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Fractal fluctuations and complexity: Current debates and future challenges

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Abstract

Complexity is maybe one of the less understood concepts, even within the scientific community. Recent theoretical and experimental advances, however, based on the close relationship between the complexity of the system and the presence of 1/*f* fluctuations in its macroscopic behavior, have opened new domains of investigation, which consider fundamental questions as well as more applied perspectives. These approaches allow a better understanding of how essential macroscopic functions could emerge from complex interactive networks. In this review we present the current state of the theoretical debate about the origins of 1/*f* fluctuations, with a special focus on recent hypotheses that establish a direct link between complexity and fractal fluctuations, and clarify some lines of opposition, especially between idiosyncratic vs nomothetic conceptions, and global vs componential approaches. Finally, we discuss the deep questioning that this approach can generate with regard to current theories of motor control and psychological processes, and some future developments which may be evoked, especially in the domain of physical medicine and rehabilitation.

Keywords: Complexity, 1/*f* fluctuations, interaction-dominant dynamics, variability, adaptability, degeneracy, scale invariance.

I. COMPLEXITY AND COMPLICATION

Complexity is maybe one of the less properly understood concepts, even in the scientific community. Complexity is often defined only in opposition to other concepts, such as simplicity, or predictability. A complex object is just difficult to understand, to model, to predict. This obviously does not allow us to take advantage of the power of this concept.

One of the most powerful approaches to complexity was developed by the French philosopher and sociologist Edgar Morin, and perhaps his most fundamental message was the necessity to clearly distinguish between complication and complexity.[1,2] Complication refers to the idea that a situation, a system, is composed of a huge number of components. Interactions between components are not of prior importance, and knowledge about the parts is sufficient for understanding the whole. The situation or the system is complicated because of the number of components that have to be analyzed or mastered. However, one can argue that with time and application, a full understanding of the system remains possible. The system can then be decomposed for analysis, and this was the basic principle of the analytic approach proposed by Descartes in *Le discours de la méthode*: « *Diviser chacune des difficultés que j'examinerais, en autant de parcelles qu'il se pourrait, et qu'il serait requis pour les mieux résoudre* [To divide each of the difficulties I examined into as many parts as possible and as

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might be required in order to resolve them better] ».[3(p. 8)]

A complex system, in contrast, is composed of a huge number of *infinitely entangled* components. Such a system cannot be decomposed in elementary components. Each element within the system is dependent on the other elements, and each level is dependent on the other levels. Interactions between components are more important than components themselves, and the whole is greater than the sum of its parts.

Beyond this essential distinction, Morin developed a powerful framework for *complex thought*, a new epistemological posture for conceiving complex phenomena. Complex thought takes into account essential properties of complex systems, especially the principle of emergence: the behavior of a complex system is a macroscopic property that is not dictated by a specific component within the system, but emerges from a wide set of interactions between its many components and levels, and cannot be reduced to any deterministic model. The dynamics of a complex system are essentially unpredictable, at least in the long term. Complex thought must abandon traditional assumptions such as the linearity of the relationship between causes and effects, or the possibility to isolate a given system of its environment, or a sub-system of the broader system to which it belongs.[1,2]

II. THE PROGRESSIVE CONSIDERATION OF COMPLEXITY

The analytic conception of Descartes had a strong influence on the development of modern sciences and especially the positivist and experimental approaches at the end of the 19th century, with the idea that scientific objects should be divided and subdivided to gain intelligibility, and studied through highly targeted and compartmentalized experiments. This analytic conception also had a strong influence on the classical representation and modeling of scientific objects. The central nervous system, for example, was for a long time considered as a set of separate entities (stages, processes, structures), linked by simple causal and linear relationships. The illusion that complex systems could be mastered, with sufficient application and calculation power, legitimized the computational perspective to motor control, supposing that a central instance, using symbolic representations and calculations, might be able to fully control the complexity of the sensorimotor system.[4]

One of the most powerful objections towards this computational conception was formulated by Bernstein, through the degrees of freedom problem: "*it is clear that the basic difficulties for co-ordination consist precisely in the extreme abundance of degrees of freedom, with which the centre is not at first in a position to deal*".[5(p. 216)]

The development of the dynamical systems approach to coordination represented a marked evolution.[6,7] The main claim of this approach was that stable patterns of coordination emerge from the interplay of the multiple components that compose the system. The multiple degrees of freedom self-organize into coherent units, and complex systems can give rise to rather simple macroscopic behaviors, governed by very general principles that explain the relative stability of coordination modes, the adoption of preferred coordination modes according to situations, and the transition between modes under specific constraints.[8]

The dynamical systems approach then sought to evidence the suitable collective variables that captured this macroscopic behavior, and to establish equations of motion that accounted for its dynamics. These equations of motion generally included a determinism part, supposed to represent the macroscopic properties that emerged from the interplay of constraints within the system, and a stochastic part arising from the background noise common to all complex systems, and essential for the dynamics of the collective variable.[9] Note that in this approach complexity is not directly considered, but represents a prerequisite for emergence.

Currently, a number of authors are trying to more directly assess complexity and its influence on behavior. In this approach, detailed below, complexity is supposed to be directly related to the properties of stability and adaptability that characterized healthy, efficient and perennial systems. Complexity is conceived as an essential property, which could be lost with aging and disease, yielding maladaptation and lack of flexibility, and regained with rehabilitation and learning.

