. They also observed a characteristic change in the shape of the phase portrait, with the appearance of an anchoring phenomenon (i.e., a thinning of trajectories in the phase plane, see also Assisi, Jirsa, & Kelso, 2005; Byblow, Carson, & Goodman, 1994; Fink, Foo, Jirsa, & Kelso, 2000) in the vicinity of the occurrence of the metronome, and a specific asymmetry of the limit cycle trajectory.

The authors proposed a modified version of the parametric driving model by Jirsa, Fink, Foo, and Kelso (2000), obeying the following equation:

$$\ddot{x} = \alpha\dot{x} - \beta\dot{x}x^2 - \gamma\dot{x}^3 - \omega_i^2 x + \varepsilon_1 \cos\Omega t + \varepsilon_3 \dot{x}\sin\Omega t + \sqrt{Q}\xi_t. \quad (26)$$

This model is an extension of the preceding hybrid model, with a non-linear parametric coupling function. In this equation, Ω represents the frequency of the metronome and ε_1 and ε_3 the strength of the linear and parametric driving terms, respectively. Note that the stiffness parameter is now indexed, indicating cycle-to-cycle changes in stiffness. The authors showed that this model was able to adequately account for the experimentally observed pattern of correlations and for the specific changes in the limit cycle dynamic. These results suggest that in contrast to the discrete correction process involved in synchronized tapping, the synchronization of oscillations to a metronome involves a continuous form of coupling. More importantly for the present purpose, they show that the complex pattern of serial correlation obtained in self-paced and synchronized oscillations could be generated by classical dynamical models, on condition that one specific parameter in the equation was provided with fractal properties.

In opposition to the nomothetic perspective promoted by Van Orden, Kello, and collaborators, Torre and Wagenmakers (2009) proposed to designate as *mechanistic* this approach that seeks for task-specific models. This mechanistic approach suggests that it is possible to account for the presence of fractal fluctuations in series of performances by injecting 1/*f* noise in precise locations in classical models. This approach has the advantage to respect fundamental aspects of previous theories that are accounted for by these classical models.

Note that this approach does not contradict one of the main hypotheses of Kello and Van Orden's approach, supposing that 1/*f* noise represents the hallmark of complex systems. The originality of Torre and Delignières' approach is to suppose that 1/*f* sources could be localized in some sub-systems within the whole system. Each of these sub-systems is supposed to possess the properties of complexity, self-critical organization, metastability, which underlie the generation of 1/*f* fluctuations. West and Scafetta (2003), for example, developed the idea that Central Pattern Generators, conceived as complex neurons networks, could represent this kind of local fractal source, generating the fractal nature of gait.

Importantly, this localization hypothesis should not be understood as that of a precise localization of fractal sources, in a specified zone of the brain for example. The nature of fractal fluctuations, suggesting the cooperative interaction of multiple components acting over different time scales, rather supposes that these sub-systems generating 1/*f* noise represent independent complex networks distributed within the whole system. In other words, the mechanistic approach claims for a *statistical* localization of the fractal sources rather than for a *structural* localization in the brain or the body.

Delignières et al. (2008) introduced some "simple" modeling solutions for generating 1/*f* fluctuations. They proposed, for example, to account for fractal fluctuations in the timekeeper component of the WK model by a stochastic version of the activation threshold model (i.e., the 'shifting strategy model'). As well, they proposed to account for 1/*f* noise in the evolution of oscillator's stiffness over time by means of the 'hopping model' initially introduced by West and Scafetta (2003). The choice of these modeling solutions was motivated by previous works suggesting their theoretical, biological, or psychological plausibility (Ashkenazy, Hausdorff, Ivanov, & Stanley, 2002; Wagenmakers, Farrell, & Ratcliff 2004). However, their precise architectures are not of central interest. Alternative fractal generators could have been used and provide similar results (for an example, see Diniz et al., 2010). These modeling solutions should be essentially conceived as formal mathematical tools for injecting 1/*f* noise in precise statistical locations in the global models.

6. Conclusion

Over the past decades, scientists have been able to develop clear intuitions about the behavioral correlates of 1/*f* noise in human behavior. More healthy, stable, and coordinated behaviors seem to go with a clearer presence of 1/*f* noise. Less skilled and deficient behaviors show more random trial-to-trial fluctuations. Along with other dynamical measures, these dynamical system properties now become accepted as a sensitive metric for coordination, and as an indication for system complexity. Although the gained intuitions are based on a large number of empirical studies, there is still a considerable debate about the cognitive architecture that enables fractal dynamics to serve as a coordinative basis for behavior.