III. FRACTAL FLUCTUATIONS AND SYSTEM COMPLEXITY

Complexity is closely linked to organization, or coordination. Complexity requires a certain level of coordination between the multiple components that compose the system. When components are independent, the system is unable to exhibit any kind of coordinated activity, and its behavior remains erratic and unpredictable. This corresponds to a state of complete disorder. In contrast, when organization is too strict, or coordination too rigid between the components, the system tends to behave in a very predictable and deterministic way. Complexity, in the present context, can be defined as a compromise between order and disorder. As eloquently expressed by Atlan, complex systems evolve between the crystal and the smoke (*entre le cristal et la fumée*).[10] In this metaphor, the crystal corresponds to rigid order, and the smoke to total disorder. In other words, complexity is a compromise between simplicity and complication.

On the basis of the previous definitions, one could conceive that it is difficult to characterize a given system as complex or not. A system can be more or less complex; moreover, complexity can be lost in two opposite ways, either towards order, or towards disorder.

This definition of complexity as a compromise between order and disorder found an interesting correspondence in time series modeling, especially through the monofractal model. This model was initially introduced by Mandelbrot and van Ness, as an extension of two well-known stochastic processes: white noise (series of independent, uncorrelated data points), and Brownian motion (series with independent, uncorrelated increments).[11] White noise series represent an instance of complete disorder: the current value cannot be predicted from the past history of the process. In contrast, in Brownian motion, which is the accumulation of successive, uncorrelated displacements, the current value keeps the memory of preceding values, and its evolution remains roughly predictable.

Mandelbrot and van Ness proposed a continuum of processes, between white noise and Brownian motion.[11] This continuum is characterized by a scaling law, in the frequency domain, relating power to frequency:

$$S(f) \propto f^{-\beta} \quad (1)$$

This scaling law can be revealed by the log-log plot of the power spectrum, which is characterized by a linear shape of slope $-\beta$.

We present in Figure 1 some example series from this continuum. The top row presents representative time series (512 data points), for β exponents of 0, 0.5, 1, 1.5, and 2, the middle row the corresponding autocorrelation functions (up to lag 30), and the bottom row the log-log power spectra.

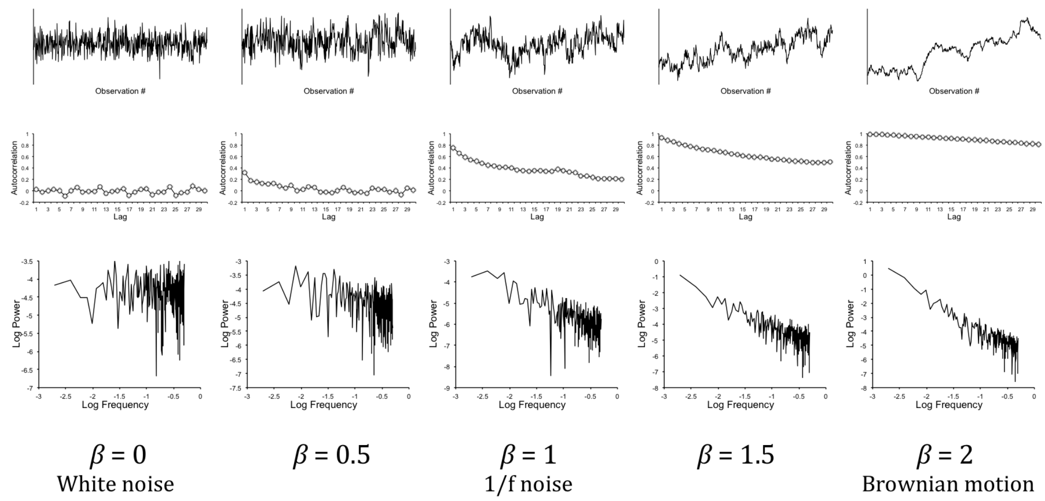

Figure 1: Fractional Gaussian noises and fractional Brownian motions. Top row: example time series, middle row: auto-correlation functions (up to lag 30), bottom row: log-log power spectra. The first two columns refer to fractional Gaussian noises ($\beta = 0$ *and* $\beta = 0.5$) and the last two columns to fractional Brownian motions ($\beta = 1.5$ and $\beta = 2$). The central column corresponds to exact $1/f$ noise ($\beta = 1$), which represents the frontier between the two families. $1/f$ fluctuations are usually considered to occur in the range [$\beta = 0.5$; $\beta = 1.5$]. From white noise to Brownian motion, correlations became more and more persistent over time, and the slope of the power spectrum steeper and steeper.

The central column, for $\beta = 1$, represents the center of the continuum. In this case, power is inversely proportional to frequency. This property led to the popular denomination of "$1/f$ noise". $1/f$ noise appears at equal distance between white noise and Brownian motion. Usually one considered that $1/f$ fluctuations occur over a wider range, from $\beta = 0.5$ to $\beta = 1.5$.[12]

A number of other methods have been proposed for estimating the scaling exponent that governs fluctuations in experimental time series. The most frequently used method is Detrended Fluctuation Analysis (DFA),[13] which provides an exponent α, linearly related to the spectral exponent β ($\alpha = (\beta + 1) / 2$). The range of $1/f$ fluctuations is then characterized by α exponents ranging from 0.75 to 1.25. Note that the application of these methods, for providing reliable results, requires some methodological precautions, regarding the nature, the length or the stationarity of the series.[14-16]

$1/f$ noise has been discovered in the time series collected in a number of situations and covering a diversity of natural and physical systems, including for example the series of discharges of the Nile River,[17] the series of magnitudes of earthquakes,[18] the evolution of traffic in Ethernet networks,[19] or the dynamics of self-esteem over time.[20] In the domain of human movement, $1/f$ noise has been evidenced in serial reaction time,[21,22] in finger tapping,[23,24] in stride duration during walking,[25] and in relative phase in a bimanual coordination task.[26]

IV. NEW CONCEPTIONS ABOUT HEALTH, STABILITY AND ADAPTABILITY

This link between complexity and fractality motivated a new approach to health and disease. $1/f$ fluctuations have been discovered in a number of experiments analyzing the behavior of young and healthy systems. In contrast, the analysis of series produced by deficient systems (i.e., patients suffering from diverse pathologies, elderly) revealed a clear alteration of fractal properties, often towards disorder, and sometimes towards order. Hausdorff and colleagues, for example, showed that the series of stride intervals during walking typically presented $1/f$ fluctuations in young and healthy adults. In contrast, stride series appeared less correlated in elderly, and in Huntington's or Parkinson's patients.[27,28] Goldberger and colleagues reported similar results in their analysis of cardiac inter-beat interval series: the series produced by healthy patients typically exhibited a correlated variability, close to $1/f$ noise. In patients suffering from arrhythmia, the series were less correlated, closer to white noise. Conversely, in patients with severe heart failure, series appeared less variable, and highly predictable.[29]

These results supported the idea of a direct link between complexity and health, and the theory of the loss of complexity with aging and disease.[30,31] Healthy systems are supposed to present a rich set of interactions between their many components and levels. In contrast, aging or disease are characterized either by a loss of interactions, or by the dominance of few residual components.

Complexity provides systems with essential capacities of adaptation and flexibility. The architecture of complex systems is characterized by redundancy and degeneracy (see below), allowing them to protect against the possible deficiency of a given component or sub-system, to maintain stability despite external perturbations, and permitting the emergence of innovative solutions when faced with a novel problem.[29]

V. THE ORIGINS OF FRACTAL FLUCTUATIONS

A. Idiosyncratic models and nomothetic hypotheses

$1/f$ noise has been discovered in a number of systems and situations, and several authors have proposed specific models for explaining the emergence of such fluctuations in the system they studied. For example, Ward suggested that $1/f$ fluctuations in psychological processes could arise from the combination of conscious, preconscious and unconscious processes.[32] The fluctuations of performance in tapping tasks have been accounted for by models based on shifts in attention,[12,33] or delayed feedback.[34]

These models and explanations have been described as *idiosyncratic*, because they suggest that $1/f$ fluctuations could arise from very different and specific mechanisms, according to each system or situation. In the preceding examples, each model invokes specific processes (i.e., levels of processing, attention, or delayed feedback) for explaining the emergence of fractal fluctuations. One could conceive that this kind of specific explanation cannot be transposed from one system to the other.

Kello and colleagues criticized these conceptions, arguing that « 1/*f* scaling is too pervasive to be idiosyncratic ».[35(p.551)] According to these authors, the ubiquity of 1/*f* fluctuations in natural and living systems suggests the presence of generic principles, common to all complex systems.

B. Abstract models

Several authors have proposed models that could be considered generic solutions for the production of 1/*f* fluctuations. As a matter of fact, a number of mathematical algorithms seem able to generate genuine fractal fluctuations, or, at least, fluctuations that mimic 1/*f* noise.[12] For example, the aggregation of auto-regressive short-range processes leads to the emergence of such long-term structured variability.[12,36,37] Pressing suggested that the aggregation of moving-average short-term processes could yield similar results.[38] Hausdorf and Peng showed that multiscaled randomness would give rise to such behavior under some conditions.[39] Diniz, Barreiros and Crato used the very generic auto-regressive-fractionally-integrated-moving-average (ARFIMA) model for accounting for the timekeeper involved in rhythmic tapping.[40] Kaulakys developed a very simple, analytically solvable model suggesting that the appearance of 1/*f* noise in a series of pulses could be due to the fact that the successive recurrence times obey an autoregressive process with very small damping.[41]

These models are generally able to generate series with properties close to those of 1/*f* noise, but they never allow to draw any causal link between the complexity of the system under study and the presence of 1/*f* fluctuations in the series produced. Some of these models suggest the presence of multiple levels in the system, acting on different time scales.[36,38,39] However, these models are based on a simple additive combination of

these levels, and do not suggest any interaction between them, a central assumption for complex systems.

C. Self-organized criticality

The first hypothesis that clearly explained 1/*f* fluctuations by a generic property of complex systems was proposed by Van Orden, Kello and colleagues.[22,35,42] The central concept of their hypothesis is criticality, a state where systems are in delicate balance between multiple behavioral solutions. Near critical states, systems are metastable, and a small and local perturbation can result in a global change in the system's behavior. Critical states make new options for behavior available, and then provide the system with adaptability and flexibility to cope with environmental constraints.

Living systems tend to spontaneously remain near critical states.[43] This so-called *self-organized criticality* is possible when multiple interactions occur between multiple levels and individual components, a condition that directly refers to the system's complexity. Self-organized criticality supposes that the dynamics of the system are dominated by interactions, and not by some dominant components within the system.[44] Finally, these interaction-dominant dynamics are known to produce statically self-similar, fractal fluctuations.[43]

Self-organized criticality appears to be a good candidate for a generic explanation of the ubiquity of 1/*f* fluctuations in natural and living systems. Obviously, this kind of hypothesis has an important impact at the theoretical level, as it severely challenges traditional points of view, especially in psychological theories.[22,45]

D. Multiplicative cascade dynamics

Cascade dynamics have recently been proposed as an alternative to the self-organized criticality hypothesis.[46,47] Similar to self-organized criticality, cascade dynamics models incorporate interactions across multiple time scales. Multiplicative cascade dynamics were especially developed for modeling energy transfer across scales in complex systems.[48] Energy transfer works from the coarsest to the finest scales, and variation on the finer scales is given by multiplying the variation on the coarser scales with random multipliers.