In this article, two theoretical approaches to 1/*f* noise in human behavior have been presented; interaction–dominant and domain-specific dynamics. According to the interaction–dominance

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approach, 1/*f* noise is the natural signature of a complex system in which the coordination among degrees-of-freedom emerges from the dynamical interdependency of the system's constituents. A domain-specific account at the other hand, seeks for encapsulated sources of the observed fractal dynamics, and views the cognitive system as an aggregate of multiple complex systems; encapsulated sources of 1/*f* noise may interact but the revealed intrinsic system dynamics are idiosyncratic, not interdependent.

From the former point of view, intrinsic dynamics emerge from the multiplicative interactions that constitute the entire system. From the latter point of view, intrinsic dynamics emerge from localized parts of the underlying system. Because of this theoretical distinction, these accounts for 1/*f* noise in human behavior have previously been described as opposed and incompatible (Kello et al., 2007; Torre & Wagenmakers, 2009).

In this final section, the question is raised whether these approaches could represent complementary points of view regarding the 1/*f* phenomenon. The answer to this question is twofold, however. From one stance (1), it appears to be hardly plausible for system dynamics to emerge both from the irreducible interdependency of elemental system constituents and from elemental and reducible idiosyncratic sources. At the other hand (2), it is unlikely for one theory to yield all pragmatic answers to any question concerning human cognition (Dale, Dietrich, & Chemero, 2009). Therefore, it is essential to embrace multiple positions in debates about enigmatic empirical phenomena like 1/*f* noise in human behavior.

Based upon the second part of the answer, it is inconvenient that a pluralist position cannot easily be achieved based on empirical observations. Van Orden and colleagues have shown that changes in the presence of 1/*f* noise with learning occur across domains, principally in similar ways as the coordination of bodily and physiological processes. They also showed that 1/*f* is prevalent in dozens of parallel signals simultaneously, which would require an endless number of idiosyncratic sources of 1/*f* noise to account for the general nature of these findings (also see Kello et al., 2007; Wijnants, Bosman, Hasselman, et al., 2009). Delignières and colleagues have empirically connected the presence and relative change of 1/*f* noise with well-established theories of motor control. Their research develops an elegant approach in which 1/*f* noise can be realistically inserted in statistically well-defined, local parts of cognitive models. Are these empirical results incompatible, and is it awkward to promote pluralism in these matters? It cannot be excluded before the fact that, under some circumstances, the cognitive system coordinates its internal degrees-of-freedom in more idiosyncratic ways, while in other contexts behavior may require feedback from its entire underlying system; hence, brain, body, environment, and their mutual history. However, such a postulate would definitely require further experimental and philosophical exploration.

More convenient is that an integration of both approaches is not a requisite for pluralism. In contrast, the road to pluralism is necessarily paved with meta-theoretical distinctions. The Van Orden camp conceives of the localization of interdependent dynamics much like "[a] drunk looking for lost keys under the lamppost because the light is better there" (Kello et al., 2007, p. 551). The Delignières camp finds that "[unlike nomothetic accounts] mechanistic accounts offer the advantages of specific, experimentally testable and thus falsifiable models of human behavior" (Torre & Wagenmakers, 2009, p. 314). The question of the 'complementarity' of the approaches boils down to axiomatic premises.

Axiomatic premises, whether they deal with the interdependence or the idiosyncrasy of the system constituents, are inevitable and essential in scientific research (Carello & Moreno, 2005). They clarify assumptions that stem from meta-theoretical intuitions, which have profound consequences on the practice of cognitive science. "They influence the phenomena we choose to study, the questions we ask about these phenomena, the experiments we perform, and the ways in which we interpret the results of these experiments" (Beer, 2000, p. 91). Whether one accepts the premise that 1/*f* originates from system complexity in the sense of emergent coordination or the premise that 1/*f* originates from an idiosyncratic source, such a priori assumptions need further elaboration (perhaps experimentally when testable predictions are derived, perhaps through post hoc explanations) in order to avoid circular reasoning (Van Orden, Pennington, & Stone, 2001). In our opinion, 1/*f* noise is both intriguing and telling of system performance. In order to develop a further understanding of the role of 1/*f* noise in human behavior, the phenomenon must be studied pragmatically, preferably from a variety of different perspectives; hence, from a variety of starting premises.

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