As previously stated, self-organized criticality produces monofractal fluctuations in behavior. In contrast, multiplicative cascade dynamics produce more complex patterns, called *multifractal* fluctuations. In the monofractal model, scale invariance is numerically defined by a single exponent. In multifractal series, the behavior around any point is described by a *local* scaling exponent, which varies over time, and then scaling invariance is defined by a spectrum of scaling exponents, rather than by a single average value.

Multifractals are able to explicitly define the width and shape of the spectrum of scaling exponents using the width and shape of the distribution of interaction multipliers, and there is a formal relationship between the local variation of scaling exponents and the multiplicative interactions between temporal scales. As such, seeking the presence of multifractal fluctuations in the behavior of systems could allow us to test the cascade dynamics hypothesis against that of self-organized criticality. Multifractals have been evidenced, for example, in human gait, simple and choice reaction time, word naming, and interval estimation.[46,47,49,50]

E. Degeneracy

A third promising hypothesis refers to degeneracy, another important property of complex systems. Degeneracy has been proposed as an essential property, explaining the relationships between complexity, robustness and evolvability in biological systems.[51] Degeneracy refers to a partial overlap in the functions of the multiple components within the system. In degenerate systems, distinct components can perform similar functions under certain conditions, but can also assume distinct roles in others conditions. Degeneracy is often preferred to the classical concept of redundancy for analyzing the design principles of biological systems, and fits closely to the concept of complex systems characterized by a multiscaled and hierarchical structure.[52-54]

The behavior of a complex, degenerate and multiscaled system, in a given situation, cannot be considered as resulting from the performance of a fixed and immutable network. Throughout repetitions, diverse sets of components can be recruited for achieving a given function.[51] From this point of view, variability does not result from perturbations within the system, but expresses its inherent complexity. These two lines of reasoning suggest a possible link between degeneracy and $1/f$ fluctuations.

West and Scafetta developed a model that mimics the behavior of a degenerate system.[55] This so-called “hopping” model is composed of a set of partially overlapping networks. This set of networks is modeled through a Markov chain produced by a one-order autoregressive process. The autoregressive parameter determines the degree of overlap between neighbor networks. Neighbors produce highly correlated outcomes, because they present a high degree of overlap, and correlation tends to decrease as distance between networks increases. The successive outcomes are produced by a series of random jumps over the available networks. Correlations between successive outcomes are determined by the mean amplitude of these jumps. Delignières, Torre and Lemoine showed that with appropriate settings for the two free parameters (i.e., the degree of overlapping between neighbors, and the mean amplitude of jumps), the hopping model produced $1/f$ fluctuations.[33]

In the initial formulation of the model the Markov chain was supposed to be of infinite length. However, one can suppose that degeneracy depends on the number of available networks in the model. Delignières, Marmelat and Torre analyzed the relationships between the length of the chain and the presence of $1/f$ fluctuations in the produced series. They showed that correlations increased with a length of the chain, and that at least 60 networks were required for reaching the $1/f$ range (Figure 2, left). A chain of at least 200 networks were necessary for obtaining a correlation pattern close to $1/f$ noise (Figure 2, right).[56] These results are obviously dependent on the values assigned to the parameters in the model, but they suggest that a sufficient level of degeneracy is necessary for the production of fractal fluctuations.

Note that Scafetta, Griffin and West performed multifractal analyses on their model and showed that the produced series presented multifractal properties.[49] This suggests that degeneracy could be an alternative candidate for explaining the emergence of this kind of complex fluctuations.

VI. GLOBAL VS COMPONENTIAL HYPOTHESES

The debate about the origin of $1/f$ fluctuation has been structured around two emblematic oppositions, idiosyncratic vs nomothetic explanations, and component-dominant vs interaction-dominant conceptions. These two oppositions were generally considered equivalent. However, one could argue that they refer to two distinct levels of analysis.

The first opposition, as presented in the preceding paragraphs, concerns the question of the specific or generic character of the processes that generate $1/f$ fluctuations. The idiosyncratic perspective was initially supported by the emergence of several models that were able to “mimic” $1/f$ fluctuations and that presented some theoretical plausibility, with regards to previous theories.

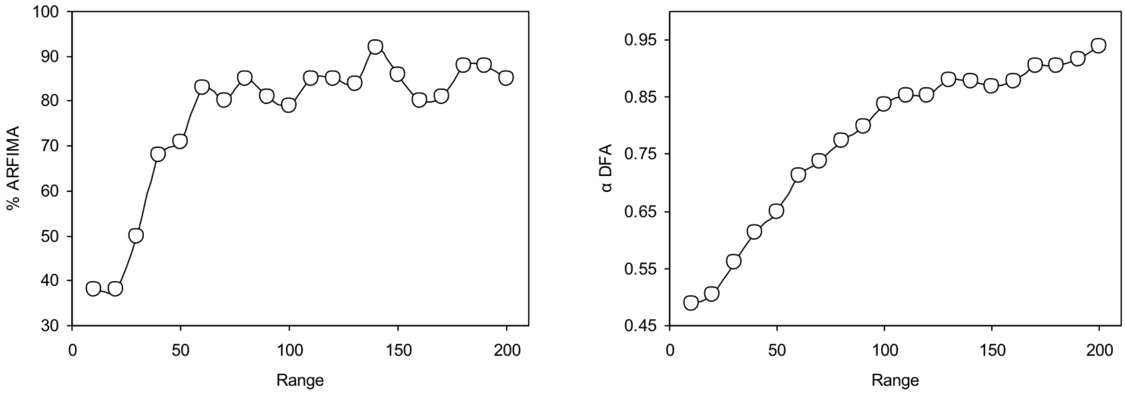

Figure 2: Degeneracy and 1/*f* fluctuations. These graphs represent the results of fractal analyses on series generated by the "hopping model".[55] One hundred series were generated for 20 range values (i.e., the length of the Markov chain), from 10 to 200, by steps of 10. The left graph reports the results of ARFIMA modeling, a method that allows testing for the presence of genuine 1/*f* fluctuations in the series. A percentage of about 90% of series best modeled by ARMIFA models is expected for accepting the 1/*f* hypothesis.[57] The present results show that this threshold is approached from a range value of about 60. The right graph shows the mean α exponent (computed by the Detrended Fluctuation Analysis), as a function of the range value. α exponents representative of 1/*f* noise (α ≐ 1) are obtained for the highest ranges.

More recently, however, the nomothetic point of view became clearly dominant, and most authors seem to now agree with the proposition that fractal fluctuations should arise from some generic principles, related to system complexity.

The second opposition refers to a distinct question: are 1/*f* fluctuations a property of the global system, or do they arise from a specific component, a *localized source* within the system? Van Orden et al. adopted a clear statement favoring the first conception: ''Pink [1/*f*] noise cannot be encapsulated; it is not the product of a particular component of the mind or body. It appears to illustrate something general about human behavior''.[22(p.345)] From this point of view, 1/*f* fluctuations emerge from interaction-dominant dynamics, and components per se do not play any role in this process.

The componential perspective, however, has received some strong supports. The first one was provided by experiments which evidenced the presence of 1/*f* fluctuations in the series of time intervals produced in finger tapping.[23,33,58] The results showed that if 1/*f* fluctuations dominated the behavior of the series on the long term, the high-frequency region was marked by the presence of short-term, anti-persistent correlations (see Figure 3).

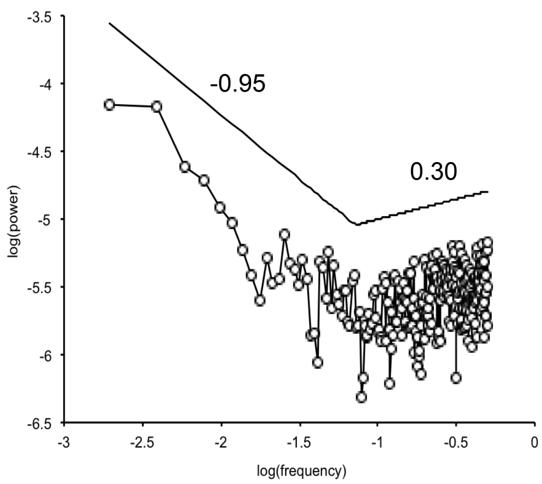

Figure 3: Averaged power spectra (in log-log coordinates) of time interval series produced in self-paced finger tapping. The negative slope in the low-frequency region reveals the presence of 1/*f* fluctuations in the long term. The positive slope in high frequencies suggests the presence of negative, short-term dependence in the series.[58]

Gilden et al. interpreted these results on the basis of the well-known model proposed by Wing and Kristofferson.[59] This model is composed of two components: a cognitive timekeeper that generates series of time intervals C_i, and a motor implementation process in charge of the execution of the tap at the end of each interval. The motor component is supposed to present a delay M_i. According to this model, the produced inter-tap interval is given by the period provided by the timekeeper, plus the difference between the motor delays that characterize the two taps that limit the interval:

$$I_i = C_i + M_i - M_{i-1} \qquad (2)$$

In the initial formulation of the model, C_i and M_i were both considered as uncorrelated white noise processes. The differenced white noise term in the model ($M_i - M_{i-1}$) is likely to produce a negative dependence in the short term, due to the presence of the same M_i term, but of opposite signs, in successive inter-tap intervals. Arguing that this differenced white noise term should be responsible for the high-frequency anti-persistent behavior in the series, Gilden et al. concluded that $1/f$ fluctuations should be produced by the lasting term in the model, C_i , i.e. the internal timekeeper.[23]

This kind of model clearly suggests that $1/f$ fluctuations could originate in localized sources in the system, and combine with other sources of variability arising from distinct components. A number of different models have been proposed, in the same vein, for accounting for the temporal structure of series collected in various situations.

Torre and Delignières, for example, proposed a model for accounting for the temporal structure of time interval series and asynchrony series observed when participants have to tap in synchrony with a regular metronome.[60] In these experimental conditions, the time interval series are characterized by anti-persistent correlations (revealed by a linear positive slope in the log-log power spectrum, see Figure 4, left graph), and the series of asynchronies by $1/f$-like fluctuations (see Figure 4, right graph).

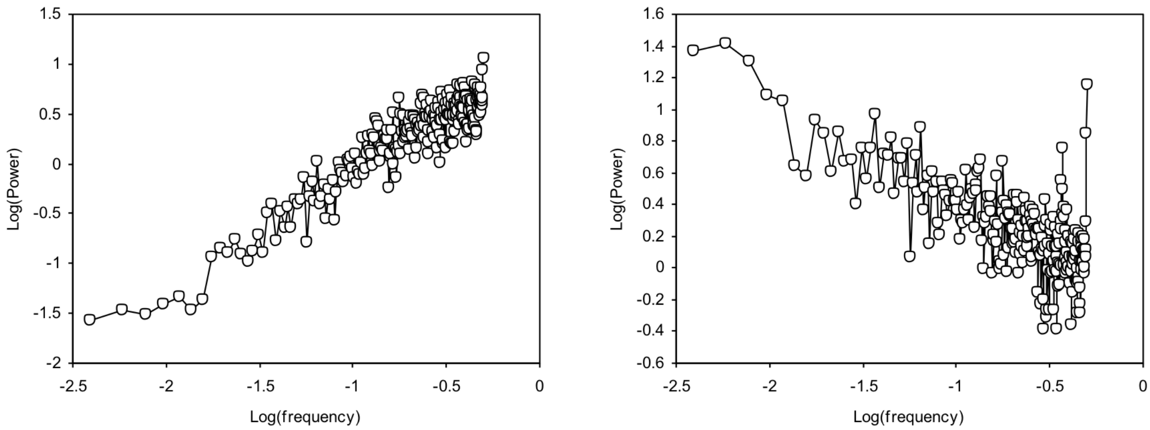

Figure 4: Log-log power spectra of series of inter-tap intervals (left) and asynchronies (right), in synchronization finger tapping.[60] The positive slope, for inter-tap interval series, reveals the presence of negative serial correlations between successive values. The negative slope for asynchrony series is indicative of the presence of $1/f$ fluctuations.

The authors started with the linear phase correction model proposed by Vorberg and Wing:[61] In synchronization tapping, each inter-tap interval (I_i) corresponds to the difference between its previous and next asynchronies (A_{i-1} and A_i), plus the period (τ) imposed by the metronome:

$$I_i = A_i - A_{i-1} + \tau. \qquad (3)$$

The main assumption of the model is that the preceding asynchrony is taken into account by a linear phase correction. The interval produced by the timekeeper is corrected by a fraction of the preceding asynchrony:

$$C^*_i = C_i - \alpha A_{i-1}. \qquad (4)$$

As proposed by the Wing-Kristofferson model (Eq. 2), the produced interval results from the combination of this corrected cognitive interval and the two successive motor delays:

$$I_i = C^*_i + (M_i - M_{i-1}). \qquad (5)$$

Combining Eq. 3, 4, and 5 leads to the following expression for current asynchrony:

$$A_i = (1 - \alpha)A_{i\text{-}1} + C_i + (M_i - M_{i\text{-}1}) - \tau. \qquad (6)$$

Torre and Delignières proposed to enrich the Vorberg-Wing model by providing C_i with fractal properties, and showed that this "1/*f*-linear phase correction model" was able to adequately reproduce the complex correlation structures of period and asynchrony series.[60] It is most important to note that the Wing-Kristofferson model (Eq. 2) and the Vorberg-Wing model (Eq. 6) are able to account for the complex patterns of serial correlations in continuation and synchronization tapping, respectively, from the moment that a single component in both models is provided with fractal properties.[60]

Some other studies have proposed similar models for accounting for interval and/or asynchrony series, for example in forearm oscillations,[33] in forearm oscillations performed in synchrony with a periodic metronome,[62] and in self-paced and metronomic walking.[55,63] In each of these models, fractal fluctuations arise from specific "sources" within the system, and combine with other sources (random fluctuations, corrective or coupling processes, etc.).

These models have been considered emblematic of the so-called component-dominant approach, and were criticized by Kello et al.[35] It is important to note, however, that in this approach the "1/*f* source" was never conceived as a local, simple component, but, rather, as a complex (sub)system.[64] For example the "timekeeper" is not considered as a specific and localized structure, somewhere in the brain, but rather as a complex, distributed network, characterized by multilevel, interaction-dominant dynamics.[23] The concept of a 1/*f* source doesn't refer to a *structural localization* in the system, but to a *statistical localization* in a formal model.[64]

The main contribution of Gilden, Delignières and their colleagues was to support the idea of a possible combination of 1/*f* fluctuations and short-term correlated processes in the series produced by the system. Those who oppose this idea have primarily criticized the principle of localization, but the most important concept remains that of combination, that allowed for explanation of the diverse alterations of the correlation structure observed in certain experimental conditions (e.g., synchronization with a metronome).

Finally, it seems important to keep in mind that when we analyze the series of performance collected in a given experimental situation, we don't get information about the "global" system (i.e., the organism), but rather about the sub-system of processes and functions that is involved in the designed situation for producing the observed outcome. For example Kello et al. analyzed separately, in a reaction time experiment, the series of reaction time and the series of key-press duration.[35] They found in both cases 1/*f* fluctuations, but there was no significant correlation between the two samples of scaling exponents. This suggests that both outcomes were produced by two independent, complex sub-systems.

VII. NEW INSIGHTS ABOUT OLD THEORIES

This approach of complexity has important implications in movement sciences. The most fundamental one entails a complete reconsideration of the meaning and role of variability. For a long time variability was considered either an error measurement, or a trace of residual background noise. Variability was then often excluded before analysis, deleted by averaging or filtering, in order to focus on means, considered as the "true" values and the best descriptors of the system.[65]

The present approach considers variability, in contrast, as the expression of the intrinsic dynamics of the system. Variability is not considered insignificant, but becomes a central focus of interest, providing essential information about the system, its functioning, its organization, and its state.

Several basic concepts have received new attention when analyzed from this point of view, for example effort,[66,67] learning,[68] or visual search.[69] These approaches to complexity and variability could have a strong impact on the core of psychological theories.[22,45] The potential renewal of conceptions is also evident in biology, for example with the questioning of homeostasis models.[70]

Some well-established models failed to account for the presence of fractal fluctuations in outcome series. Torre and Delignières,[71] for example, showed that the Multiple Timer model[72] was unable to account for the presence of 1/*f* fluctuations in bimanual tapping. The authors also showed that the well-known Haken-Kelso-Bunz model[73] was able to simulate stable in-phase coordination, when enriched by 1/*f* fluctuations. However this model was unable to sustain anti-phase coordination, even at low frequencies, in contradiction with experimental observations.

One of the most promising issues for future research and practical application focuses on the relationships between an organism and its environment, both considered as complex, fractal systems. We evoke this thematic in the following section.

VIII. WITH WHAT KIND OF WORLD DO WE ADAPT?

In a first step, the focus has been put on the complexity of the organism. However, another level of complexity seems particularly interesting for current and future research: the complexity of the environment to which individuals have to adapt. The situations we are facing in real life, the problems we have to cope with, are obviously highly complex: natural objects, landscapes, possess (geometric) fractal properties,[74] the movements of our partners present $1/f$ fluctuations,[75] etc. For a long time, adaptation to environment has been studied in standardized and simplified situations, and one could question whether analyzing the adaptation to such simple environments can provide relevant information about adaptation to real-life situations.

An example of this issue is offered by research on synchronization. For a long time, synchronization has been studied in laboratory experiments where participants had to synchronize their movements (e.g., finger tapping) with the beats of a regular metronome. A vast and rich literature has been produced on this topic, and a variety of models have been proposed and tested.[76]

In real life, however, synchronization with regular rhythms is more the exception than the rule. When we walk with another person, we spontaneously synchronize our gait with that of our companion, which is fractal in nature. Musicians synchronize with the fractal rhythms produced by their drummer; dancers synchronize with the fractal movements of their partner. Does synchronization with a regular metronome represent a relevant model for the study of real-life synchronization?

In a recent pilot study, we analyzed the series of stride intervals produced during walking on a treadmill, in synchronization with (1) a regular metronome and (2) with a metronome presenting fractal fluctuations around a mean value. The fluctuations of the fractal metronome were close to $1/f$ noise (β = 1), and the coefficient of variation of the series of inter-beat intervals was of about 2%, with respect to a mean value of 1100 ms. We present in Figure 5 (top panel) an example series of inter-beat intervals for the fractal metronome.

The two bottom graphs represent the average log-log power spectra of stride interval series, for regular metronomes (left) and fractal metronomes (right). The spectrum obtained for the condition of synchronization with a regular metronome is consistent with that reported in a previous experiment,[63] with anti-persistent behavior in a large band of low frequencies, and persistent behavior in high frequencies. This pattern of fluctuation has been adequately modeled by a forced van der Pol oscillator obeying the following equation:[55,63]

$$\ddot{x} = \mu \dot{x} - \lambda \dot{x} x^2 - \omega_i^2 x + A\sin(\omega_0 t), \qquad (7)$$

where x represents position and the dot notation differentiation with respect to time. In this equation ω_i is the virtual inner frequency of the oscillator during the i^{th} cycle. ω_i is supposed to vary discretely from one cycle to the next, and the ω_i series possesses fractal properties. The forcing function represents the external driving of the metronome, ω_0 representing frequency and A the strength of forcing.

For negligible values of forcing strength, this model produces series of stride durations exhibiting $1/f$ fluctuations, as observed in self-paced walking. The increase of A induces a gradual alteration of the correlation structure, and the experimental pattern of correlation illustrated in Figure 5 (left graph) is reproduced for $A = 10$.[63] This result is a typical example of combination of a $1/f$ source with a short-term coupling process in a complex system.

As can be seen, synchronization with a fractal metronome yielded a completely distinct correlation structure, with $1/f$-like behavior over the entire spectrum (Figure 5, right graph).

Stephen, Stepp, Dixon and Turvey obtained a similar result, in an experiment where participants had to tap their index finger on a table in synchrony with a chaotic metronome.[77] The authors evidenced a close correlation between the scaling exponents of the series of inter-tap intervals produced by participants and those of the series of inter-beat intervals of their respective metronomes.

The authors interpreted these results in terms of *strong anticipation*. Dubois distinguished two forms of anticipation:[78] anticipation can be performed on the basis of a prediction of future events, based on an internalized model of the environment. This form of anticipation, referred to as *weak anticipation*, supposes that the regularities of the stimulus are symbolically coded, and that this internal model can support an effective prediction of future events. Most cognitive theories on anticipation refer to this computational perspective. The model of synchronization presented in Eq. 6 is typical of this approach: based on the knowledge of the regularities of the metronome (i.e. the regular period τ), one can locally correct errors for anticipating the next signals.

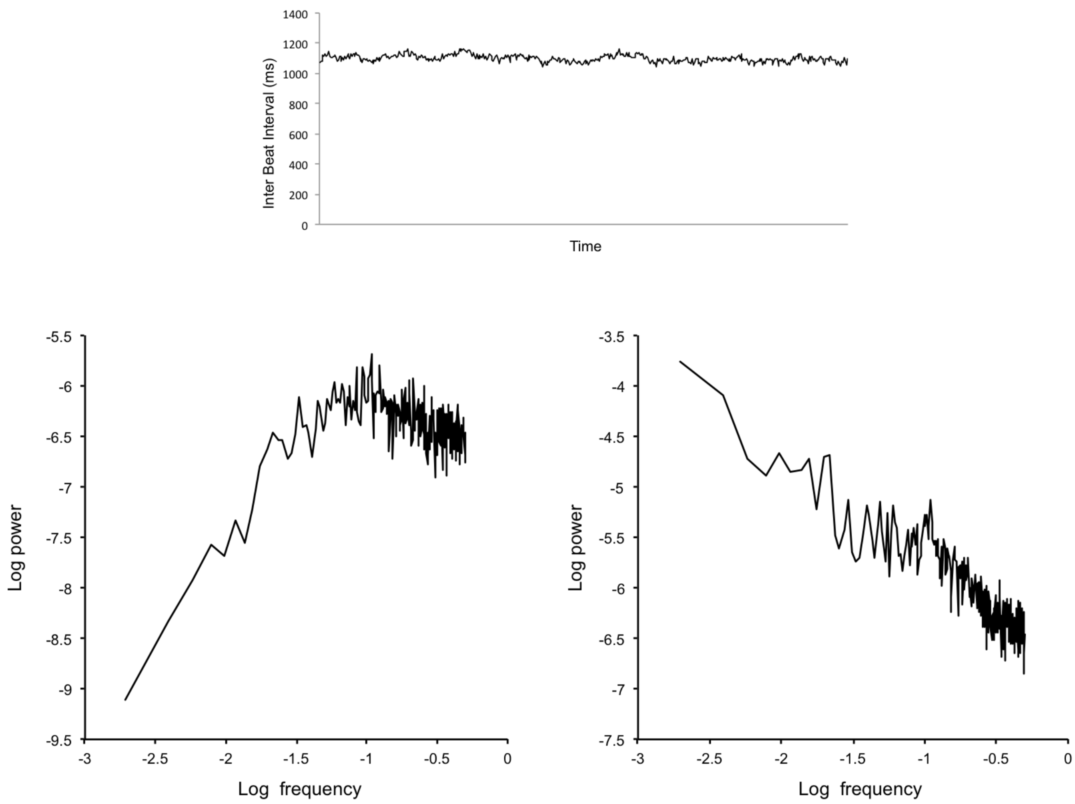

Figure 5: Top graph: an example series of inter-beat intervals provided by the fractal metronome. Bottom graphs, left: average log-log power spectrum for series of stride intervals during synchronization with a regular metronome; right: average log-log power spectrum for series of stride intervals during synchronization with a fractal metronome.

In contrast, strong anticipation does not require any internal model or representation: strong anticipation is based on the embedding of the organism within its environment, which creates a new, organism-environment system, possessing lawful regularities that allow the emergence of anticipation.[79,80] The presence of $1/f$ fluctuations in the environment is essential for allowing strong anticipation: the long-term, multiscale persistence of signals represents a resource that the organism can exploit by matching its own statistical structure to that of the environment with which it has to adapt. In Stephen et al.'s experiment, local predictions were impossible, due to the chaotic fluctuations of the metronome.[77] However, anticipation remained partially possible by an adaptation of the global temporal structure of responses to that of the stimulus.

We reported other evidence for strong anticipation in inter-personal coordination.[75] In this experiment participants worked in dyads: each participant performed oscillations with a hand-held pendulum, and had to synchronize the oscillations of his/her pendulum with those of his/her partner. Results revealed a close relationship between the scaling exponents of the series of periods produced by the two partners in each dyad (Figure 6).

This result could be simply and trivially explained by the fact that the oscillations of one pendulum closely mimic those of the other. However, a close examination of windowed cross-correlations showed that the series of periods produced by the two participants of a dyad were rarely correlated on local scales. The close matching of the scaling exponents is not the result of local mutual adaptations, but seems related to a more global process of multiscaled coupling between the two partners. These results are conceptually close to the *Complexity Matching* hypothesis, which suggests that information exchange between systems is easier when systems present equivalent levels of complexity.[81]

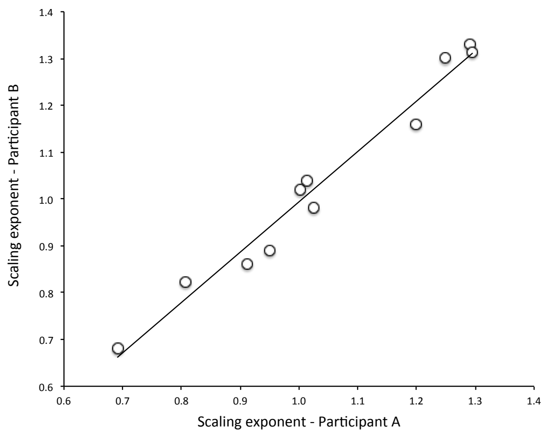

Figure 6: Relationship between the scaling exponents of the series of periods produced by the two participants of each dyad, in an interpersonal coordination task. Scaling exponents were estimated through Detrended Fluctuation Analysis.[80]

In summary, these experiments suggest that synchronization, or adaptation with complex environments is based on processes that are completely different from those involved in the adaptation to simple and predictable signals. This supports the reconsideration if a vast amount of literature, and could have important implications in the design of learning or rehabilitation protocols. One could think that complex environments might allow more useful adaptations and better transfer to real-life situations.

XI. CONCLUSION

$1/f$ fluctuations were for a long time considered at best an intriguing phenomenon, at worst just a mathematical curiosity. Their ubiquity over a wide range of physical or living systems has sometimes been advocated for minimizing their scientific interest: if fractal fluctuations are everywhere, are they useful for distinguishing between situations, individuals, or experimental conditions? It could be more interesting to delete these superfluous perturbations, and to focus analyses on average values, or deterministic trends.

The recent theoretical and experimental advances, and especially the direct relationship that is envisaged between complexity and $1/f$ fluctuations, open new perspectives for the understanding of the functioning of complex systems, their architecture, and the mechanisms that provide them with the essential properties of stability, adaptability and flexibility. This approach allows for fundamental reconsideration of classical assumptions and theories, and new perspectives about essential functions of the brain and the central nervous system, such as perception, decision-making, and the formation of new knowledge and representations. In the domain of movement science, this approach provides new insights into the control of the many degrees of freedom of the human body, and the capability to maintain relatively stable performance over time.

Finally, complexity and $1/f$ fluctuations offer promising perspectives in more applied domains, especially in physical medicine, for example for diagnosis and prognosis, or for the design of efficient protocols of rehabilitation.

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